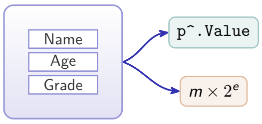
Data Representation

A-Level Computer Science ▪ Topic 13

A-Level Computer Science



What we will cover

1 User-defined types

Why define your own types?

Built-in types (`INTEGER`, `STRING`, ...) are too loose for real data.

Restrict
限制
a `STRING` will hold nonsense; an enumerated type will not

Group
组合
a taxi is a name *and* a capacity *and* a flag

Name
自说明
`DECLARE Taxi : Vehicle` reads as what it is

User-defined types 用户定义类型 make the compiler stricter and the code self-documenting.

Enumerated type – a fixed named list

An **enumerated type** 枚举类型 has values that are a fixed list of named constants.

A fleet of taxis

```
TYPE Vehicle = (M100, M230, T101,
     T102, T120, T150)
DECLARE MyTaxi : Vehicle
MyTaxi ← T102
```

You cannot assign anything outside the list. Internally the names are small integers.

Vehicle

M100
M230
T101
T102
T120

An enumerated type, drawn

TYPE Vehicle =

M100	M230	T101	T102	T120	T150

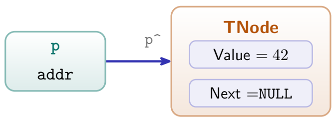
`MyTaxi` may only hold one of these named values

The values are named constants in a fixed order, so they can be compared and counted — but they are not strings and not integers.

Pointer type – an address, not a value

A **pointer** 指针 holds the memory address of another variable (or NULL).

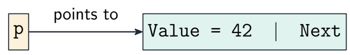
- builds dynamic structures: lists, trees
- passes a reference without copying
- **dereference** 解引用 p^ reaches the target



Point at a node

```
TYPE PNode = ^TNode
DECLARE p : PNode
p ← NEW TNode
p^.Value ← 42
```


A pointer holds an address



p^ dereferences — reach the node's fields, e.g. p^.Value

The variable stores where the data is, not the data — which is what lets a linked structure grow without moving anything.

Composite types – several values, one name

A **composite type** 复合类型 groups several values under one name.

record
记录
fields of *different* types in a TYPE …ENDTYPE block

set
集合
unordered, unique values: add, remove, membership, union

class
类
attributes *and* methods – an object is an instance

Enumerated and pointer are non-composite. Record, set and class are composite.

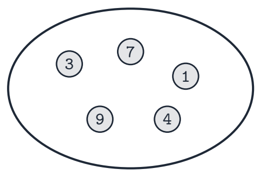
Two composite shapes

record Student

Name : STRING
Age : INTEGER
Grade : CHAR
Enrolled : BOOLEAN

one name, several fields of different types

A **record**: named fields of different types, held together.



unordered, every value unique

A **set**: unordered, no duplicates, tested with membership.

Choosing a type

Pick the type that matches the kind of value, not the first one that compiles.

a value from a fixed list	**enumerated**
a group of fields	**record**
an unordered unique collection	**set**
indirection / a linked structure	**pointer**
state *and* behaviour together	**class**

2 File organisation

Three ways to lay records out

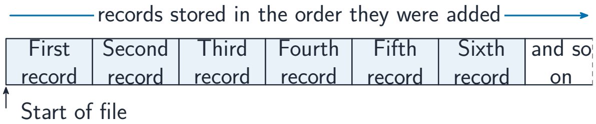
File organisation 文件组织 is the layout; file access is how you reach a record.

serial
串行文件
records in the *order added*. Append is fast; search is slow. Logs.

sequential
顺序文件
sorted on a *key*. Search can stop early; insert must shift.

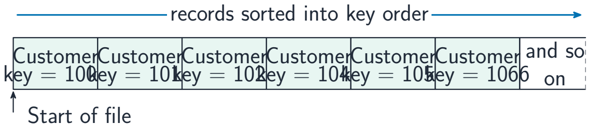
random
随机文件
position from the key (often a hash). Direct lookup is fast.

Serial – records in arrival order



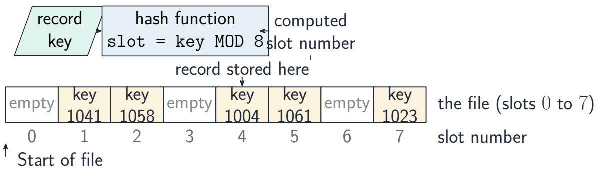
New records go on the end. Simple to write, but any search reads from the start.

Sequential – ordered by a key



Records are kept in key order, so a search can stop early — but an insertion means rewriting the file.

Random – the key computes the address



A **hash** 散列 of the key gives the record's position, so one read finds it — direct access, at the cost of managing collisions.

Sequential access vs direct access

- **sequential access** 顺序存取 – read from the start, one record after another
- **direct access** 直接存取 – jump straight to a known position

Serial and sequential files need sequential access. A random file allows direct access.

Match the layout to the dominant operation: single-key lookups favour random; in-order reports favour sequential.

serial / sequential — walk from the start

random — hash, then jump

3 Hashing

A hash function turns a key into an address

A **hash function** 散列函数 takes a record key and produces the slot where the record is stored. A good one is fast, **deterministic** 确定性, and spreads keys evenly. For N slots:

- modulo: `key MOD N`
- folding: split, add the pieces, `MOD N`
- string: sum character codes, `MOD N`

`key T102` → `hash(key)` → slot **4**

Keep the **load factor** 装填因子 below about 70%.

Collisions – two keys, one slot

A **collision** 冲突 is when two keys hash to the same address.

linear probing
线性探测
use the next free slot, wrapping around. Simple, but keys *cluster*.

chaining
链接法
each slot points to a **linked list** 链表. No clustering; uses more memory.

rehashing
再散列
apply a second hash function. Spreads keys, more work.

Worked example – resolve a collision

Keys A and B both hash to slot 2

Linear probing: B goes in the next free slot (3).

Chaining: slot 2 holds a list $A \to B$.

To search: hash the key, read that slot; if the keys do not match, follow the resolution strategy until a match or an empty slot.

		A	B	
0	1	2	3	4

probing: B sits in 3

Empty slot = "not found" when probing.

4 Floating-point

Mantissa × two-to-the-exponent

A **floating-point** 浮点 number is binary scientific notation:

$$\text{value} = \text{mantissa} \times 2^{\text{exponent}}$$

Both fields are **two's complement** 补码 integers.

- more mantissa bits ⇒ more precision
- more exponent bits ⇒ more range

The mantissa is a binary *fraction*: the first bit after the point is $\frac{1}{2}$, then $\frac{1}{4}$, $\frac{1}{8}$, ...

mantissa
`0.1010000`

exponent
`00000010`

$0.625 \times 2^2 = 2.5$

Worked example – binary to denary

Mantissa `10110000`, exponent `00000011`

Exponent `00000011` $= +3$.

Mantissa starts with 1, so it is negative. Sign bit $= -1$, fraction $\frac{1}{4} + \frac{1}{8} = 0.375$:

$$m = -1 + 0.375 = -0.625$$

$$\text{value} = -0.625 \times 2^3 = -5.0$$

A negative mantissa follows two's-complement rules. Convert the exponent first – it is the easy part.

Worked example – store $+2.5$

Write $+2.5$ in this 8+8-bit format

In binary $2.5 = 10.1$.

As a normalised fraction: $2.5 = 0.101 \times 2^2$.

mantissa 01010000 (sign 0, then .101)
exponent 00000010 ($= 2$)

10.1_2 → 0.101×2^2 → 01010000 00000010

Normalisation maximises precision

A number is **normalised** 规格化 when the first significant bit sits immediately after the binary point – no wasted leading zeros.

- positive: starts $0.1\ldots$
- negative (two's complement): starts $1.0\ldots$

Shift the mantissa left and decrease the exponent (or right and increase it) until that pattern holds. The value is unchanged.

Normalise 0.0101

Shift left until it reads $0.1\ldots$:

$$0.0101 \to 0.101 \times 2^{-1}$$

mantissa 0.101, exponent -1

Each left shift subtracts one from the exponent.

What normalising does to the bits

before	`0.0011010`	exponent 4 (two wasted leading zeros)
	↓ shift left 2, exponent -2	
after	`0.1101000`	exponent 2 (normalised — same value)

Shifting until the first two bits differ uses every mantissa bit for precision instead of for leading zeros.

Why $0.1 + 0.2$ is not 0.3

Many denary reals cannot be stored exactly in binary – $0.1_{10} = 0.000110011\ldots_2$ must be truncated.

rounding
舍入误差
errors build up over many operations

overflow
溢出
result too large for the exponent's range

underflow
下溢
result too small, rounds to zero

Never test reals with =. Use a tolerance, or use **fixed-point** 定点 / BCD for money.

5 Recap

Topic 13 – key takeaways

1. **Types** — enumerated, pointer, record, set, class
2. **Files** — serial, sequential, random; sequential vs direct access
3. **Hashing** — key → address; probing / chaining / rehash
4. **Float** — $m \times 2^e$; normalise; rounding / overflow

Check yourself

1. Which file type gives direct access by key?
2. Linear probing vs chaining – what is the trade-off?
3. How does a normalised *positive* mantissa start?
4. Why can 0.1_{10} not be stored exactly?

Answers **1.** random **2.** probing clusters; chaining uses extra memory **3.** $0.1\ldots$ **4.** it is a repeating binary fraction