A-LEVEL COMPUTER SCIENCE • EXERCISE SHEET

19.1 Algorithms

Name: _____ Class: _____ Date: _____ **Total: 31 marks**

KEY WORDS bubble sort 冒泡排序 • binary search 二分查找 • insertion sort 插入排序 • linear search 线性查找 • array 数组

Objective

Build the skills to answer exam questions on **algorithms** — linear and binary search, bubble and insertion sort, and comparing algorithms.

You must be able to:

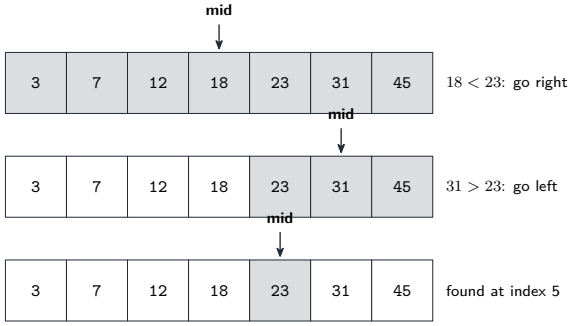
- trace a **linear** and a **binary** search
- trace a **bubble sort** and an **insertion sort**
- compare algorithms by **time** and **memory** (Big-O)

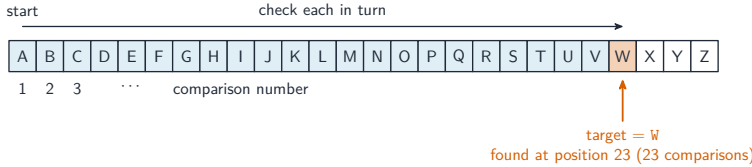
1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

Searching

A **linear search** checks each element in turn — works on **any** list, worst case **O(n)**. A **binary search** needs the data **sorted**: check the middle, then keep the half that could contain the target, halving the range each step — worst case **O(log n)**:





Linear search compared with binary search

Bubble sort

Repeatedly walk the array, **swapping** adjacent out-of-order pairs, so the largest "bubbles" to the end each pass:

```
FOR pass ← 1 TO n - 1
    FOR i ← 1 TO n - pass
        IF A[i] > A[i+1] THEN swap A[i], A[i+1]
    NEXT i
NEXT pass
```

Insertion sort

Build a sorted prefix; insert each new element by **shifting larger ones right**:

```
FOR i ← 2 TO n
    key ← A[i]
    j ← i - 1
    WHILE j >= 1 AND A[j] > key DO
        A[j+1] ← A[j]; j ← j - 1
    ENDWHILE
    A[j+1] ← key
NEXT i
```

Both are **O(n²)** worst case; both are **O(n)** on already-sorted data (with an early exit / no shifts). Compare algorithms by **time taken** and **memory used**.

2 Practice

Now apply the methods above.

2.1 State the **precondition** that a binary search needs. [1]

2.2 State the **worst-case** time complexity of a **linear** search and a **binary** search. [2]

2.3 In a **bubble sort**, describe what happens during **one pass**. [1]

2.4 State **one** criterion used to compare two algorithms. [1]

3 Exam-style questions

3.1 Trace a **binary search** for 23 in `[3, 7, 12, 18, 23, 31, 45]` (indices 1–7). State the **index** examined at each step. [3]

3.2 The array `[5, 2, 8, 1]` is sorted with a **bubble sort**. Show the array **after the first complete pass**. [2]

3.3 State **two** advantages of a binary search over a linear search, and **one** disadvantage. [3]

3.4 State why an **insertion sort** is efficient on a list that is **already nearly sorted**. [1]

3.5 The algorithm below is intended to find the position of the first negative value in an array `Data` of 10 integers, or to output a message if there is none.

```
Found ← FALSE
Index ← 1
FOR Index ← 1 TO 10
    IF Data[Index] < 0
      THEN
        Found ← TRUE
        Position ← Index
    ENDIF
NEXT Index
IF Found = TRUE
  THEN OUTPUT Position
  ELSE OUTPUT "None found"
ENDIF
```

The array holds: `8, 3, -6, 11, -2, 7, 4, 9, 1, 5`.

(a) **Complete** the trace table by dry running the algorithm for the first **five** iterations. [4]

Index	`Data[Index]`	Found	Position

(b) **State** the value the algorithm outputs, and **explain** why it is **not** the position of the *first* negative value. [3]

(c) The outer loop structure is **not** the most appropriate one for this task. **Explain** why, and **name** the loop you would use instead. [3]

(d) **Write** corrected pseudocode using that loop, so the algorithm stops as soon as it finds the first negative value. [4]

(e) **Calculate** how many comparisons your corrected version makes on this data, and how many the original makes. **Describe** what this shows about the two loop choices when the target is near the start. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the data must be **sorted** (in order)

2.2

- linear search: **O(n)**
- binary search: **O(log n)**

2.3

- adjacent pairs are compared along the array and **swapped if out of order**, moving the largest remaining value to the end

2.4

- any one: **time taken** (number of operations); **memory used**

3.1

- mid = index **4** (value 18, $< 23 \rightarrow$ go right)
- mid = index **6** (value 31, $> 23 \rightarrow$ go left)
- mid = index **5** (value 23 $\rightarrow$ found)

3.2

- first pass: `5`↔`2` → `[2,5,8,1]`; `5,8` ok; `8`↔`1` → `[2,5,1,8]`
- **`[2, 5, 1, 8]`**

3.3

- advantages (any two): **far fewer comparisons** on large lists (O(log n) vs O(n)); much faster when searching repeatedly
- disadvantage: the data must be **sorted first** (a one-off cost)

3.4

- most elements are already in place, so the inner `WHILE` does **few or no shifts** — close to O(n)

3.5

(a) rows for `Index` 1 to 5: `Data[Index]` $= 8, 3, -6, 11, -2$

- `Found` stays `FALSE` until `Index` = 3, then `TRUE`
- `Position` is unset until `Index` = 3, when it becomes 3, then becomes 5 at `Index` = 5

(b) it outputs **5**

- the `FOR` loop always runs all ten times

- so every later negative value **overwrites** `Position`, leaving the position of the **last** one rather than the first

(c) a `FOR` loop is a **count-controlled** loop and cannot stop early, but here the number of iterations is not known in advance — it depends on the data

- a **condition-controlled** loop is appropriate: a `WHILE` loop

(d)

```
Found ← FALSE
Index ← 1
WHILE Index <= 10 AND Found = FALSE
    IF Data[Index] < 0
      THEN
        Found ← TRUE
        Position ← Index
      ELSE
        Index ← Index + 1
    ENDIF
ENDWHILE
IF Found = TRUE
  THEN OUTPUT Position
  ELSE OUTPUT "None found"
ENDIF
```

- `WHILE` with both conditions; the flag set and the index advanced correctly; the loop cannot run past element 10; output unchanged and correct

(e) the corrected version stops at the third element, so it makes **3** comparisons

- the original always makes **10**
- a condition-controlled loop can leave as soon as the answer is known, so on data where the target appears early it does a fraction of the work; a count-controlled loop pays the full cost every time