

15.2 Boolean Algebra and Logic Circuits

Name: _____ Class: _____ Date: _____

Total: 17 marks

Objective

Build the skills to answer exam questions on **Boolean algebra and logic circuits** — simplification, Karnaugh maps, adders and flip-flops.

You must be able to:

- simplify **Boolean** expressions (De Morgan, absorption)
- use a **Karnaugh map** to simplify a truth table
- give the truth tables of a **half/full adder** and describe **SR/JK flip-flops**

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Boolean algebra

$+$ = OR, $\cdot$ = AND, overbar = NOT. Useful laws:

- **De Morgan:** $\overline{A + B} = \overline{A} \cdot \overline{B}$ and $\overline{A \cdot B} = \overline{A} + \overline{B}$
- **absorption:** $A + AB = A$
- $A + \overline{A} = 1$, so $Z = AB + A\overline{B} = A(B + \overline{B}) = A$.

Fewer terms $\rightarrow$ **fewer gates**.

■ Sum-of-products from a truth table

For **each row** where the output is 1, AND the inputs (a variable as itself if it is 1, or NOT it if 0); then OR these product terms together. If the 1s are at rows $\overline{A}\overline{B}C$ and ABC :

$$Z = \overline{A}\overline{B}C + ABC = AC(\overline{B} + B) = AC.$$

This is the **sum-of-products (SOP)** form; simplify it with Boolean algebra or a Karnaugh map.

■ Karnaugh map

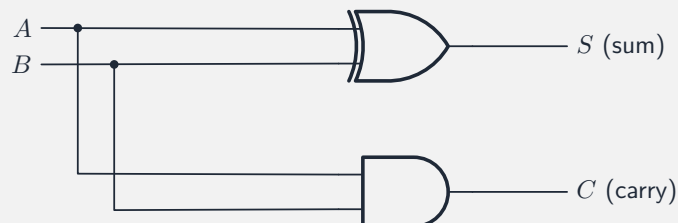
Place each truth-table 1 in the map (Gray-code order), then **group adjacent 1s** in blocks of 1, 2, 4 or 8. A larger group = a simpler term —variables that **change** within the group disappear:

		AB				
		00	01	11	10	
C	00	1	1	0	0	group of 4 = $\overline{A}\overline{C}$
	01	1	1	0	0	
	11	0	0	0	0	
	10	0	0	0	0	

A group of 2 drops 1 variable; a group of 4 drops **2**; a group of 8 drops 3.

■ Half adder and flip-flops

A **half adder** gives $S = A \oplus B$ and $C = A \cdot B$:



A **full adder** also adds a carry-in. A **flip-flop** stores **one bit**: an **SR** flip-flop sets/resets/holds ($S=1, R=1$ is invalid); a **JK** uses 1,1 to **toggle**, so it suits counters.

2 Practice

Now apply the methods above.

2.1 Simplify $A + AB$.

[1]

2.2 Use **De Morgan's law** to rewrite $\overline{A \cdot B}$. [1]

2.3 State the **sum** and **carry** outputs of a half adder as Boolean expressions. [2]

2.4 State what a **flip-flop** stores. [1]

3 Exam-style questions

3.1 Simplify $Z = AB + A\overline{B}$ using Boolean algebra. Show your steps. [2]

3.2 A K-map has four 1s forming a group where $A = 0$ and $C = 0$ throughout. Write the simplified term, and state how many variables a **group of 4** removes. [2]

3.3 Complete the truth table for a **half adder**. [3]

A	B	S	C
0	0		
0	1		
1	0		
1	1		

3.4 State **one** difference between an **SR** flip-flop and a **JK** flip-flop. [2]

3.5 A two-input logic circuit gives output 1 only on the rows $A=0$, $B=1$ and $A=1$, $B=1$.

Write the Boolean expression as a **sum-of-products**, then **simplify** it.

[3]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **15.2 Boolean Algebra and Logic Circuits** past-paper sheet in the Library, or try these in **Practice mode**:

- 9618/32 J25 —Q7 (Karnaugh map and sum-of-products)
- 9618/33 J25 —Q3 (Boolean simplification)
- 9618/32 N24 —Q6 (logic circuit and truth table)
- 9618/33 N25 —Q6 (Boolean algebra)

Solutions

2.1 $A + AB = A$ (absorption law).

2.2 $\overline{A \cdot B} = \overline{A} + \overline{B}$.

2.3 $S = A \oplus B$ (A XOR B); $C = A \cdot B$ (A AND B).

2.4 One bit (it is bistable —it remembers its state).

3.1 $Z = AB + A\overline{B} = A(B + \overline{B}) = A \cdot 1 = A$.

3.2 Term = $\overline{A}\overline{C}$; a group of 4 removes **2** variables.

3.3

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

3.4 In an SR flip-flop the input **S=1, R=1** is **invalid**; a JK flip-flop uses **J=1, K=1** to **toggle** (flip) the output —so JK suits counters.

3.5 SOP: $Z = \overline{A} \cdot B + A \cdot B; = B(\overline{A} + A) = B \cdot 1 = B$.