A-LEVEL COMPUTER SCIENCE • EXERCISE SHEET

3.2.2 Building logic circuits from expressions, problems and truth tables

Name: _____ Class: _____ Date: _____ **Total: 41 marks**

KEY WORDS logic circuit 逻辑电路 • logic expression 逻辑表达式 • problem statement 问题陈述 • truth table 真值表 • logic gate 逻辑门

Objective

Learn to **build** logic: draw a **logic circuit** 逻辑电路 from a **logic expression** 逻辑表达式, turn a **problem statement** 问题陈述 into an expression and a circuit, and write an expression from a **truth table** 真值表. On sheet 3.2.1 you read circuits that were given to you; here you make them. This is the last sheet of 3.2.

You must be able to:

- construct a logic circuit from a logic expression
- construct a logic expression and a logic circuit from a problem statement
- construct a logic expression from a truth table, and a circuit from that expression
- check your circuit with a truth table

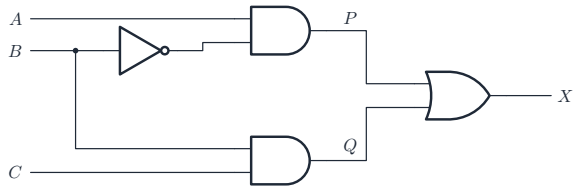
1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

From an expression to a circuit: three rules

1. **One gate for each operator.** Count the operators (NOT counts too).
2. **Brackets first.** The gate for a bracket is drawn on the **left**, and its output goes into the gate outside the bracket.
3. **An input that is used twice is one input.** Draw one wire, with a **junction dot** where it branches.

X = (A AND NOT B) OR (B AND C): four operators, so four gates; B is used twice, so its wire branches.



Draw the correct **shape** of each gate (small circle on NOT, NAND and NOR). Every gate except NOT has **two** inputs. Label the inputs and the output.

From a problem statement to an expression

Give each condition a letter, and translate the sentence piece by piece.

words in the problem	operator
"… **and** …", "both"	AND
"… **or** …", "either"	OR
"is **not**", "is off", "is 0"	NOT
"one or the other **but not both**"	XOR
"**neither** … **nor** …"	NOR

An alarm X sounds when the door is open ($A = 1$) and the system is on ($B = 1$), or when the panic button is pressed ($C = 1$).

- "door open **and** system on" → A AND B; "**or** the panic button" → OR C.
- X = (A AND B) OR C: two operators, two gates. C goes straight into the OR gate.

Watch the **value** in the problem: "the tank is **not** full ($A = 0$)" is true when A is 0, so write NOT A.

From a truth table to an expression

1. Use only the rows where $X = 1$.
2. In a row, write an input that is **1** as itself and an input that is **0** as **NOT** that input; join them with **AND**.
3. Join the terms of all the rows with **OR**.

$X = 1$ on the rows $ABC = 001$, 010 and 111:

X = (NOT A AND NOT B AND C) OR (NOT A AND B AND NOT C) OR (A AND B AND C)

> One term for each 1-row, and every term has **all three** inputs. **Check** a circuit or an expression by testing two or three rows against the problem.

2 Practice

Now use the methods above. The first questions need one idea each.

2.1 Name the logic operator for the words in bold. [4]

(a) The pump runs when the tank is empty **and** the switch is on.

(b) The light is on when it is **not** daytime.

(c) The door opens when **either** button is pressed.

(d) The signal is sent when **one or the other, but not both**, of the keys is turned.

2.2 A circuit is drawn for $X = (A \text{ OR } B) \text{ AND } (\text{NOT } C)$.

State how many gates the circuit has, and name each gate. [2]

2.3 Draw the logic circuit for $X = (A \text{ NOR } B) \text{ AND } C$. [2]

2.4 A light X is on when it is dark ($D = 1$) **and** the switch is on ($S = 1$), **or** when the test button is pressed ($T = 1$).

Write the logic expression for X. [2]

2.5 A truth table gives $X = 1$ on the rows $AB = 01$ and 10, and $X = 0$ on the other two rows.

(a) Write the logic expression for X. Use only AND, OR and NOT. [2]

(b) Name the **single** gate that gives the same output. [1]

2.6 A circuit uses input B as an input to two different gates. State how this is shown on the circuit diagram. [1]

3 Exam-style questions

3.1 A chemical tank has three sensors. An alarm X sounds when the temperature sensor A reads 1 **and** the pressure sensor B reads 0, **or** when the manual override C reads 1.

(a) Write the logic expression for X. [2]

(b) Draw the logic circuit for X. [3]

3.2 A truth table gives $X = 1$ on the rows $ABC = 000$, 011 and 110, and $X = 0$ on every other row.

Write the logic expression for X. [3]

3.3 Draw the logic circuit for $X = (A \text{ XOR } B) \text{ AND } (\text{NOT } C)$. [3]

3.4 A student is asked to draw the circuit for $X = A \text{ AND } (B \text{ OR } C)$, but wires it as $(A \text{ AND } B) \text{ OR } C$.

Explain, naming **one** row of the truth table, how the two circuits behave differently. [3]

3.5 A heater H in a greenhouse is switched on ($H = 1$) when:

- the greenhouse is cold ($C = 1$)
- **and** either the window is closed ($W = 0$) **or** the override switch is on ($V = 1$).

(a) Write the logic expression for H. [3]

(b) Draw the logic circuit for H. [4]

(c) Complete the truth table for H. [4]

C	W	V	**Working space**	H
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

(d) On a cold day the window is open and the override switch is off. Use your truth table to state whether the heater is on, and explain why this is sensible. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

(a) both conditions are needed → **AND**

(b) the condition must be false → **NOT**

(c) one is enough, and both is also allowed → **OR**

(d) exactly one → **XOR**

2.2

- the expression has three operators: OR, AND and NOT, so the circuit has **three** gates
- an **OR** gate, a **NOT** gate and an **AND** gate

2.3

- a NOR gate with A and B as its inputs, drawn on the left because it is in the brackets
- the NOR output and C go into an AND gate, whose output is labelled X

2.4

- "dark and the switch is on" is D AND S
- "or the test button" adds OR T: $X = (D \text{ AND } S) \text{ OR } T$

2.5

(a) row 01 gives NOT A AND B; row 10 gives A AND NOT B

- $X = (\text{NOT } A \text{ AND } B) \text{ OR } (A \text{ AND NOT } B)$

(b) the output is 1 when the inputs are different → **XOR**

2.6

- the wire from B **branches**, and a **junction dot** is drawn where it branches; B is still one input

3.1

(a) "A reads 1 and B reads 0" is A AND NOT B

- "or C reads 1" adds OR C: $X = (A \text{ AND NOT } B) \text{ OR } C$

(b) a NOT gate with B as its input

- an AND gate that takes A and the output of the NOT gate
- an OR gate that takes the AND output and C, and gives X
- C goes straight into the OR gate: nothing in the sentence acts on it first

3.2

- row 000: NOT A AND NOT B AND NOT C

- row 011: NOT A AND B AND C
- row 110: A AND B AND NOT C, and the three terms joined with OR

$X = (\text{NOT } A \text{ AND NOT } B \text{ AND NOT } C) \text{ OR } (\text{NOT } A \text{ AND } B \text{ AND } C) \text{ OR } (A \text{ AND } B \text{ AND NOT}$

3.3

- an XOR gate with A and B as its inputs
- a NOT gate with C as its input
- both outputs go into an AND gate, whose output is X
- the XOR symbol is the OR shape with a **second curved line** at its back; a plain OR shape loses the first mark

3.4

- the brackets decide which gate comes first: in A AND (B OR C) the OR is worked out first and A must be 1 for X to be 1
- in (A AND B) OR C, the input C reaches the output on its own, so $C = 1$ gives $X = 1$ whatever A is
- any row with $A = 0$ and $C = 1$ shows the difference, for example $A = 0$, $B = 0$, $C = 1$: the correct circuit gives $X = 0$ and the student's circuit gives $X = 1$

3.5

(a) "the window is closed" is $W = 0$, so write NOT W

- "either the window is closed or the override is on" is (NOT W) OR V
- $H = C$ AND ((NOT W) OR V)

(b) a NOT gate with W as its input

- an OR gate that takes the NOT output and V
- an AND gate that takes C and the OR output
- the output of the AND gate is labelled H, and all three inputs are labelled

(c) working column NOT W: 1, 1, 0, 0, 1, 1, 0, 0

- working column (NOT W) OR V: 1, 1, 0, 1, 1, 1, 0, 1
- AND this column with C, which is 1 only on the last four rows
- H = **0, 0, 0, 0, 1, 1, 0, 1**

(d) the row is $C = 1$, $W = 1$, $V = 0$, and it gives $H = 0$: the heater is **off**

- this is sensible because the heat would escape through the open window, so heating would waste energy