A-LEVEL COMPUTER SCIENCE • EXERCISE SHEET

3.2.2 Building logic circuits from expressions, problems and truth tables

Name: _____ Class: _____ Date: _____ **Total: 41 marks**

KEY WORDS logic circuit 逻辑电路 • logic expression 逻辑表达式 • problem statement 问题陈述 • truth table 真值表 • logic gate 逻辑门

Objective

Learn to **build** logic: draw a **logic circuit** 逻辑电路 from a **logic expression** 逻辑表达式, turn a **problem statement** 问题陈述 into an expression and a circuit, and write an expression from a **truth table** 真值表. On sheet 3.2.1 you read circuits that were given to you; here you make them. This is the last sheet of 3.2.

You must be able to:

- construct a logic circuit from a logic expression
- construct a logic expression and a logic circuit from a problem statement
- construct a logic expression from a truth table, and a circuit from that expression
- check your circuit with a truth table

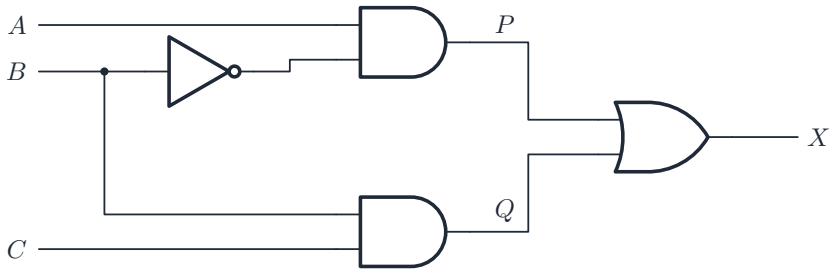
1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

■ **From an expression to a circuit: three rules**

1. **One gate for each operator.** Count the operators (NOT counts too).
2. **Brackets first.** The gate for a bracket is drawn on the **left**, and its output goes into the gate outside the bracket.
3. **An input that is used twice is one input.** Draw one wire, with a **junction dot** where it branches.

$X = (A\ AND\ NOT\ B)\ OR\ (B\ AND\ C)$*:* four operators, so four gates; B is used twice, so its wire branches.



Draw the correct **shape** of each gate (small circle on NOT, NAND and NOR). Every gate except NOT has **two** inputs. Label the inputs and the output.

From a problem statement to an expression

Give each condition a letter, and translate the sentence piece by piece.

words in the problem	operator
"… **and** …", "both"	AND
"… **or** …", "either"	OR
"is **not**", "is off", "is 0"	NOT
"one or the other **but not both**"	XOR
"**neither** … **nor** …"	NOR

An alarm X sounds when the door is open ($A = 1$) and the system is on ($B = 1$), or when the panic button is pressed ($C = 1$).

- "door open **and** system on" $\rightarrow$ A AND B; "**or** the panic button" $\rightarrow$ OR C.
- $X = (A \text{ AND } B) \text{ OR } C$: two operators, two gates. C goes straight into the OR gate.

Watch the **value** in the problem: "the tank is **not** full ($A = 0$)" is true when A is 0, so write NOT A.

From a truth table to an expression

1. Use only the rows where $X = 1$.
2. In a row, write an input that is **1** as itself and an input that is **0** as **NOT** that input; join them with **AND**.
3. Join the terms of all the rows with **OR**.

$X = 1$ on the rows $ABC = 001$, 010 and 111:

$X = (\text{NOT } A \text{ AND NOT } B \text{ AND } C) \text{ OR } (\text{NOT } A \text{ AND } B \text{ AND NOT } C) \text{ OR } (A \text{ AND } B \text{ AND } C)$

> One term for each 1-row, and every term has **all three** inputs. **Check** a circuit or an expression by testing two or three rows against the problem.

2 Practice

Now use the methods above. The first questions need one idea each.

2.1 Name the logic operator for the words in bold. [4]

(a) The pump runs when the tank is empty **and** the switch is on.

(b) The light is on when it is **not** daytime.

(c) The door opens when **either** button is pressed.

(d) The signal is sent when **one or the other, but not both**, of the keys is turned.

2.2 A circuit is drawn for $X = (A \text{ OR } B) \text{ AND } (\text{NOT } C)$.

State how many gates the circuit has, and name each gate. [2]

2.3 Draw the logic circuit for $X = (A \text{ NOR } B) \text{ AND } C$. [2]

2.4 A light X is on when it is dark ($D = 1$) **and** the switch is on ($S = 1$), **or** when the test button is pressed ($T = 1$).

Write the logic expression for X. [2]

2.5 A truth table gives $X = 1$ on the rows $AB = 01$ and 10, and $X = 0$ on the other two rows.

(a) Write the logic expression for X. Use only AND, OR and NOT. [2]

(b) Name the **single** gate that gives the same output. [1]

2.6 A circuit uses input B as an input to two different gates. State how this is shown on the circuit diagram. [1]

3 Exam-style questions

3.1 A chemical tank has three sensors. An alarm X sounds when the temperature sensor A reads 1 **and** the pressure sensor B reads 0, **or** when the manual override C reads 1.

(a) Write the logic expression for X. [2]

(b) Draw the logic circuit for X. [3]

3.2 A truth table gives $X = 1$ on the rows $ABC = 000$, 011 and 110, and $X = 0$ on every other row.

Write the logic expression for X. [3]

3.3 Draw the logic circuit for $X = (A \text{ XOR } B) \text{ AND } (\text{NOT } C)$. [3]

3.4 A student is asked to draw the circuit for $X = A$ AND (B OR C), but wires it as (A AND B) OR C.

Explain, naming **one** row of the truth table, how the two circuits behave differently. [3]

3.5 A heater H in a greenhouse is switched on ($H = 1$) when:

- the greenhouse is cold ($C = 1$)
- **and** either the window is closed ($W = 0$) **or** the override switch is on ($V = 1$).

(a) Write the logic expression for H. [3]

(b) Draw the logic circuit for H. [4]

(c) Complete the truth table for H. [4]

C	W	V	**Working space**	H
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

(d) On a cold day the window is open and the override switch is off. Use your truth table to state whether the heater is on, and explain why this is sensible. [2]