3.2.1 Drawing logic circuits, three-input truth tables and reading an expression back out

Name: ______________ Class: __________ Date: ____________ **Total: 31 marks**

KEY WORDS logic circuit 逻辑电路 • truth table 真值表 • working column 工作列 • ram 随机存取存储器 • rom 只读存储器

Objective

Build the skills to answer the **logic circuit 逻辑电路** questions that sheet 3.2 does not reach. That sheet names the six gates and writes short expressions; this one is about the three things the syllabus asks you to **construct** and that carry most of the marks: **drawing a circuit**, filling a three-input **truth table 真值表** with a **working column 工作列** for each gate, and reading a **logic expression** back out of a completed truth table.

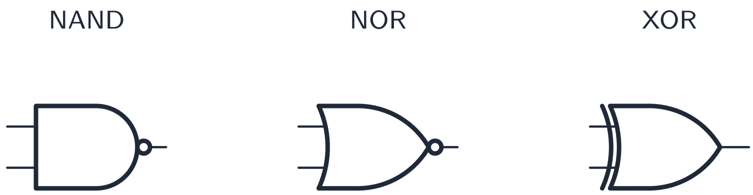
You must be able to:

- construct a **logic circuit** from a logic expression, from a problem statement, and from a truth table
- construct a **truth table** from a logic circuit, using a working column for each gate's output
- construct a **logic expression** from a truth table, from a circuit, and from a problem statement
- use **NAND**, **NOR** and **XOR** inside a larger circuit

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

■ From an expression to a circuit

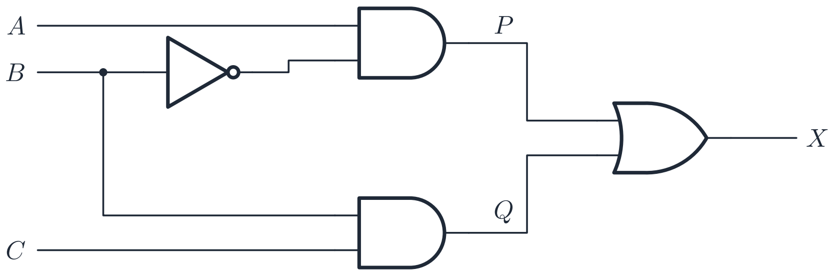


Three rules, and they settle every drawing question:

1. **One gate per operator.** Count the operators in the expression and you know how many gates to draw.
2. **Innermost brackets first.** Whatever is inside a bracket is computed before it can be fed into the gate outside it, so it sits to the **left** in the drawing.
3. **A variable used twice is still one input.** Draw a single wire from that input and put a **junction dot** where it branches. Two separate wires labelled B is the drawing error the mark scheme looks for.

Worked example. *Draw the circuit for $X = (A\ AND\ NOT\ B)\ OR\ (B\ AND\ C)$.*

Four operators, so four gates: a NOT, two ANDs and an OR. B appears twice, so it is one input with a junction.



- **Label each gate's output** (P, Q) as you draw. It costs nothing and it is what makes the truth table below easy.

■ A three-input truth table, with working columns

Three inputs means $2^3 = \mathbf{8}$ rows. Write them by **counting up in binary** from `000` to `111` – that way none is missed and none repeats.

Then add one column per gate. Work left to right across the circuit, filling a whole column at a time rather than a whole row.

A	B	C	$P = A$ **AND NOT** B	$Q = B$ **AND** C	$X = P$ **OR** Q
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	1	**1**
1	0	0	1	0	**1**
1	0	1	1	0	**1**
1	1	0	0	0	0
1	1	1	0	1	**1**

- The working columns are worth marks of their own when the question prints a **working space**. Fill it in: a wrong final column with correct working usually still earns something, and a right answer with nothing shown can earn nothing on a "show your working" question.

From a truth table to an expression

Look only at the rows where $X = 1$. Each one becomes a term:

- write each input that is **1** as itself, and each input that is **0** as **NOT** that input,
- **AND** the three together,
- then **OR** all the terms from all the 1-rows.

Worked example. *A truth table gives $X = 1$ on the rows $ABC = 001$, 010 and 111, and 0 everywhere else.*

$$X = (\text{NOT } A \text{ AND NOT } B \text{ AND } C) \text{ OR } (\text{NOT } A \text{ AND } B \text{ AND NOT } C) \text{ OR } (A \text{ AND } B \text{ AND } C)$$

- One term per 1-row, and every term carries **all three** inputs. Leaving one out changes the expression.
- Rows where $X = 0$ contribute nothing. Do not write terms for them.

From a problem statement

Read the sentence and translate it piece by piece: each **condition** becomes an input, each **“and”** an AND gate, each **“or”** an OR gate, and each **“not”**, **“is off”** or **“is 0”** a NOT gate. Keep the order of the sentence, and bracket whatever the sentence groups together.

”A pump runs when the tank is not full and the switch is on” becomes $X = (\text{NOT } A)$ AND B, then one NOT gate feeding one AND gate.

2 Practice

Now use the methods above.

2.1 Draw the logic circuit for $X = (A \text{ OR } B) \text{ AND } (\text{NOT } C)$. [3]

2.2 Complete the truth table for the circuit in 2.1. Use the working columns. [3]

A	B	C	A **OR** B	**NOT** C	X
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

2.3 Name the **single** gate that does the same job as each of these. [2]

(a) NOT (A AND B)

(b) NOT (A OR B)

2.4 A truth table gives $X = 1$ on the rows $ABC = 001$ and 110, and $X = 0$ on every other row.

Write the logic expression for X. [2]

2.5 A circuit uses input B as an input to two different gates. State how this is shown on the circuit diagram. [1]

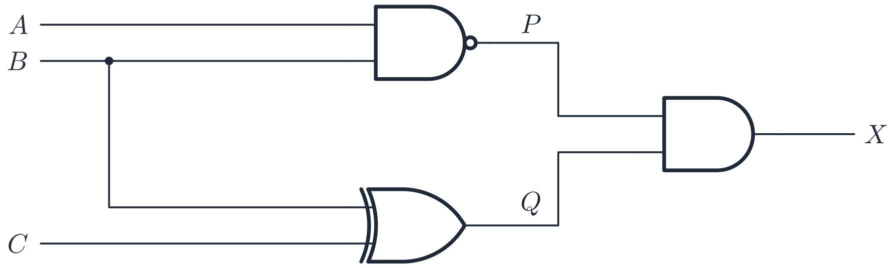
3 Exam-style questions

3.1 A chemical tank has three sensors. An alarm X sounds when the temperature sensor A reads 1 **and** the pressure sensor B reads 0, **or** when the manual override C reads 1.

(a) Write the logic expression for X. [2]

(b) Draw the logic circuit for X. [3]

3.2 A logic circuit is shown. P and Q are the outputs of the first two gates.



(a) Complete the truth table, filling the working columns P and Q. [4]

A	B	C	P	Q	X
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

(b) State the values of A, B and C for which $X = 1$. [2]

3.3 A truth table gives $X = 1$ on the rows $ABC = 000$, 011 and 110, and $X = 0$ on every other row.

Write the logic expression for X. [3]

3.4 Draw the logic circuit for $X = (A \text{ XOR } B) \text{ AND } (\text{NOT } C)$. [3]

3.5 A student is asked to draw the circuit for $X = A \text{ AND } (B \text{ OR } C)$, but wires it as $(A \text{ AND } B) \text{ OR } C$.

Explain, naming **one** row of the truth table, how the two circuits behave differently. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- an OR gate with A and B as its inputs
- a NOT gate with C as its input
- both outputs feeding an AND gate, whose output is labelled X
- the OR and the NOT must both come **before** the AND: the brackets say so

2.2

- A OR B column correct: 0, 0, 1, 1, 1, 1, 1, 1
- NOT C column correct: 1, 0, 1, 0, 1, 0, 1, 0
- X column correct

A	B	C	A **OR** B	**NOT** C	X
0	0	0	0	1	0
0	0	1	0	0	0
0	1	0	1	1	**1**
0	1	1	1	0	0
1	0	0	1	1	**1**
1	0	1	1	0	0
1	1	0	1	1	**1**
1	1	1	1	0	0

- $X = 1$ only when $C = 0$ and at least one of A, B is 1

2.3

(a) **NAND**

(b) **NOR**

2.4

- row 001 gives NOT A AND NOT B AND C; row 110 gives A AND B AND NOT C
- X = (NOT A AND NOT B AND C) OR (A AND B AND NOT C)

2.5

- the wire from B **branches**, with a **junction dot** drawn at the branch point; B is still a single input

3.1

(a) "A reads 1 and B reads 0" is A AND NOT B

- "or C reads 1" adds an OR with C: X = (A AND NOT B) OR C

(b) a NOT gate with B as its input

- an AND gate taking A and the NOT gate's output

- an OR gate taking that AND output and C directly, giving X
- C goes straight into the OR gate: nothing in the sentence acts on it first

3.2

(a) P column correct – P is NOT (A AND B), so it is 1 everywhere except the last two rows

- Q column correct – Q is 1 when B and C **differ**
- X column correct, $X = P$ AND Q

A	B	C	P	Q	X
0	0	0	1	0	0
0	0	1	1	1	**1**
0	1	0	1	1	**1**
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	1	**1**
1	1	0	0	1	0
1	1	1	0	0	0

(b) $X = 1$ on $ABC = 001$, 010 and 101

- in words: B and C must differ, and A and B must not both be 1

3.3

- row 000: NOT A AND NOT B AND NOT C
- row 011: NOT A AND B AND C
- row 110: A AND B AND NOT C, and the three ORed together

$$X = (\text{NOT } A \text{ AND NOT } B \text{ AND NOT } C) \text{ OR } (\text{NOT } A \text{ AND } B \text{ AND } C) \text{ OR } (A \text{ AND } B \text{ AND NOT}$$

3.4

- an XOR gate with A and B as its inputs
- a NOT gate with C as its input
- both feeding an AND gate whose output is X
- the XOR symbol is the OR shape with a **second curved line** across its back; drawing a plain OR loses the first mark

3.5

- the brackets change which gate comes first: A AND (B OR C) ORs B and C **before** the AND, while (A AND B) OR C ANDs first and lets C reach the output on its own
- so in the second circuit $C = 1$ forces $X = 1$ whatever A is, which the first circuit does not do
- any row with $A = 0$ and $C = 1$ shows it, for example $A = 0$, $B = 0$, $C = 1$: the correct circuit gives $X = 0$ and the student's gives $X = 1$

- $A = 0$, $B = 1$, $C = 1$ is also accepted; a row where the two agree proves nothing