

1.1 Data Representation

Name: _____ Class: _____ Date: _____

Total: 28 marks

Objective

Build the skills to answer the real exam questions on **data representation** 数据表示—number bases, binary arithmetic, two's complement, BCD, and **character sets** 字符集.

You must be able to:

- convert between **binary** 二进制, **denary** 十进制 and **hexadecimal** 十六进制, and use **BCD** 二进制十进数
- state the **minimum number of bits** needed to store a value
- add 8-bit binary numbers and **name and describe overflow** 溢出
- use **two's complement** 补码: write a negative denary number in binary, read a two's-complement value, and subtract
- represent character data with **ASCII** and **Unicode**, and explain why Unicode is used

1 Worked examples

Study these first. Each one shows the method for an exam question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ Binary → denary

Multiply each bit by **2 raised to its position** (positions count from 0 at the right), then add. This same method works for *any* base —for hexadecimal you use powers of 16.

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	1	1	0	0	1	0

$$\begin{aligned}
 & (1 \times 2^7) + (1 \times 2^5) + (1 \times 2^4) + (1 \times 2^1) \\
 & = 128 + 32 + 16 + 2 = \mathbf{178}
 \end{aligned}$$

To go **denary** → **binary**, divide by 2 again and again; the **remainders** 余数, read from **bottom to top**, are the bits.

$178 \div 2 = 89$	remainder 0	↑ ↑ read up = 10110010
$89 \div 2 = 44$	remainder 1	
$44 \div 2 = 22$	remainder 0	
$22 \div 2 = 11$	remainder 0	
$11 \div 2 = 5$	remainder 1	
$5 \div 2 = 2$	remainder 1	
$2 \div 2 = 1$	remainder 0	
$1 \div 2 = 0$	remainder 1	

■ Hexadecimal

Each hex digit is exactly 4 bits (a **nibble** 半字节): 0–9, then A = 10 up to F = 15. **Hex** → **denary** uses the powers method with base 16: $2F = (2 \times 16^1) + (15 \times 16^0) = 32 + 15 = 47$. **Shortcut binary hex**: group the bits into nibbles of 4 from the right and convert each.



■ Number of bits to store a value

To store the values 0 to N , you need the smallest number of bits b with $2^b > N$. Example: to store 0–200, $2^7 = 128$ is too small but $2^8 = 256 > 200$, so **8 bits**. (An 8-bit byte holds 0–255.)

■ Binary addition and overflow

Add one column at a time from the right; when a column makes 2, write 0 and **carry** a 1.

0101 1100	(92)
+ 0011 0110	(54)
1001 0010 (146)	

Overflow is when the answer is too big for the register (over 255 for 8 bits): a 1 carries out of the leftmost (bit-7) column, so the stored 8-bit result is wrong.

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    1100 1000    (200)
+   0101 0000    (80)
-----
    1 0001 1000    (carry out of bit 7 → overflow)

```

In words: **overflow** is when a calculation gives a result with more bits than the register can hold, so the carry out of the most-significant bit is lost and the stored answer is incorrect.

■ Two's complement: negative numbers

In two's complement the **most significant bit** is the **sign bit** (0 = positive, 1 = negative). For 8 bits the range is -128 to $+127$.

- **Write -18 :** take $+18 = 00010010$, **invert** every bit $\rightarrow 11101101$, **add 1** $\rightarrow 11101110$.
- **Read 11101110 :** the sign bit is 1, so it is negative. Invert $\rightarrow 00010001$, add 1 $\rightarrow 00010010 = 18$, so the value is -18 .
- **Subtract** by adding a negative. $45 - 18$: add 00101101 (45) and 11101110 (-18), then **discard** the carry out of bit 7:

```

    0010 1101    (45)
+   1110 1110   (-18)
-----
    1 0001 1011    (discard carry → 27)

```

■ Binary Coded Decimal (BCD)

BCD stores each denary digit on its own as a 4-bit code —*not* the plain binary of the whole number. For 59: $5 \rightarrow 0101$ and $9 \rightarrow 1001$, so 59 is $0101\ 1001$. Each nibble uses only 0–9; patterns 1010–1111 are invalid.

■ Character sets: ASCII and Unicode

A computer stores text as numbers: each character has a **code point** 码点 fixed by a character set.

- **ASCII** uses **7 bits** $\rightarrow$ 128 characters (basic English letters, digits, punctuation, control codes). Example: 'A' = $65 = 1000001$.
- **Unicode** is a universal set covering (almost) every script plus symbols and emoji, so it needs **more bits per character**.

A character's stored code is just a binary number, so you convert it like any other: the value $0010\ 0111 = 32 + 4 + 2 + 1 = 39$.

Why Unicode? It represents **far more characters** than ASCII (every language, not just English), at the cost of **larger files** for plain English text.

2 Practice

Now apply the methods above.

2.1 Convert binary 01101010 to denary. [1]

2.2 Convert denary 200 to 8-bit binary. [2]

2.3 Convert binary 11110001 to (a) denary and (b) hexadecimal. [2]

2.4 State the **minimum number of bits** needed to store the denary value 500. Show how you decided. [2]

2.5 Represent the denary number 59 in **BCD**. [2]

2.6 Add the 8-bit numbers 01011100 and 00110110. State whether overflow occurs. [2]

3 Exam-style questions

3.1 Add the 8-bit binary numbers 10110100 and 01101101. Show your working, then **name and describe** the error that occurs. [4]

3.2 (a) Convert the hexadecimal number 3D to denary. Show your working. [2]

(b) Convert the denary number 158 to hexadecimal. Show your working. [2]

3.3 (a) Write the denary number -45 as an 8-bit two's complement binary number. Show your working. [2]

(b) The 8-bit two's complement number 11101110 is stored. Calculate the denary value it represents. [2]

3.4 A computer stores text using a character set.

(a) The text is stored using the **Unicode** character set instead of **ASCII**. Give **two** advantages of using Unicode. [2]

(b) A character has the binary code 0010 0111. Convert this code to denary. [1]

3.5 A program stores currency amounts using **BCD** rather than ordinary binary.

(a) State what **BCD** stands for. [1]

(b) Give one **advantage** of using BCD for this purpose. [1]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **1.1 Data Representation** past-paper sheet in the Library, or try these in **Practice mode**:

- 9618/11 N25 —Q1 (binary, hexadecimal, BCD and two's complement)
- 9618/12 N25 —Q6 (8-bit binary addition and overflow)
- 9618/12 J25 —Q2 (number of bits to store data; two's complement)
- 9618/13 N24 —Q8 (character sets and hexadecimal)

Solutions

2.1 $01101010 = 64 + 32 + 8 + 2 = 106$.

2.2 Repeated division by 2 gives remainders, read bottom-up $\rightarrow 11001000$.

2.3 (a) $11110001 = 128 + 64 + 32 + 16 + 1 = 241$.

(b) split into $1111 \mid 0001 = F1$.

2.4 $2^8 = 256$ is too small for 500, but $2^9 = 512 > 500$, so **9 bits**.

2.5 $5 \rightarrow 0101$, $9 \rightarrow 1001$, so $0101 \ 1001$.

2.6 01011100 (92) + 00110110 (54) = 146 = 10010010 ; $146 \leq 255$, so **no overflow**.

3.1 Adding the columns: \

1011 0100	(180)
+ 0110 1101	(109)

0010 0001	(carry out of bit 7)

$180 + 109 = 289 > 255$. The error is **overflow**: the result needs 9 bits but only 8 are stored, so the carry out of the most-significant bit is lost and the stored answer is wrong.

3.2 (a) $3D = (3 \times 16) + 13 = 48 + 13 = 61$.

(b) $158 = (9 \times 16) + 14$, and $14 = E$, so **9E**.

3.3 (a) $+45 = 00101101$; invert $\rightarrow 11010010$; add 1 $\rightarrow 11010011$.

(b) sign bit is 1 $\rightarrow$ negative; invert $11101110 \rightarrow 00010001$; add 1 $\rightarrow 00010010 = 18$; so the value is -18 .

3.4 (a) Any **two**: Unicode can represent characters from (almost) every language / many more characters than ASCII; it includes symbols and emoji; it avoids the regional clashes of extended ASCII.

(b) $0010 \ 0111 = 32 + 4 + 2 + 1 = 39$.

3.5 (a) Binary Coded Decimal.

(b) Each denary digit is stored exactly, so values like money are not changed by rounding errors when converting fractions to binary (or: BCD digits map directly to a display).