

3.2.2 Building logic circuits from expressions, problems and truth tables

Name: _____ Class: _____ Date: _____

Total: 41 marks

KEY WORDS logic circuit 逻辑电路 • logic expression 逻辑表达式 • problem statement 问题陈述
• truth table 真值表 • logic gate 逻辑门

Objective

Learn to **build** logic: draw a **logic circuit** 逻辑电路 from a **logic expression** 逻辑表达式, turn a **problem statement** 问题陈述 into an expression and a circuit, and write an expression from a **truth table** 真值表. On sheet 3.2.1 you read circuits that were given to you; here you make them. This is the last sheet of 3.2.

You must be able to:

- construct a logic circuit from a logic expression
- construct a logic expression and a logic circuit from a problem statement
- construct a logic expression from a truth table, and a circuit from that expression
- check your circuit with a truth table

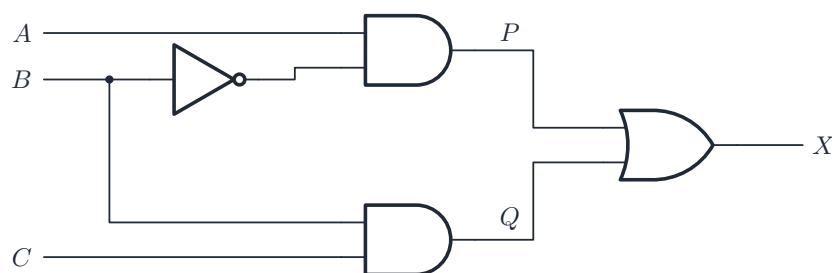
1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

■ From an expression to a circuit: three rules

1. **One gate for each operator.** Count the operators (NOT counts too).
2. **Brackets first.** The gate for a bracket is drawn on the **left**, and its output goes into the gate outside the bracket.
3. **An input that is used twice is one input.** Draw one wire, with a **junction dot** where it branches.

$X = (A \text{ AND } NOT\ B) \text{ OR } (B \text{ AND } C)$: four operators, so four gates; B is used twice, so its wire branches.



Draw the correct **shape** of each gate (small circle on NOT, NAND and NOR). Every gate except NOT has **two** inputs. Label the inputs and the output.

■ From a problem statement to an expression

Give each condition a letter, and translate the sentence piece by piece.

words in the problem	operator
"... and ...", "both"	AND
"... or ...", "either"	OR
"is not ", "is off", "is 0"	NOT
"one or the other but not both "	XOR
" neither ... nor ..."	NOR

An alarm X sounds when the door is open ($A = 1$) and the system is on ($B = 1$), or when the panic button is pressed ($C = 1$).

- "door open **and** system on" $\rightarrow A$ AND B ; "or the panic button" $\rightarrow$ OR C .
- $X = (A$ AND $B)$ OR C : two operators, two gates. C goes straight into the OR gate.



Watch the **value** in the problem: "the tank is **not** full ($A = 0$)" is true when A is 0, so write NOT A .

■ From a truth table to an expression

- Use only the rows where $X = 1$.
- In a row, write an input that is **1** as itself and an input that is **0** as **NOT** that input; join them with **AND**.
- Join the terms of all the rows with **OR**.

$X = 1$ on the rows $ABC = 001, 010$ and 111 :

$$X = (\text{NOT } A \text{ AND NOT } B \text{ AND } C) \text{ OR } (\text{NOT } A \text{ AND } B \text{ AND NOT } C) \text{ OR } (A \text{ AND } B \text{ AND } C)$$

One term for each 1-row, and every term has **all three** inputs. **Check** a circuit or an expression by testing two or three rows against the problem.

2 Practice

Now use the methods above. The first questions need one idea each.

2.1 Name the logic operator for the words in bold. [4]

- (a) The pump runs when the tank is empty **and** the switch is on.
- (b) The light is on when it is **not** daytime.
- (c) The door opens when **either** button is pressed.
- (d) The signal is sent when **one or the other, but not both**, of the keys is turned.

2.2 A circuit is drawn for $X = (A \text{ OR } B) \text{ AND } (\text{NOT } C)$.

State how many gates the circuit has, and name each gate. [2]

2.3 Draw the logic circuit for $X = (A \text{ NOR } B) \text{ AND } C$. [2]

2.4 A light X is on when it is dark ($D = 1$) **and** the switch is on ($S = 1$), **or** when the test button is pressed ($T = 1$).

Write the logic expression for X . [2]

2.5 A truth table gives $X = 1$ on the rows $AB = 01$ and 10 , and $X = 0$ on the other two rows.

(a) Write the logic expression for X . Use only AND, OR and NOT. [2]

(b) Name the **single** gate that gives the same output. [1]

2.6 A circuit uses input B as an input to two different gates. State how this is shown on the circuit diagram. [1]

3 Exam-style questions

3.1 A chemical tank has three sensors. An alarm X sounds when the temperature sensor A reads 1 **and** the pressure sensor B reads 0, **or** when the manual override C reads 1.

(a) Write the logic expression for X . [2]

(b) Draw the logic circuit for X . [3]

3.2 A truth table gives $X = 1$ on the rows $ABC = 000, 011$ and 110 , and $X = 0$ on every other row.

Write the logic expression for X . [3]

3.3 Draw the logic circuit for $X = (A \text{ XOR } B) \text{ AND } (\text{NOT } C)$. [3]

3.4 A student is asked to draw the circuit for $X = A \text{ AND } (B \text{ OR } C)$, but wires it as $(A \text{ AND } B) \text{ OR } C$.

Explain, naming **one** row of the truth table, how the two circuits behave differently. [3]

3.5 A heater H in a greenhouse is switched on ($H = 1$) when:

- the greenhouse is cold ($C = 1$)
- **and** either the window is closed ($W = 0$) **or** the override switch is on ($V = 1$).

(a) Write the logic expression for H . [3]

(b) Draw the logic circuit for H . [4]

(c) Complete the truth table for H . [4]

C	W	V	Working space	H
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

(d) On a cold day the window is open and the override switch is off. Use your truth table to state whether the heater is on, and explain why this is sensible. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- (a) both conditions are needed $\rightarrow$ **AND**
- (b) the condition must be false $\rightarrow$ **NOT**
- (c) one is enough, and both is also allowed $\rightarrow$ **OR**
- (d) exactly one $\rightarrow$ **XOR**

2.2

- the expression has three operators: OR, AND and NOT, so the circuit has **three** gates
- an **OR** gate, a **NOT** gate and an **AND** gate

2.3

- a NOR gate with A and B as its inputs, drawn on the left because it is in the brackets
- the NOR output and C go into an AND gate, whose output is labelled X

2.4

- "dark and the switch is on" is D AND S
- "or the test button" adds OR T : $X = (D \text{ AND } S) \text{ OR } T$

2.5

- (a) row 01 gives NOT A AND B ; row 10 gives A AND NOT B
 - $X = (\text{NOT } A \text{ AND } B) \text{ OR } (A \text{ AND NOT } B)$
- (b) the output is 1 when the inputs are different $\rightarrow$ **XOR**

2.6

- the wire from B **branches**, and a **junction dot** is drawn where it branches; B is still one input

3.1

- (a) " A reads 1 and B reads 0" is A AND NOT B
 - "or C reads 1" adds OR C : $X = (A \text{ AND NOT } B) \text{ OR } C$
- (b) a NOT gate with B as its input
 - an AND gate that takes A and the output of the NOT gate
 - an OR gate that takes the AND output and C , and gives X
 - C goes straight into the OR gate: nothing in the sentence acts on it first

3.2

- row 000: NOT A AND NOT B AND NOT C

- row 011: NOT A AND B AND C
- row 110: A AND B AND NOT C , and the three terms joined with OR

$$X = (\text{NOT } A \text{ AND NOT } B \text{ AND NOT } C) \text{ OR } (\text{NOT } A \text{ AND } B \text{ AND } C) \text{ OR } (A \text{ AND } B \text{ AND NOT } C)$$

3.3

- an XOR gate with A and B as its inputs
- a NOT gate with C as its input
- both outputs go into an AND gate, whose output is X
- the XOR symbol is the OR shape with a **second curved line** at its back; a plain OR shape loses the first mark

3.4

- the brackets decide which gate comes first: in A AND $(B$ OR $C)$ the OR is worked out first and A must be 1 for X to be 1
- in $(A$ AND $B)$ OR C , the input C reaches the output on its own, so $C = 1$ gives $X = 1$ whatever A is
- any row with $A = 0$ and $C = 1$ shows the difference, for example $A = 0$, $B = 0$, $C = 1$: the correct circuit gives $X = 0$ and the student's circuit gives $X = 1$

3.5

(a) "the window is closed" is $W = 0$, so write NOT W

- "either the window is closed or the override is on" is $(\text{NOT } W) \text{ OR } V$
- $H = C \text{ AND } ((\text{NOT } W) \text{ OR } V)$

(b) a NOT gate with W as its input

- an OR gate that takes the NOT output and V
- an AND gate that takes C and the OR output
- the output of the AND gate is labelled H , and all three inputs are labelled

(c) working column NOT W : 1, 1, 0, 0, 1, 1, 0, 0

- working column $(\text{NOT } W) \text{ OR } V$: 1, 1, 0, 1, 1, 1, 0, 1
- AND this column with C , which is 1 only on the last four rows
- $H = 0, 0, 0, 0, 1, 1, 0, 1$

(d) the row is $C = 1$, $W = 1$, $V = 0$, and it gives $H = 0$: the heater is **off**

- this is sensible because the heat would escape through the open window, so heating would waste energy