HOLIDAY HOMEWORK 假期作业

AS-Level Further Mathematics

Exercise sheets and past-paper practice, topic by topic.

每个单元：练习卷 + 历年真题。

For class or self-study • no answer keys included.

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1 Further Pure Mathematics 1 – Part 1

Name: ________ Class: ______ Date: ________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
roots	根			
coefficients	系数			
matrix	矩阵			
determinant	行列式			
inverse matrix	逆矩阵			
identity matrix	单位矩阵			

Recall

In AS Maths you solved quadratics and worked with their coefficients. Bring this with you:

- a quadratic $ax^2 + bx + c$ has coefficients a, b, c;
- you solved $ax^2 + bx + c = 0$ by factorising.

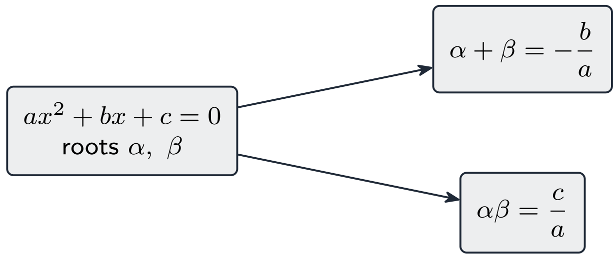
Further Pure 1 links a polynomial's **roots** to its coefficients, and adds a new object — the **matrix**.

New ideas

1 Roots of a quadratic

For $ax^2 + bx + c = 0$ with **roots** 根 α and β, the **coefficients** 系数 give the sum and product directly:

$$\alpha + \beta = -\frac{b}{a}, \qquad \alpha\beta = \frac{c}{a}.$$



Worked example. For $x^2 - 5x + 6 = 0$: $\alpha + \beta = 5$ and $\alpha\beta = 6$ (the roots are 2 and 3).

Micro-check. For $x^2 - 7x + 10 = 0$, write down $\alpha + \beta$ and $\alpha\beta$. [2]

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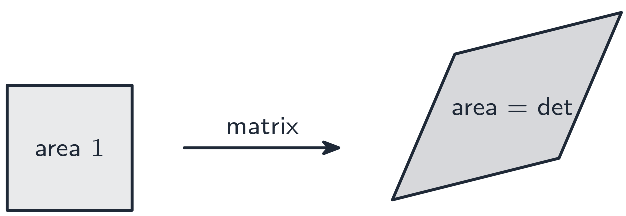
2 Matrices

A **matrix** 矩阵 is a block of numbers. For a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the **determinant** 行列式 is $ad - bc$. When $ad - bc \neq 0$, the **inverse matrix** 逆矩阵 is

$$\frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

The **identity matrix** 单位矩阵 I leaves any matrix unchanged. A 2×2 matrix maps the unit square to a parallelogram whose area is the determinant.



Worked example. The determinant of $\begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$ is $3 \times 4 - 1 \times 2 = 10$.

Micro-check. Find the determinant of $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$. [1]

Watch out: the sum of the roots is $-\frac{b}{a}$ —keep the minus sign. For the inverse, swap the diagonal entries a and d, negate b and c, then divide by the determinant $ad - bc$ (which must not be 0).

Practice

1.1 For $x^2 - 8x + 15 = 0$ with roots α, β, write down (a) $\alpha + \beta$ and (b) $\alpha\beta$. [2]

1.2 A quadratic $2x^2 + 6x + 4 = 0$ has roots α, β. Find $\alpha + \beta$ and $\alpha\beta$. [2]

1.3 Find the determinant of $\begin{pmatrix} 4 & 3 \\ 2 & 5 \end{pmatrix}$. [2]

1.4 Find the inverse of $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$. [2]

1.5 State what the **identity matrix** I does when it multiplies another matrix. [1]

1.6 Challenge. The equation $x^2 - 6x + 8 = 0$ has roots α and β. Without solving it, find $\alpha^2 + \beta^2$. [2]

2 The cubic equation $x^3 + 2x + 1 = 0$ has roots α, β, γ.

(a) Find a cubic equation whose roots are $\alpha^3 - 1$, $\beta^3 - 1$, $\gamma^3 - 1$. [3]

(b) Find the value of $\left(\alpha^3-1\right)^2+\left(\beta^3-1\right)^2+\left(\gamma^3-1\right)^2$. [2]

(c) Find the value of $\left(\alpha^3-1\right)^3+\left(\beta^3-1\right)^3+\left(\gamma^3-1\right)^3$. [2]

6 The curve C has equation $y = \frac{x^2+a}{x+a}$, where a is a positive constant.

(a) Find the equations of the asymptotes of C. [3]

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(b) Find, in terms of a, the x-coordinates of the stationary points on C. [3]

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(c) Sketch C, stating the coordinates of any intersections with the axes. [3]

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(d) Sketch the curve with equation $y = \left|\dfrac{x^2 + a}{x + a}\right|$. [1]

(e) Find the set of values of a for which $\left|\dfrac{x^2+a}{x+a}\right| = a$ has two real solutions. [4]

1 **(a)** Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^{n}(8r^3+12r^2+4r+3)$ in terms of n, simplifying your answer. [3]

(b) Show that

$$\frac{1}{r^4} - \frac{1}{(r+1)^4} = \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$$

and hence use the method of differences to find $\sum_{r=1}^{n} \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$. [5]

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$. [1]

1 Further Pure Mathematics 1 – Part 2

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	**中文**	**Write it 3 times** (English)		
Mathematical induction	数学归纳法			
Polar coordinates	极坐标			
Cartesian coordinates	直角坐标			

Recall

In **Part 1** you linked a quadratic's roots to its coefficients, and met the 2×2 matrix. Part 2 adds a way to **prove** a result for every n (induction), and a new way to give a point's position (**polar coordinates**).

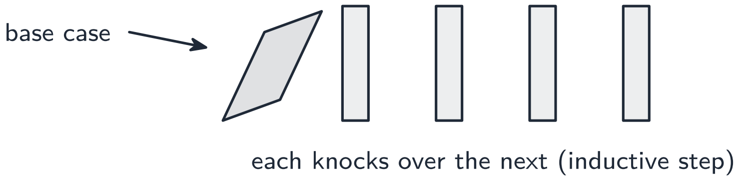
New ideas

■ 1 Proof by induction

Mathematical induction 数学归纳法 proves a result is true for every positive integer n in two steps:

1. **base case** —show the result is true for $n = 1$;
2. **inductive step** —assume it is true for $n = k$, then prove it for $n = k + 1$.

If both steps work, the result holds for **all** n. It is like a row of dominoes: knock the first over, and each one knocks over the next.



Micro-check. Name the two steps of a proof by induction. [2]

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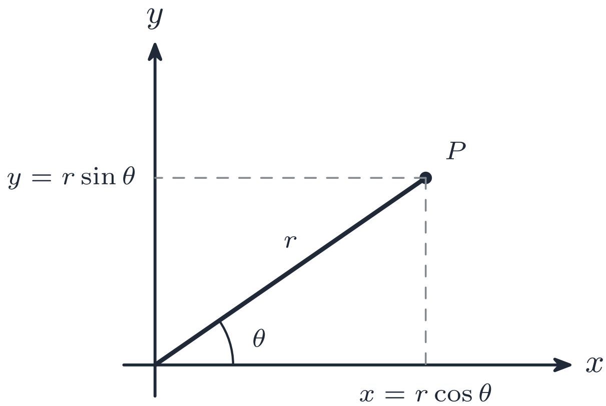
2 Polar coordinates

Polar coordinates 极坐标 give a point by its distance r from the origin and its angle θ. They link to **Cartesian coordinates** 直角坐标 by

$$x = r\cos\theta, \qquad y = r\sin\theta, \qquad r^2 = x^2 + y^2.$$



Worked example. The polar point $(r, \theta) = (2, 90^\circ)$ has $x = 2\cos 90^\circ = 0$ and $y = 2\sin 90^\circ = 2$.

Micro-check. A point has $x = 3$ and $y = 4$. Find its distance r from the origin. [1]

Watch out: induction needs **both** steps —assuming the result for $n = k$ and then proving it for $n = k + 1$; the base case alone proves nothing. For polar form, $r = \sqrt{x^2 + y^2}$ is always positive, and $x = r\cos\theta$ (not $r\sin\theta$).

Practice

1.1 State, in order, the two steps of a proof by **mathematical induction**. [2]

1.2 A result is being proved by induction. State what you do in the **base case**. [1]

1.3 Convert the polar point $(r, \theta) = (4, 0°)$ to Cartesian coordinates (x, y). [2]

1.4 A point has Cartesian coordinates $(6, 8)$. Find its distance r from the origin. [2]

1.5 Write the formulae linking x and y to r and θ. [2]

1.6 Challenge. Convert the Cartesian point $(3, 4)$ to polar coordinates (r, θ), giving θ to the nearest degree. [3]

4 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, where $0 < \theta < 2\pi$.

(a) The matrix **M** represents a sequence of two geometrical transformations in the x–y plane.

State the type of each transformation, and make clear the order in which they are applied. [2]

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(b) Find the value of θ for which the transformation represented by **M** has a line of invariant points. [7]

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6 The curve C has polar equation $r = \sin 3\theta$, for $0 \leqslant \theta \leqslant \frac{1}{3}\pi$.

(a) Sketch C and state the equation of the line of symmetry. [3]

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(b) Find the exact value of the area of the region enclosed by C. [4]

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In parts **(c)** and **(d)** you may use the identity $\sin 3\theta \equiv 3\sin\theta - 4\sin^3\theta$.

(c) Find the maximum distance of a point on C from the initial line. [5]

(d) Find a Cartesian equation for C. [3]

6 The plane Π has equation $x + 3y + 2z = 1$.

(a) Find the perpendicular distance from the origin O to the plane Π. [2]

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Relative to O, the points A, B, C have position vectors

$$-\mathbf{j} + 2\mathbf{k}, \quad 2\mathbf{i} - \mathbf{k}, \quad 2\mathbf{i} - \mathbf{j} - \mathbf{k},$$

respectively.

(b) Find the acute angle between the planes OAB and Π. [4]

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(c) Find an equation for the common perpendicular to the lines OC and AB. [8]

2 Prove by mathematical induction that $2025^n + 47^n - 2$ is divisible by 46 for all positive integers n. [6]

3 Further Mechanics – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
projectile	抛射体			
angular speed	角速度			
centripetal acceleration	向心加速度			

Recall

In AS Mechanics you used the suvat equations and $F = ma$. Bring this with you:

- free fall has a constant downward acceleration g;
- a velocity has a size and a direction (it is a vector).

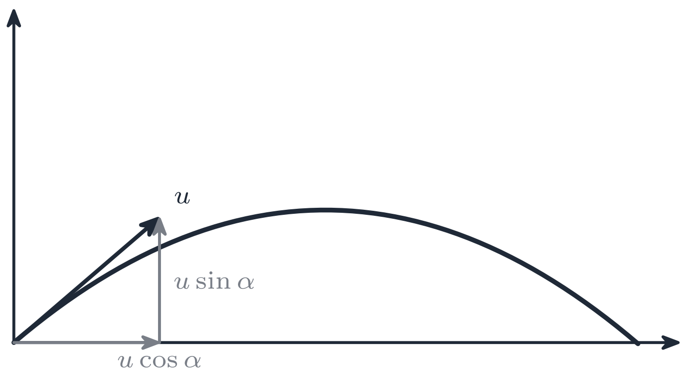
Further Mechanics adds motion in **two directions** (projectiles) and motion in a **circle**. (Take $g = 10$ m s^{-2}.)

New ideas

1 Projectile motion

A **projectile** 抛射体 moves freely under gravity: a constant downward acceleration g, and **no** horizontal acceleration. Treat the horizontal and vertical motions **separately** (they share the same time t).

Launched at speed u and angle α: the path is a parabola, the range on level ground is $\dfrac{u^2 \sin 2\alpha}{g}$, and the greatest height is $\dfrac{u^2 \sin^2 \alpha}{2g}$.



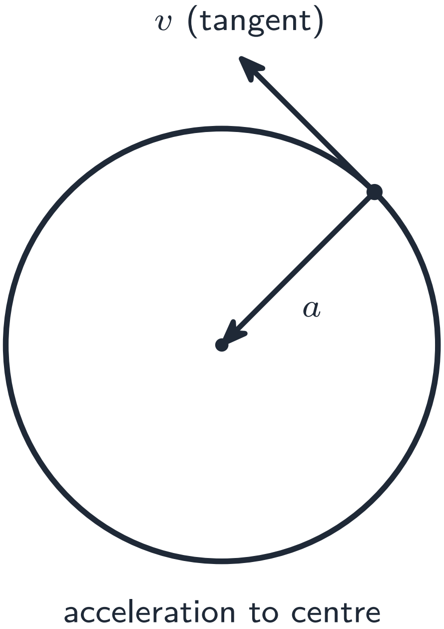
Worked example. $u = 20$ m s^{-1} at $\alpha = 30°$: range $= \dfrac{20^2 \sin 60°}{10} = 34.6$ m.

Micro-check. A projectile is launched. State its horizontal acceleration. [1]

■ 2 Circular motion

For a particle moving in a circle of radius r, the **angular speed 角速度** ω links to the speed by $v = r\omega$. The acceleration points **towards the centre** —the **centripetal acceleration 向心加速度**:

$$a = r\omega^2 = \frac{v^2}{r}.$$



Worked example. $r = 2$ m, $\omega = 3$ rad s^{-1}: $v = r\omega = 6$ m s^{-1} and $a = r\omega^2 = 18$ m s^{-2}.

Micro-check. A particle moves in a circle with $r = 4$ m and $\omega = 2$ rad s^{-1}. Find its speed v. [1]

Watch out: a projectile's **horizontal** acceleration is zero —gravity acts only vertically, so the horizontal speed never changes. The centripetal acceleration points **towards the centre**, not along the motion, and $v = r\omega$ needs ω in radians per second.

Practice

3.1 Explain why the horizontal and vertical motions of a projectile can be treated **separately**. [2]

3.2 A ball is launched at $u = 30$ m s^{-1} at $\alpha = 30°$. Find the range ($\sin 60° = 0.866$). [2]

3.3 A particle moves in a circle of radius 3 m with angular speed 4 rad s^{-1}. Find its speed. [2]

3.4 Using the particle in 3.3, find its centripetal acceleration ($a = r\omega^2$). [2]

3.5 State the **direction** of the centripetal acceleration. [1]

3.6 Challenge. A ball is thrown **horizontally** at 15 m s^{-1} from a height of 20 m. Find the time it takes to land and how far it travels horizontally. (Take $g = 10$ m s^{-2}.) [3]

2 A particle P is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle θ, where $\tan\theta = 2$, above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a time t s are denoted by x m and y m respectively.

(a) Use the equation of the trajectory given in the list of formulae (MF 19) to show that

$$y = 2x - \frac{25x^2}{u^2}.$$

[1]

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(b) In the subsequent motion, P passes through the point with coordinates $(8,12)$. The particle then hits a fixed vertical barrier 7 m high that is at a horizontal distance of D m from the point of projection.

Find the set of possible values of D. [5]

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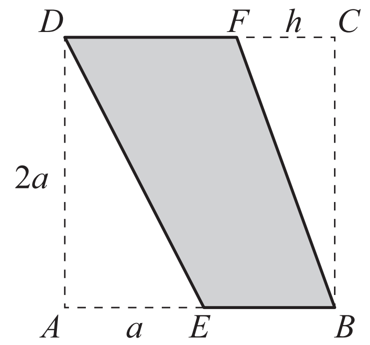


The lamina $BFDE$ is obtained by removing triangles AED and BCF from a uniform square lamina $ABCD$ of side $2a$. The length of side AE is a and the length of side FC is h (see diagram). The centre of mass of $BFDE$ is at a distance $\overline{x}$ from AD, and at a distance $\overline{y}$ from AB.

(a) Show that $\overline{x} = \dfrac{h^2 - 6ah + 11a^2}{3(3a-h)}$ and find a corresponding expression for $\overline{y}$. [5]

(b) The lamina *BFDE* is placed vertically on its edge *EB* on a smooth horizontal surface.

Find, in terms of a, the set of possible values of h for which the lamina remains in equilibrium. [2]

1 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle moves in a horizontal circle with constant angular speed ω and with the string inclined at an angle of θ to the downward vertical.

Given that $\tan\theta = \frac{4}{3}$, find ω in terms of a and g. [3]

3 Further Mechanics – Part 2

Name: ______ Class: ______ Date: ______

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
Hooke's law	胡克定律			
modulus of elasticity	弹性模量			
elastic potential energy	弹性势能			
coefficient of restitution	恢复系数			

Recall

In **Part 1** you met projectiles and circular motion. Part 2 adds stretchy **elastic strings** (Hooke's law) and **bouncy collisions** (the coefficient of restitution).

New ideas

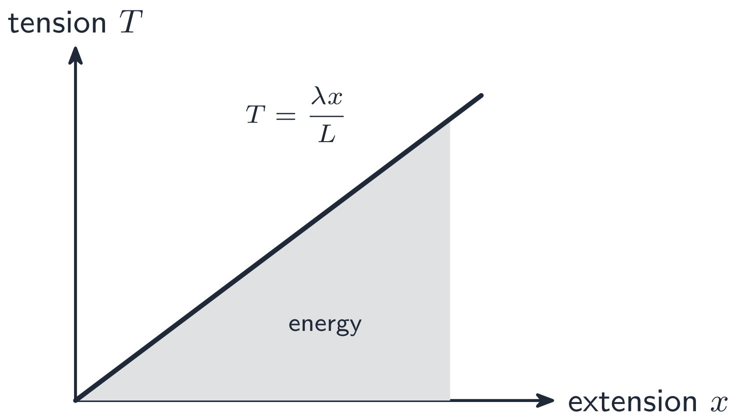
■ 1 Hooke's law

Hooke's law 胡克定律 says the tension in an elastic string or spring is proportional to its **extension** x:

$$T = \frac{\lambda x}{L},$$

where L is the natural length and λ is the **modulus of elasticity** 弹性模量. Stretching it stores **elastic potential energy** 弹性势能:

$$E = \frac{\lambda x^2}{2L}.$$



Worked example. $L = 2$ m, $\lambda = 50$ N, $x = 0.5$ m: $T = \dfrac{50 \times 0.5}{2} = 12.5$ N.

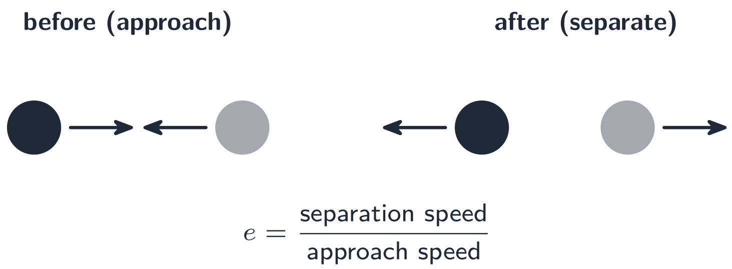
Micro-check. An elastic string has $L = 1$ m, $\lambda = 20$ N and extension $x = 0.2$ m. Find the tension. [1]

■ 2 The coefficient of restitution

In a collision, momentum is conserved. The bounciness is measured by the **coefficient of restitution 恢复系数** e:

$$e = \frac{\text{speed of separation}}{\text{speed of approach}}, \qquad 0 \leqslant e \leqslant 1.$$

Here $e = 1$ is perfectly elastic (no energy lost) and $e = 0$ is perfectly inelastic (the bodies stay together).



Worked example. Two balls approach at 4 m s^{-1} and separate at 2 m s^{-1}: $e = \dfrac{2}{4} = 0.5$.

Micro-check. Two bodies approach at 6 m s^{-1} and separate at 3 m s^{-1}. Find e. [1]

Watch out: in $T = \dfrac{\lambda x}{L}$, x is the **extension** —the extra length, not the total stretched length. The stored energy uses x^2, so doubling the stretch **quadruples** the energy. The coefficient of restitution is separation ÷ approach, always between 0 and 1.

Practice

3.1 An elastic string has natural length 2 m, modulus 40 N, and is stretched by 0.5 m. Find the tension. [2]

3.2 For the string in 3.1, find the stored elastic energy ($E = \dfrac{\lambda x^2}{2L}$). [2]

3.3 Two balls approach at 8 m s^{-1} and separate at 2 m s^{-1}. Find the coefficient of restitution. [2]

3.4 State the value of e for (a) a perfectly elastic collision; (b) a perfectly inelastic one. [2]

3.5 State what is conserved in **every** collision. [1]

3.6 Challenge. An elastic string of natural length 1.5 m and modulus 60 N is stretched to a total length of 2 m. Find the tension and the stored elastic energy. [3]

5 One end of a light elastic string of natural length 0.5 m and modulus of elasticity 14 N is attached to a fixed point A on a smooth plane. The plane makes an angle α to the horizontal, where $\tan\alpha = \frac{7}{24}$. A particle P of mass 2 kg is attached to the other end of the string. The string lies along a line of greatest slope of the plane. The particle P is initially held on the plane above the level of A, where $AP = 0.8$ m. The particle P is then released from rest.

Find the maximum velocity of P during the subsequent motion. [6]

1 A particle P of mass 8 kg is moving in a straight horizontal line. At time t s, P has displacement x m from a fixed point O on the line and velocity $v\,\mathrm{m\,s^{-1}}$. The only horizontal force acting on P has magnitude $(x^3 + 4x)$ N and acts in the direction OP. When $t = 0$, $x = 0$ and $v = 1$.

(a) Find an expression for v in terms of x, giving your answer in the form $v = ax^2 + b$, where a and b are constants to be determined. [3]

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(b) Find an expression for x in terms of t. [2]

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1 Two uniform smooth spheres A and B of equal radii have masses $4m$ and m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u and collides directly with sphere B. After the collision, the momentum of A is three times the momentum of B.

Find the value of the coefficient of restitution e. [4]

4 Further Probability & Statistics – Part 1

Name: ________ Class: ______ Date: ______

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
probability density function	概率密度函数			
cumulative distribution function	累积分布函数			
median	中位数			

Recall

In AS Statistics you used discrete random variables and the normal distribution. Bring this with you:

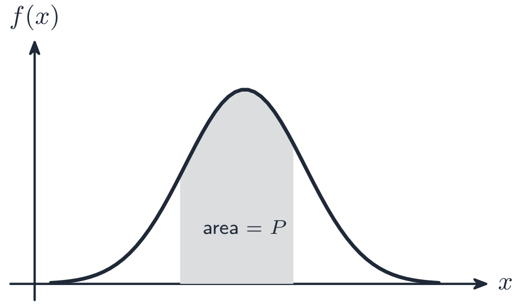
- a discrete variable takes separate values, each with a probability;
- the normal distribution is a bell-shaped curve.

Further Statistics adds **continuous** random variables, described by a curve.

New ideas

■ 1 The probability density function

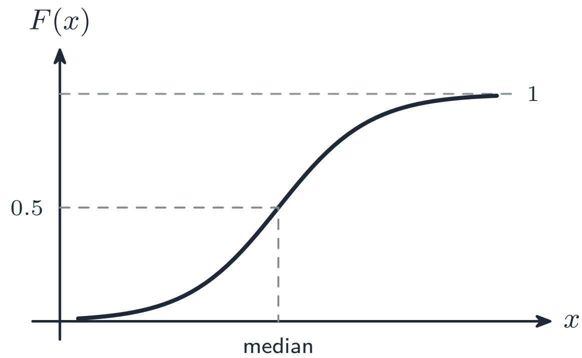
A continuous variable X is described by a **probability density function** 概率密度函数 $f(x)$. The probability that X lies in a range is the **area** under f over that range —and the **total** area under f is 1.



Micro-check. What does the **total** area under a probability density function equal? [1]

■ 2 The cumulative distribution function

The **cumulative distribution function** 累积分布函数 $F(x) = P(X \leqslant x)$ is the running total —the area to the **left** of x. It rises from 0 to 1. The **median** 中位数 m is the value where the area is split in half: $F(m) = 0.5$.



Worked example. If $f(x) = \frac{1}{2}x$ for $0 \leqslant x \leqslant 2$, then $F(x) = \frac{1}{4}x^2$. Setting $F(m) = 0.5$ gives $m^2 = 2$, so $m = \sqrt{2} = 1.41$.

Micro-check. The median m of a continuous variable satisfies $F(m) = ?$ [1]

Watch out: $f(x)$ is a **density**, not a probability —a probability is the **area** under it (an integral), and the total area is 1. The cumulative function $F(x)$ only ever **rises**, from 0 up to 1.

Practice

4.1 State what the **area** under a probability density function over a range represents. [1]

4.2 State the value of the **total** area under a probability density function. [1]

4.3 State what $F(x) = P(X \leqslant x)$ is called, and what value it rises **to**. [2]

4.4 A continuous variable has $F(x) = \frac{1}{4}x^2$ for $0 \leqslant x \leqslant 2$. Find the median (solve $F(m) = 0.5$). [2]

4.5 State the equation the median m must satisfy. [1]

4.6 Challenge. A continuous variable has $f(x) = kx$ for $0 \leqslant x \leqslant 3$ (and 0 elsewhere). Find the value of k. [3]

5 A continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \dfrac{1}{16}\sqrt{x} & 0 \leqslant x < 4, \\ \dfrac{1}{k\sqrt{x}} & 4 \leqslant x \leqslant 9, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Show that $k = 3$. [2]

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(b) Find the median value of X. [3]

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The random variable Y is defined by $Y = \sqrt{X}$.

(c) Find the probability density function of Y. [5]

1 A group of 10 school children are asked to estimate the size of an angle $\theta°$ in a given acute angled triangle. These estimates, in degrees, are as follows.

$$84 \quad 85 \quad 77 \quad 85 \quad 84 \quad 87 \quad 86 \quad 88 \quad 83 \quad 85$$

(a) Stating any assumptions you make, calculate a 95% confidence interval for θ. [5]

(b) Give a reason why the assumptions made in part **(a)** may not be appropriate in this case. [1]

4 Further Probability & Statistics – Part 2

Name: ______ Class: ______ Date: ______

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
chi-squared test	卡方检验			
probability generating function	概率母函数			

Recall

In **Part 1** you met continuous random variables (the density $f(x)$, the cumulative $F(x)$, and the median). Part 2 adds a **test** that compares observed and expected counts, and a **function** that packs a whole distribution into one.

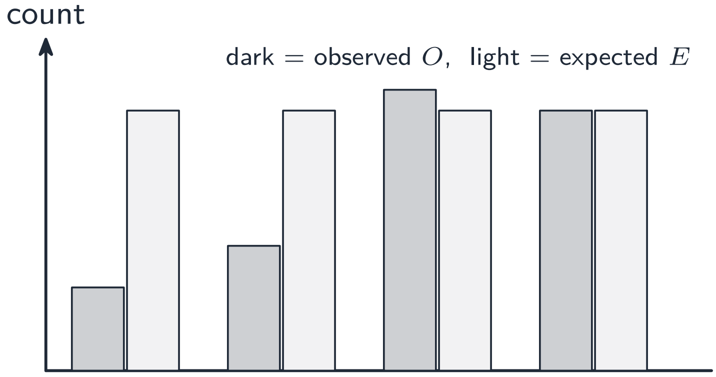
New ideas

■ 1 The chi-squared test

A **chi-squared test** 卡方检验 compares **observed** counts O with the **expected** counts E from a model:

$$\chi^2 = \sum \frac{(O - E)^2}{E}.$$

A **small** χ^2 means the data fits the model well; a **large** χ^2 (above the table value) means it fits badly.



Worked example. Observed $20, 30, 25, 25$ with each expected $= 25$: $\chi^2 = \dfrac{(20-25)^2 + (30-25)^2 + 0 + 0}{25} = \dfrac{50}{25} = 2$.

Micro-check. In a chi-squared test, does a **large** value mean the model fits well or badly? [1]

■ 2 Probability generating functions

The **probability generating function** 概率母函数 of a discrete variable X is

$$G(t) = E(t^X) = \sum_x P(X = x)\, t^x.$$

It packs the whole distribution into one function. The mean comes from its derivative at $t = 1$: $E(X) = G'(1)$.

Worked example. If $P(X{=}0) = 0.5$, $P(X{=}1) = 0.3$, $P(X{=}2) = 0.2$, then $G(t) = 0.5 + 0.3t + 0.2t^2$, so $G'(t) = 0.3 + 0.4t$ and $E(X) = G'(1) = 0.7$.

Micro-check. The PGF gives the mean by $E(X) = G'(?)$ —state the value. [1]

Watch out: in χ^2, divide each $(O - E)^2$ by its **own** E before adding —not by the total. A **large** χ^2 means a **bad** fit. The PGF gives the mean from $G'(1)$, not $G(1)$ (which is always 1).

Practice

4.1 Write the formula for the chi-squared statistic χ^2. [1]

4.2 Observed counts are $10, 20, 30$ with expected counts $20, 20, 20$. Find χ^2. [2]

4.3 State what a **large** value of χ^2 tells you about the model. [1]

4.4 A variable has $G(t) = 0.4 + 0.4t + 0.2t^2$. Find $G'(t)$ and hence $E(X) = G'(1)$. [2]

4.5 State what the probability generating function $G(t)$ "packs" into one function. [1]

4.6 Challenge. A variable has $G(t) = \frac{1}{4} + \frac{1}{2}t + \frac{1}{4}t^2$. Find $E(X)$. [3]

2 The manager of a car park claims that the number of cars entering the car park follows a Poisson distribution with mean 2.8. The numbers of cars entering the car park are recorded on a working day during successive 5-minute periods. The following table contains the observed frequencies, together with most of the expected frequencies and their contributions to the χ^2-test statistic.

Number of cars	0	1	2	3	4	5	$\geqslant 6$
Observed frequency	2	15	31	29	13	3	7
Expected frequency	6.081	17.03	23.84	p	15.57	8.721	6.511
χ^2-test statistic	2.739	0.241	2.152	q	0.425	3.753	0.037

(a) Find the value of p and the value of q. [2]

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(b) Carry out a goodness of fit test at the 5% significance level to investigate the manager's claim. [4]

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1 A large company claims that the median salary of its employees is \$32 500. The salaries (\$) of 15 randomly selected employees are listed below.

18 750	30 500	125 000	42 500	25 000
26 000	52 500	23 000	27 500	19 500
25 500	33 000	30 000	21 500	29 000

(a) Explain why a Wilcoxon signed-rank test may not be appropriate to test the company's claim in this case. [1]

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(b) Carry out a sign test at the 10% significance level to investigate the company's claim. [5]

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4 The random variable X takes values 1 and 2 with probabilities $\frac{2}{5}$ and $\frac{3}{5}$ respectively.

(a) Write down the probability generating function $G_X(t)$ of X. [1]

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The random variable Y is the sum of four independent observations of X.

(b) Find the probability generating function $G_Y(t)$ of Y. Give your answer in the form $G_Y(t) = at^m(b+ct)^n$, where a, b, c, m and n are constants to be determined. [2]

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(c) Use $G_Y(t)$ to find $P(Y = 6)$. [2]

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(d) Find Var(Y). [5]