Exercise walk-through

Logic Gates and Logic Circuits ▪ 3.2

eXercise

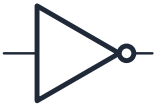
You must be able to

- recognise and define **NOT, AND, OR, NAND, NOR** and **XOR**
- construct the **truth table** **真値表** for any gate or circuit
- build a **logic expression** and a **logic circuit** from a problem statement, and convert between expression, circuit and truth table

Method — The six gates

- **NOT** — output is the opposite of the input.
- **AND** — output 1 only if **all** inputs are 1. **OR** — output 1 if **at least one** input is 1.
- **NAND** = NOT AND; **NOR** = NOT OR (so their outputs are the AND/OR output flipped).
- **XOR** — output 1 only when the inputs are **different**.

Method — The six gates



NOT

AND

OR

NAND

NOR

XOR

Method — Building a truth table

List **every** input combination (2 inputs $\rightarrow 2^2 = 4$ rows; 3 inputs $\rightarrow 2^3 = 8$ rows), then work out the output for each. For example, AND and XOR:

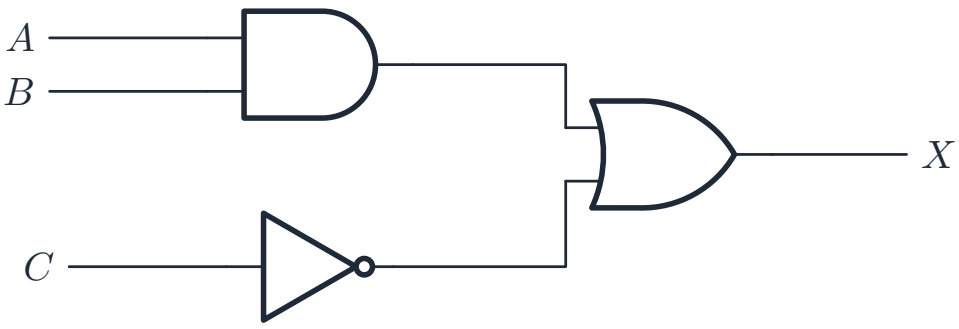
A	B	A AND B	A XOR B
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Method — Circuit, expression and truth table

These three describe the same logic. From this circuit you can read the expression $X = (A \text{ AND } B) \text{ OR } (\text{NOT } C)$, then build the truth table by filling each gate's output column.

A	B	C	A AND B	NOT C	X
0	0	0	0	1	1
0	0	1	0	0	0
0	1	0	0	1	1
0	1	1	0	0	0
1	0	0	0	1	1
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	1	0	1

Method — Circuit, expression and truth table



Practice 2.1 ▪ 2 marks

Complete the truth table for a **NAND** gate.

A	B	A NAND B
0	0	
0	1	
1	0	
1	1	

Solution 2.1

Method: Circuit, expression and truth table

Worked steps

NAND outputs: $1, 1, 1, 0$ (top to bottom) M1 A1 *for 0/1 grouping.*

Practice 2.2 ▪ 1 mark

Name the gate that outputs 1 only when its two inputs are **different**.

Solution 2.2

Method: The six gates

Worked steps

XOR B1.

Practice 2.3 ▪ 2 marks

Write the logic expression for: *"X is 1 when A is 1 OR B is 0."*

Solution 2.3

Method: Circuit, expression and truth table

Worked steps

$X = A$ OR (NOT B) M1 A1 *for the NOT.*

Practice 2.4 ▪ 2 marks

Complete the truth table for $X = A$ AND (NOT B).

A	B	X
0	0	
0	1	
1	0	
1	1	

Solution 2.4

Method: Circuit, expression and truth table

Worked steps

X outputs: $0, 0, 1, 0$ — X is 1 only when $A = 1$ and $B = 0$ M1 A1.

Exam-style 3.1 ▪ 3 marks

A circuit has the expression $X = (A \text{ OR } B) \text{ AND } (\text{NOT } C)$.
(a) Draw the logic circuit for X.
(b) Complete the truth table for X.

A	B	C	X
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Solution 3.1

Method: Circuit, expression and truth table

Worked steps

(a) A and B into an **OR** gate; C into a **NOT** gate; the OR output and the NOT output into an **AND** gate $\rightarrow$ X **M1** **M1** **A1** *OR + NOT, final AND, correct wiring.*
(b) X outputs (top to bottom): $0, 0, 1, 0, 1, 0, 1, 0$ — i.e. X $= 1$ only when (A or B) is 1 **and** C $= 0$ **M1** **M1** **A1**.

Exam-style 3.2 ▪ 2 marks

A pump X turns on when the tank is **not full** $(A = 0)$ **or** the manual switch B is on. Write the logic expression for X.

Solution 3.2

Method: Circuit, expression and truth table

Worked steps

$X = (\text{NOT } A) \text{ OR } B$ **M1** **A1** *for NOT A.*

Exam-style 3.3 ▪ 3 marks

Construct the truth table for A XOR B, and give one use of an XOR gate.

Solution 3.3

Method: Circuit, expression and truth table

Worked steps

A XOR B: $0, 1, 1, 0$ M1 A1. Use: e.g. compare two bits (equal $\rightarrow$ 0), or add two bits in a half-adder B1.

Exam-style 3.4 ▪ 2 marks

Write the Boolean expression for the logic described by this truth table.

A	B	X
0	0	1
0	1	1
1	0	0
1	1	1

Solution 3.4

Method: Circuit, expression and truth table

Worked steps

$X = (\text{NOT } A)$ OR B — X is 0 only on the row $A = 1, B = 0$ **M1** **A1** *identifies that row, correct expression.*