A-LEVEL COMPUTER SCIENCE • EXERCISE SHEET

13.3 Floating-point numbers, representation and manipulation

Name: _____ Class: _____ Date: _____ **Total: 21 marks**

KEY WORDS exponent 指数 • mantissa 尾数 • floating-point 浮点 • two's complement 补码 • floating-point numbers 浮点数

Objective

Build the skills to answer exam questions on **floating-point numbers** 浮点数 — the format, conversions, normalisation and rounding error.

You must be able to:

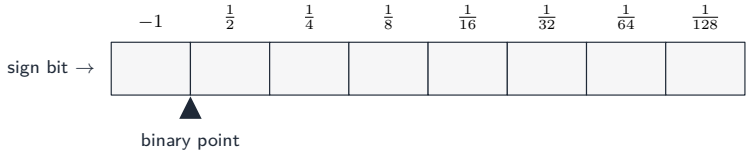
- describe the **mantissa** + **exponent** format
- convert **binary** ↔ **denary** floating-point
- **normalise** a number and explain **rounding errors**

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

The format

A floating-point number is mantissa $\times 2^{\text{exponent}}$. Both are stored in **two's complement**. The mantissa is a fraction with the binary point just after the **sign bit**:



Binary → denary

Mantissa `01010000`, exponent `00000010`:

- sign bit `0` → positive; fraction `.1010000` $= \frac{1}{2} + \frac{1}{8} = 0.625$
- exponent `00000010` $= +2$
- value $= 0.625 \times 2^2 = 0.625 \times 4 =$ **2.5**

Denary → binary (normalised)

Convert `6.0` (8-bit mantissa, 8-bit exponent):

- `6` in binary $= 110.0$
- move the point left until it is just after the first `1`: 0.110×2^3
- mantissa = `01100000`, exponent $= 3 =$ `00000011`

Normalising

A positive mantissa is **normalised** when it begins `0.1…` (no wasted leading zeros — maximum precision). Shift left and decrease the exponent:

- `00011000` exp `00000101` $= 0.0011 \times 2^5$
- shift mantissa left 2 → `01100000`, exponent $5 - 2 = 3 =$ `00000011` (same value).

Negative numbers

A **negative** mantissa is in **two's complement** and is normalised when it begins `1.0…` (the first two bits differ). To represent -0.75: take $+0.75 =$ `0.1100000`, then two's-complement it (invert, add 1) → `1.0100000`. The sign/integer bit has place value -1, so `1.0100000` $= -1 + \frac{1}{4} = -0.75$.

Rounding error

Some denary reals (like 0.1) are **repeating** binary fractions, so they are stored only **approximately**. Errors build up, so test `ABS(x - 0.3) < 1e-9`, not `x = 0.3`.

2 Practice

Now apply the methods above.

2.1 State the **two** parts of a floating-point number and what each represents. [2]

2.2 A mantissa `01000000` has exponent `00000011`. State its **denary** value. [2]

2.3 State what it means for a number to be **normalised**. [1]

2.4 State **one** reason some denary real numbers cannot be stored exactly. [1]

3 Exam-style questions

A system uses an **8-bit mantissa** and an **8-bit exponent**, both in two's complement.

3.1 Convert mantissa `01010000`, exponent `00000010` to **denary**. Show your working. [3]

3.2 Convert `6.0` to its **normalised** floating-point representation. Show your working. [4]

3.3 Normalise the mantissa `00011000` with exponent `00000101`. Give the new mantissa and exponent. [3]

3.4 Explain why `0.1 + 0.2` might **not** equal exactly `0.3` when stored in floating-point. [2]

3.5 Convert the floating-point number with mantissa `10110000` and exponent `00000010` to **denary**. Show your working. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the **mantissa** holds the **significant digits**
- the **exponent** is the **power of 2** to multiply by

2.2

- `0.1000000` = 0.5; exponent = 3
- $0.5 \times 2^3 = \mathbf{4}$

2.3

- the first significant bit is **immediately after the binary point** (`0.1…` for a positive number) — no wasted leading zeros, so precision is maximised

2.4

- some denary fractions (e.g. 0.1) are **repeating/infinite** in binary, so they must be **truncated** to fit the mantissa

3.1

- sign bit `0` → positive
- `.1010000` = $\frac{1}{2} + \frac{1}{8} = 0.625$, exponent = 2
- $0.625 \times 2^2 = \mathbf{2.5}$

3.2

- `6` = 110.0_2
- normalise to 0.110×2^3
- mantissa `01100000`
- exponent `00000011`

3.3

- `00011000` = 0.0011×2^5; shift mantissa **left 2**, exponent $5 - 2 = 3$
- new mantissa `01100000`
- new exponent `00000011`

3.4

- `0.1` and `0.2` cannot be stored **exactly** in binary (they are repeating fractions)
- the small stored errors **add up**, so the result is very close to but **not exactly** `0.3`

3.5

- sign bit `1` → negative
- `1.0110000` = $-1 + \frac{1}{4} + \frac{1}{8} = -0.625$, exponent = 2
- $-0.625 \times 2^2 = -\mathbf{2.5}$