A-LEVEL COMPUTER SCIENCE • EXERCISE SHEET

10.2.1 Bubble sort, array bounds and the standard array routines

Name: _____ Class: _____ Date: _____ **Total: 30 marks**

KEY WORDS index 索引 • bubble sort 冒泡排序 • lower bound 下界 • upper bound 上界 • linear search 线性查找

Objective

Build the skills to answer the **array** 数组 questions that sheet 10.2 does not reach. That sheet declares 1-D and 2-D arrays and processes them with a **linear search** 线性查找 and a running maximum; this one covers the algorithm the syllabus names alongside it and that sheet never shows – the **bubble sort** 冒泡排序 – together with counting elements from the **bounds** 边界, choosing 1-D or 2-D for a task, and the standard routines that move data about inside an array.

You must be able to:

- use the technical terms associated with arrays, including **index** 索引, **lower bound** 下界 and **upper bound** 上界
- select a suitable data structure, a 1-D or a 2-D array, for a given task
- write pseudocode for 1-D and 2-D arrays
- write pseudocode to process array data: sort using a bubble sort, search using a linear search

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

■ Bounds, and how many elements there are

In `DECLARE Data : ARRAY[1:120] OF REAL`, the **lower bound** is 1 and the **upper bound** is 120. The count is

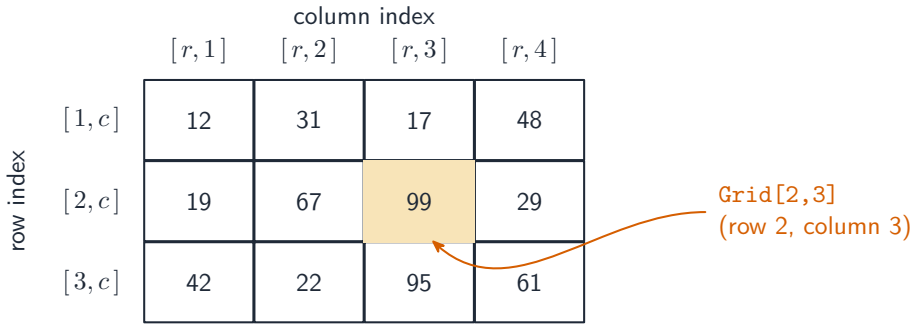
$$\text{upper bound} - \text{lower bound} + 1$$

so this array holds **120** elements. The "+1" is what catches people out: `ARRAY[0:8]` holds **9**, not 8, and `ARRAY[5:14]` holds **10**.

For a 2-D array, multiply the two counts: `ARRAY[1:150, 1:2] OF STRING` is $150 \times 2 =$ **300** elements, and `ARRAY[0:4, 0:5]` is $5 \times 6 =$ **30**.

- A declaration needs the **bounds and the data type**. Give both, or the mark is not there.

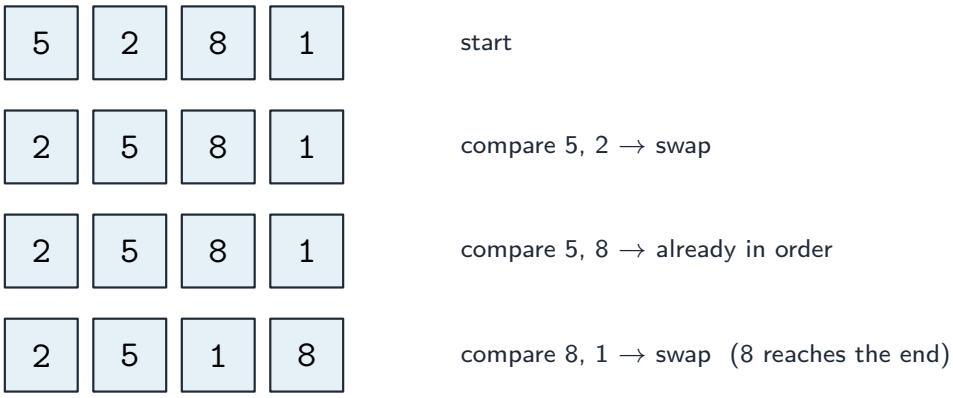
1-D or 2-D?



- Use **1-D** for a single sequence of values: the marks of 30 students, the readings from one sensor.
- Use **2-D** where the data has **two natural dimensions**: a grid, or rows against columns – a seating plan, a noughts-and-crosses board, monthly sales for each of several shops.
- The first index is the **row**, the second the **column**. Visit every cell with **nested** loops, the outer over rows and the inner over columns. To search one row only, fix the row index and loop over the column.

An array can also replace a chain of selection statements: `DaysInMonth[Month]` looks the answer up directly instead of twelve `IF` clauses, which is shorter to write, faster to read and easier to maintain.

The bubble sort, one pass at a time



A bubble sort compares **adjacent pairs** and swaps any that are out of order. After one pass, the largest value has reached the end – which is the fact the efficient version exploits.

Worked example. *Sort* `7, 3, 9, 2`.

After pass	Array	Swaps in that pass
1	`3, 7, 2, 9`	2
2	`3, 2, 7, 9`	1
3	`2, 3, 7, 9`	1
4	`2, 3, 7, 9`	0 – so stop

```
REPEAT
    Swapped <- FALSE
    FOR Index <- 1 TO Limit - 1
        IF Data[Index] > Data[Index + 1] THEN
            Temp <- Data[Index]
            Data[Index] <- Data[Index + 1]
            Data[Index + 1] <- Temp
            Swapped <- TRUE
        ENDIF
    NEXT Index
    Limit <- Limit - 1
UNTIL Swapped = FALSE
```

The marks are given for four separate things, so make each one visible:

1. the **outer loop** that repeats until a pass makes no swap,
2. the **`Swapped` flag**, set inside the `IF`,
3. the **three-line swap** through a temporary variable – `Data[Index] <- Data[Index + 1]` on its own destroys a value,
4. the **shrinking limit**, because after each pass one more value is already in place.

The same sort, in steps

When a question says *do not use pseudocode*, write it as steps:

1. Repeat steps 2 to 4 until a pass makes no swaps.
2. Compare each adjacent pair of elements in turn, from the start.
3. If a pair is out of order, swap the two values.
4. After each pass the largest unsorted value has reached the end, so the next pass can stop one place earlier.

The routines that move data about

- **Position of the largest.** Set `Largest` to the first element and `Position` to 1; for each remaining element, if it is greater than `Largest`, store both the value and the index. Using a strict `>` keeps the **first** of two equal maxima; `>=` would keep the last.
- **Linear search returning a position.** Set `FoundAt <- -1` before the loop – a value that can never be a valid index – so after the loop `FoundAt = -1` means "not found". Leave the loop as soon as it matches.

- **Remove the element at index `k`.** Move every later element one place **towards the start**, so the gap closes, then mark the last element as unused. Removing index 2 from `4, 9, 2, 7, 5` gives `4, 2, 7, 5, unused`.
- **Insert into a sorted array.** Find the first index whose element is **larger** than the new value, move that element and every later one one place **towards the end**, and store the new value in the gap. Inserting `6` into `2, 4, 8, 9`: the first larger element is `8` at index 3, so the result is `2, 4, 6, 8, 9`.
- For a remove, copy **forwards** through the array; for an insert, copy **backwards** from the end. Going the wrong way overwrites the values you have not moved yet.

2 Practice

Now use the methods above.

2.1 State how many elements each of these arrays holds. [3]

(a) `DECLARE Data : ARRAY[1:120] OF REAL`

(b) `DECLARE Flags : ARRAY[0:8] OF BOOLEAN`

(c) `DECLARE Table : ARRAY[1:150, 1:2] OF STRING`

2.2 An array is declared as `ARRAY[5:14] OF INTEGER`. State its **lower bound**, its **upper bound**, and the number of elements. [2]

2.3 For each task, state whether a **1-D** or a **2-D** array is the better choice, and give a reason. [2]

(a) the marks of 30 students in one test

(b) a noughts-and-crosses board

2.4 An array holds `7, 3, 9, 2`. Write the contents of the array after **one complete pass** of a bubble sort. [2]

2.5 Before a linear search, a programmer sets `FoundAt <- -1` rather than `0`. Explain why. [2]

3 Exam-style questions

3.1 A 1-D array `Data[1:n]` holds `n` integers. Write pseudocode to sort it into ascending order using a **bubble sort**. [5]

3.2 The sort in 3.1 always makes the same number of passes, whatever the data.

(a) Rewrite it so that it stops as soon as a pass makes no swap, and so that each pass examines one fewer element. [3]

(b) Explain what **each** of those two changes saves. [2]

3.3 Describe a bubble sort as a series of steps. Do **not** use pseudocode. [4]

3.4 A 1-D array `Stock[1:100]` holds product codes as strings. An unused element holds `""`.

Write pseudocode for a procedure `Remove(Position)` that deletes the element at `Position` by moving every later element one place towards the start, and marks the last element as unused. [5]

Solutions

Each answer shows the steps, then the **result**.

2.1

(a) $120 - 1 + 1 =$ **120**

(b) $8 - 0 + 1 =$ **9** – not 8

(c) $150 \times 2 =$ **300**

2.2

- lower bound **5**, upper bound **14**
- $14 - 5 + 1 =$ **10** elements

2.3

(a) **1-D** – the marks are a single sequence of values, one per student

(b) **2-D** – the board has two natural dimensions, three rows by three columns

2.4

- compare and swap each adjacent pair in turn: `7, 3` swap; `7, 9` in order; `9, 2` swap
- after one pass: **`3, 7, 2, 9`**
- the largest value, `9`, has reached the end

2.5

- a valid index can be `0` in an array whose lower bound is `0`, so `0` could be mistaken for a genuine result
- `-1` can never be a valid index, so after the loop `FoundAt = -1` unambiguously means "not found"

3.1

- an outer loop that repeats the passes
- an inner loop over the adjacent pairs
- the comparison `Data[Index] > Data[Index + 1]`
- the swap through a **temporary** variable, all three lines
- the loop ends correctly, leaving the array sorted

```
DECLARE Index, Pass, Temp : INTEGER
FOR Pass <- 1 TO n - 1
    FOR Index <- 1 TO n - 1
        IF Data[Index] > Data[Index + 1] THEN
            Temp <- Data[Index]
            Data[Index] <- Data[Index + 1]
            Data[Index + 1] <- Temp
        ENDIF
    NEXT Index
NEXT Pass
```

3.2

(a) a `Swapped` flag, set `FALSE` at the start of each pass and `TRUE` inside the `IF`

- the outer loop becomes `REPEAT ... UNTIL Swapped = FALSE`
- a `Limit` that decreases by one after each pass, with the inner loop running to `Limit - 1`

```
Limit <- n
REPEAT
    Swapped <- FALSE
    FOR Index <- 1 TO Limit - 1
        IF Data[Index] > Data[Index + 1] THEN
            Temp <- Data[Index]
            Data[Index] <- Data[Index + 1]
            Data[Index + 1] <- Temp
            Swapped <- TRUE
        ENDIF
    NEXT Index
    Limit <- Limit - 1
UNTIL Swapped = FALSE
```

(b) the **flag** stops the sort as soon as the array is in order, so an already-sorted array costs one pass instead of $n - 1$

- the **shrinking limit** avoids re-examining the values at the end, which each pass has already put in their final place

3.3 any four, in a sensible order:

- repeat the following until a pass makes no swaps
- compare each adjacent pair of elements in turn, starting at the beginning
- if a pair is out of order, swap the two values
- after each pass the largest unsorted value has reached the end, so the next pass can stop one place earlier
- no pseudocode keywords used (`FOR`, `IF`, `<-` are pseudocode, so they lose the style mark)

3.4

- header with the parameter, and the loop counter declared
- loop from `Position` to the second-from-last element
- `Stock[Index] <- Stock[Index + 1]` – copying **forwards**, so nothing is overwritten before it is moved
- set the last element to `""` after the loop
- `ENDPROCEDURE`, with everything closed

```
PROCEDURE Remove(Position : INTEGER)
    DECLARE Index : INTEGER
    FOR Index <- Position TO 99
        Stock[Index] <- Stock[Index + 1]
    NEXT Index
    Stock[100] <- ""
ENDPROCEDURE
```

- copying backwards here would fill the rest of the array with one repeated value