Exercise walk-through

3.2.2

Building logic circuits from expressions, problems and truth tables

A-Level Computer Science

You must be able to

- construct a logic circuit from a logic expression
- construct a logic expression and a logic circuit from a problem statement
- construct a logic expression from a truth table, and a circuit from that expression
- check your circuit with a truth table

1 The method

Method — From an expression to a circuit: three rules

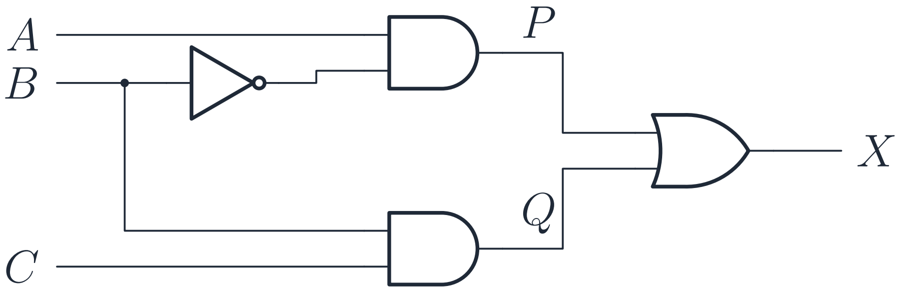
1. **One gate for each operator.** Count the operators (NOT counts too).
2. **Brackets first.** The gate for a bracket is drawn on the **left**, and its output goes into the gate outside the bracket.
3. **An input that is used twice is one input.** Draw one wire, with a **junction dot** where it branches.

$X = (A\ AND\ NOT\ B)\ OR\ (B\ AND\ C)$: four operators, so four gates; B is used twice, so its wire branches.

Draw the correct **shape** of each gate (small circle on NOT, NAND and NOR). Every gate except NOT has **two** inputs. Label the inputs and the output.

Method — From an expression to a circuit: three rules

four operators, so four gates; … is used twice, so its wire branches.



Method — From a problem statement to an expression

Give each condition a letter, and translate the sentence piece by piece.

words in the problem	**operator**
"… **and** …", "both"	AND
"… **or** …", "either"	OR
"is **not**", "is off", "is 0"	NOT
"one or the other **but not both**"	XOR
"**neither** … **nor** …"	NOR

Method — From a problem statement to an expression

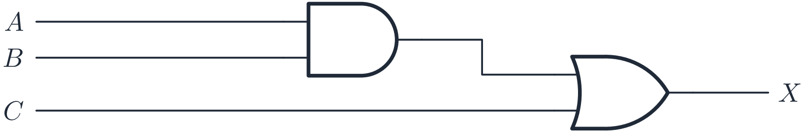
An alarm X sounds when the door is open ($A = 1$) and the system is on ($B = 1$), or when the panic button is pressed ($C = 1$).

- "door open **and** system on" $\rightarrow$ A AND B; "**or** the panic button" $\rightarrow$ OR C.
- $X = (A \text{ AND } B) \text{ OR } C$: two operators, two gates. C goes straight into the OR gate.

Watch the **value** in the problem: "the tank is **not** full ($A = 0$)" is true when A is 0, so write NOT A.

Method — From a problem statement to an expression

Watch the value in the problem: "the tank is not full (...)" is true when ... is 0, so write NOT



Method — From a truth table to an expression

1. Use only the rows where $X = 1$.
2. In a row, write an input that is **1** as itself and an input that is **0** as **NOT** that input; join them with **AND**.
3. Join the terms of all the rows with **OR**.

$X =$ (NOT A AND NOT B AND C) OR (NOT A AND B AND NOT C) OR (A AND B AND

One term for each 1-row, and every term has **all three** inputs. **Check** a circuit or an expression by testing two or three rows against the problem.

2 Practice

Practice 2.1 ▪ 4 marks

Name the logic operator for the words in bold.

Practice 2.1 (a)

The pump runs when the tank is empty **and** the switch is on.

Solution 2.1 (a)

Method: From a problem statement to an expression — p.6

Worked steps

both conditions are needed $\rightarrow$ **AND** B1

Practice 2.1 (b)

The light is on when it is **not** daytime.

Solution 2.1 (b)

Worked steps

the condition must be false $\rightarrow$ **NOT** B1

Practice 2.1 (c)

The door opens when **either** button is pressed.

Solution 2.1 (c)

Worked steps

one is enough, and both is also allowed $\rightarrow$ **OR** B1

Practice 2.1 (d)

The signal is sent when **one or the other, but not both**, of the keys is turned.

Solution 2.1 (d)

Worked steps

exactly one $\rightarrow$ **XOR** B1

Practice 2.2 ▪ 2 marks

A circuit is drawn for $X = (A \text{ OR } B) \text{ AND } (\text{NOT } C)$.
State how many gates the circuit has, and name each gate.

Solution 2.2

Method: From an expression to a circuit: three rules — p.4

Worked steps

- the expression has three operators: OR, AND and NOT, so the circuit has **three** gates B1
- an **OR** gate, a **NOT** gate and an **AND** gate B1

Practice 2.3 ▪ 2 marks

Draw the logic circuit for $X = (A \text{ NOR } B) \text{ AND } C$.

Solution 2.3

Method: From an expression to a circuit: three rules — p.4

Worked steps

- a NOR gate with A and B as its inputs, drawn on the left because it is in the brackets **B1**
- the NOR output and C go into an AND gate, whose output is labelled X **B1**

Practice 2.4 ▪ 2 marks

A light X is on when it is dark $(D = 1)$ **and** the switch is on $(S = 1)$, **or** when the test button is pressed $(T = 1)$.
Write the logic expression for X.

Solution 2.4

Method: From a problem statement to an expression — p.6

Worked steps

- “dark and the switch is on” is D AND S **M1**
- “or the test button” adds OR T: $X = (D \text{ AND } S)$ OR T **A1**

Practice 2.5 ▪ 3 marks

A truth table gives $X = 1$ on the rows $AB = 01$ and 10, and $X = 0$ on the other two rows.

Practice 2.5 (a) ▪ 2 marks

Write the logic expression for X. Use only AND, OR and NOT.

Solution 2.5 (a)

Method: From a problem statement to an expression — p.6

Worked steps

row 01 gives NOT A AND B; row 10 gives A AND NOT B **M1**

- $X = (\text{NOT } A \text{ AND } B) \text{ OR } (A \text{ AND NOT } B)$ **A1**

Practice 2.5 (b) ▪ 1 mark

Name the **single** gate that gives the same output.

Solution 2.5 (b)

Worked steps

the output is 1 when the inputs are different $\rightarrow$ **XOR** **B1**

Practice 2.6 ▪ 1 mark

A circuit uses input B as an input to two different gates. State how this is shown on the circuit diagram.

Solution 2.6

Worked steps

- the wire from B **branches**, and a **junction dot** is drawn where it branches; B is still one input **B1**

3 Exam-style

Exam-style 3.1 ▪ 5 marks

A chemical tank has three sensors. An alarm X sounds when the temperature sensor A reads 1 **and** the pressure sensor B reads 0, **or** when the manual override C reads 1.

Exam-style 3.1 (a) ▪ 2 marks

Write the logic expression for X.

Solution 3.1 (a)

Method: From a problem statement to an expression — p.6

Worked steps

"A reads 1 and B reads 0" is A AND NOT B **B1**

- "or C reads 1" adds OR C: $X = (A \text{ AND NOT } B) \text{ OR } C$ **B1**

Exam-style 3.1 (b) ▪ 3 marks

Draw the logic circuit for X.

Solution 3.1 (b)

Worked steps

a NOT gate with B as its input **B1**

- an AND gate that takes A and the output of the NOT gate **B1**
- an OR gate that takes the AND output and C, and gives X **B1**
- C goes straight into the OR gate: nothing in the sentence acts on it first

Exam-style 3.2 ▪ 3 marks

A truth table gives $X = 1$ on the rows $ABC = 000$, 011 and 110, and $X = 0$ on every other row.
Write the logic expression for X.

Solution 3.2

Method: From a truth table to an expression — p.9

Worked steps

- row 000: NOT A AND NOT B AND NOT C **B1**
- row 011: NOT A AND B AND C **B1**
- row 110: A AND B AND NOT C, and the three terms joined with OR **B1**

$X =$ (NOT A AND NOT B AND NOT C) OR (NOT A AND B AND C) OR (A AND B AND NOT

Exam-style 3.3 ▪ 3 marks

Draw the logic circuit for $X = (A \text{ XOR } B) \text{ AND } (\text{NOT } C)$.

Solution 3.3

Method: From an expression to a circuit: three rules — p.4

Worked steps

- an XOR gate with A and B as its inputs **B1**
- a NOT gate with C as its input **B1**
- both outputs go into an AND gate, whose output is X **B1**
- the XOR symbol is the OR shape with a **second curved line** at its back; a plain OR shape loses the first mark

Exam-style 3.4 ▪ 3 marks

A student is asked to draw the circuit for $X = A \text{ AND } (B \text{ OR } C)$, but wires it as $(A \text{ AND } B) \text{ OR } C$.
Explain, naming **one** row of the truth table, how the two circuits behave differently.

Solution 3.4

Method: From a truth table to an expression — p.9

Worked steps

- the brackets decide which gate comes first: in A AND $(B$ OR $C)$ the OR is worked out first and A must be 1 for X to be 1 **B1**
- in $(A$ AND $B)$ OR C, the input C reaches the output on its own, so $C = 1$ gives $X = 1$ whatever A is **B1**
- any row with $A = 0$ and $C = 1$ shows the difference, for example $A = 0$, $B = 0$, $C = 1$: the correct circuit gives $X = 0$ and the student's circuit gives $X = 1$ **B1**

Exam-style 3.5 ▪ 13 marks

A heater H in a greenhouse is switched on $(H = 1)$ when:

- the greenhouse is cold $(C = 1)$
- **and** either the window is closed $(W = 0)$ **or** the override switch is on $(V = 1)$.

Exam-style 3.5 (a) ▪ 3 marks

Write the logic expression for H.

Solution 3.5 (a)

Method: From a problem statement to an expression — p.6

Worked steps

"the window is closed" is $W = 0$, so write NOT W **B1**

- "either the window is closed or the override is on" is (NOT W) OR V **B1**
- $H = C$ AND ((NOT W) OR V) **B1**

Exam-style 3.5 (b) ▪ 4 marks

Draw the logic circuit for H.

Solution 3.5 (b)

Worked steps

a NOT gate with W as its input **B1**

- an OR gate that takes the NOT output and V **B1**
- an AND gate that takes C and the OR output **B1**
- the output of the AND gate is labelled H, and all three inputs are labelled **B1**

Exam-style 3.5 (c) ▪ 4 marks

Complete the truth table for H.

C	W	V	**Working space**	H
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

Solution 3.5 (c)

Worked steps

working column NOT W: 1, 1, 0, 0, 1, 1, 0, 0 M1

- working column (NOT W) OR V: 1, 1, 0, 1, 1, 1, 0, 1 M1
- AND this column with C, which is 1 only on the last four rows M1
- $H = \mathbf{0, 0, 0, 0, 1, 1, 0, 1}$ A1

Exam-style 3.5 (d) ▪ 2 marks

On a cold day the window is open and the override switch is off. Use your truth table to state whether the heater is on, and explain why this is sensible.

Solution 3.5 (d)

Worked steps

the row is $C = 1$, $W = 1$, $V = 0$, and it gives $H = 0$: the heater is **off** **B1**

- this is sensible because the heat would escape through the open window, so heating would waste energy **B1**