Exercise walk-through

3.2.1

Drawing logic circuits, three-input truth tables and reading an expression back out

A-Level Computer Science

You must be able to

- construct a **logic circuit** from a logic expression, from a problem statement, and from a truth table
- construct a **truth table** from a logic circuit, using a working column for each gate's output
- construct a **logic expression** from a truth table, from a circuit, and from a problem statement
- use **NAND**, **NOR** and **XOR** inside a larger circuit

1 The method

Method — From an expression to a circuit

Three rules, and they settle every drawing question:

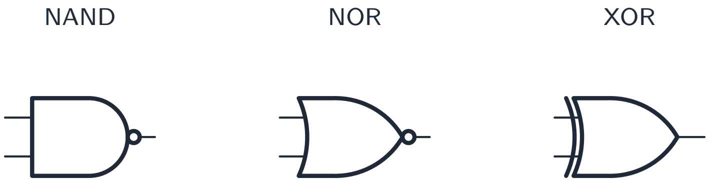
1. **One gate per operator.** Count the operators in the expression and you know how many gates to draw.
2. **Innermost brackets first.** Whatever is inside a bracket is computed before it can be fed into the gate outside it, so it sits to the **left** in the drawing.
3. **A variable used twice is still one input.** Draw a single wire from that input and put a **junction dot** where it branches. Two separate wires labelled B is the drawing error the mark scheme looks for.

Worked example. *Draw the circuit for* $X = (A\ AND\ NOT\ B)\ OR\ (B\ AND\ C)$*.* Four operators, so four gates: a NOT, two ANDs and an OR. B appears twice, so it is one input with a junction.

- **Label each gate's output** (P, Q) as you draw. It costs nothing and it is what makes the truth table below easy.

Method — From an expression to a circuit

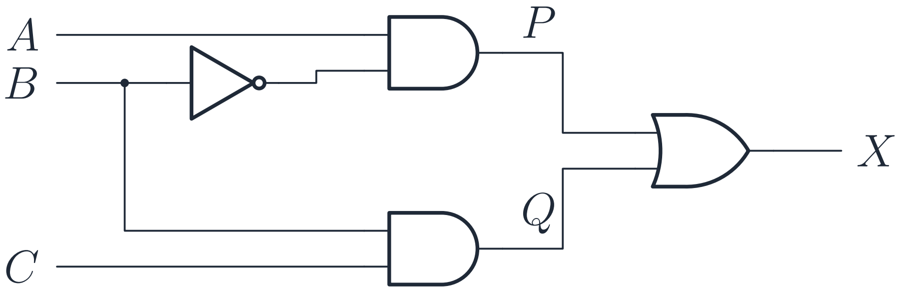
Three rules, and they settle every drawing question: Worked example.



The symbols for the NAND, NOR and XOR gates

Method — From an expression to a circuit

Three rules, and they settle every drawing question: Worked example.



Method — A three-input truth table, with working columns

Three inputs means $2^3 = \mathbf{8}$ rows. Write them by **counting up in binary** from 000 to 111 – that way none is missed and none repeats.
Then add one column per gate. Work left to right across the circuit, filling a whole column at a time rather than a whole row.

A	B	C	$P = A$ **AND NOT** B	$Q = B$ **AND** C	$X = P$ **OR** Q
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	1	**1**
1	0	0	1	0	**1**
1	0	1	1	0	**1**
1	1	0	0	0	0
1	1	1	0	1	**1**

Method — A three-input truth table, with working columns

- The working columns are worth marks of their own when the question prints a **working space**. Fill it in: a wrong final column with correct working usually still earns something, and a right answer with nothing shown can earn nothing on a "show your working" question.

Method — From a truth table to an expression

Look only at the rows where $X = 1$. Each one becomes a term:

- write each input that is **1** as itself, and each input that is **0** as **NOT** that input,
- **AND** the three together,
- then **OR** all the terms from all the 1-rows.

Worked example. *A truth table gives* $X = 1$ *on the rows* $ABC = 001$, 010 *and* 111, *and 0 everywhere else.*

$$X = (\text{NOT } A \text{ AND NOT } B \text{ AND } C) \text{ OR } (\text{NOT } A \text{ AND } B \text{ AND NOT } C) \text{ OR } (A \text{ AND } B \text{ AND}$$

- One term per 1-row, and every term carries **all three** inputs. Leaving one out changes the expression.
- Rows where $X = 0$ contribute nothing. Do not write terms for them.

Method — From a problem statement

Read the sentence and translate it piece by piece: each **condition** becomes an input, each **"and"** an AND gate, each **"or"** an OR gate, and each **"not"**, **"is off"** or **"is 0"** a NOT gate. Keep the order of the sentence, and bracket whatever the sentence groups together.
"A pump runs when the tank is not full and the switch is on" becomes
$X = (\text{NOT } A) \text{ AND } B$, then one NOT gate feeding one AND gate.

2 Practice

Practice 2.1 ▪ 3 marks

Draw the logic circuit for $X = (A \text{ OR } B) \text{ AND } (\text{NOT } C)$.

Solution 2.1

Method: From an expression to a circuit — p.4

Worked steps

- an OR gate with A and B as its inputs **B1**
- a NOT gate with C as its input **B1**
- both outputs feeding an AND gate, whose output is labelled X **B1**
- the OR and the NOT must both come **before** the AND: the brackets say so

Practice 2.2 ▪ 3 marks

Complete the truth table for the circuit in 2.1. Use the working columns.

A	B	C	A **OR** B	**NOT** C	X
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

Solution 2.2

Method: A three-input truth table, with working columns — p.7

Worked steps

- A OR B column correct: 0, 0, 1, 1, 1, 1, 1, 1 **B1**
- NOT C column correct: 1, 0, 1, 0, 1, 0, 1, 0 **B1**
- X column correct **B1**

Solution 2.2

A	B	C	A **OR** B	**NOT** C	X
0	0	0	0	1	0
0	0	1	0	0	0
0	1	0	1	1	**1**
0	1	1	1	0	0
1	0	0	1	1	**1**
1	0	1	1	0	0
1	1	0	1	1	**1**
1	1	1	1	0	0

Solution 2.2

- $X = 1$ only when $C = 0$ and at least one of A, B is 1

Practice 2.3 ▪ 2 marks

Name the **single** gate that does the same job as each of these.

Practice 2.3 (a)

NOT (A AND B)

Solution 2.3 (a)

Method: From an expression to a circuit — p.4

Worked steps

NAND B1

Practice 2.3 (b)

NOT (A OR B)

Solution 2.3 (b)

Worked steps

NOR B1

Practice 2.4 ▪ 2 marks

A truth table gives $X = 1$ on the rows $ABC = 001$ and 110, and $X = 0$ on every other row.
Write the logic expression for X.

Solution 2.4

Method: From a truth table to an expression — p.9

Worked steps

- row 001 gives NOT A AND NOT B AND C; row 110 gives A AND B AND NOT C **M1**
- $X =$ (NOT A AND NOT B AND C) OR (A AND B AND NOT C) **A1**

Practice 2.5 ▪ 1 mark

A circuit uses input B as an input to two different gates. State how this is shown on the circuit diagram.

Solution 2.5

Worked steps

- the wire from B **branches**, with a **junction dot** drawn at the branch point; B is still a single input **B1**

3

Exam-style

Exam-style 3.1 ▪ 5 marks

A chemical tank has three sensors. An alarm X sounds when the temperature sensor A reads 1 **and** the pressure sensor B reads 0, **or** when the manual override C reads 1.

Exam-style 3.1 (a) ▪ 2 marks

Write the logic expression for X.

Solution 3.1 (a)

Method: From an expression to a circuit — p.4

Worked steps

"A reads 1 and B reads 0" is A AND NOT B **B1**

- "or C reads 1" adds an OR with C: $X = (A \text{ AND NOT } B) \text{ OR } C$ **B1**

Exam-style 3.1 (b) ▪ 3 marks

Draw the logic circuit for X.

Solution 3.1 (b)

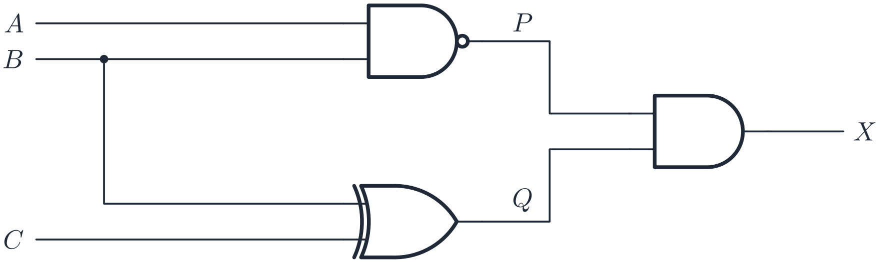
Worked steps

a NOT gate with B as its input **B1**

- an AND gate taking A and the NOT gate's output **B1**
- an OR gate taking that AND output and C directly, giving X **B1**
- C goes straight into the OR gate: nothing in the sentence acts on it first

Exam-style 3.2 ▪ 6 marks

A logic circuit is shown. P and Q are the outputs of the first two gates.



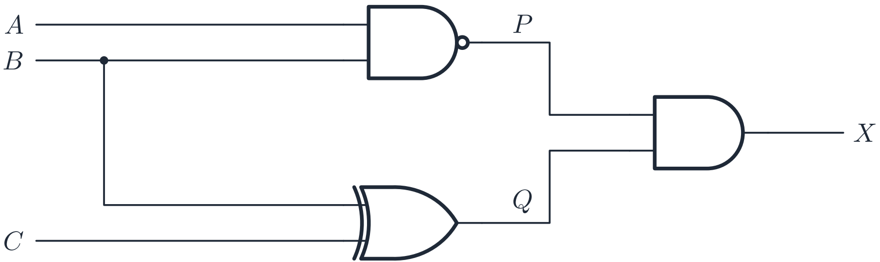
Exam-style 3.2 (a) ▪ 4 marks

Complete the truth table, filling the working columns P and Q.

A	B	C	P	Q	X
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

Exam-style 3.2 (a) ▪ 4 marks

Complete the truth table, filling the working columns ... and ... .



Solution 3.2 (a)

Method: From an expression to a circuit — p.4

Worked steps

P column correct – P is NOT (A AND B), so it is 1 everywhere except the last two rows **B1**

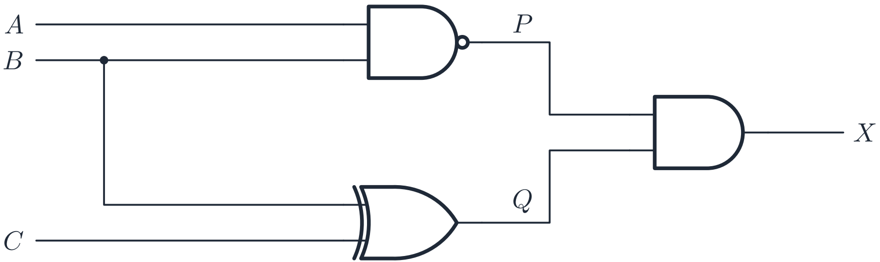
- Q column correct – Q is 1 when B and C **differ** **B1**
- X column correct, $X = P$ AND Q **B1** **B1**

Solution 3.2 (a)

A	B	C	P	Q	X
0	0	0	1	0	0
0	0	1	1	1	**1**
0	1	0	1	1	**1**
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	1	**1**
1	1	0	0	1	0
1	1	1	0	0	0

Exam-style 3.2 (b) ▪ 2 marks

State the values of A, B and C for which $X = 1$.



Solution 3.2 (b)

Worked steps

$X = 1$ on $ABC = 001$, 010 and 101 **B1**

- in words: B and C must differ, and A and B must not both be 1 **B1**

Exam-style 3.3 ▪ 3 marks

A truth table gives $X = 1$ on the rows $ABC = 000$, 011 and 110, and $X = 0$ on every other row.
Write the logic expression for X.

Solution 3.3

Method: From a truth table to an expression — p.9

Worked steps

- row 000: NOT A AND NOT B AND NOT C **B1**
- row 011: NOT A AND B AND C **B1**
- row 110: A AND B AND NOT C, and the three ORed together **B1**

$$X = (\text{NOT } A \text{ AND NOT } B \text{ AND NOT } C) \text{ OR } (\text{NOT } A \text{ AND } B \text{ AND } C) \text{ OR } (A \text{ AND } B \text{ AND NOT}$$

Exam-style 3.4 ▪ 3 marks

Draw the logic circuit for $X = (A \text{ XOR } B) \text{ AND } (\text{NOT } C)$.

Solution 3.4

Method: From an expression to a circuit — p.4

Worked steps

- an XOR gate with A and B as its inputs **B1**
- a NOT gate with C as its input **B1**
- both feeding an AND gate whose output is X **B1**
- the XOR symbol is the OR shape with a **second curved line** across its back; drawing a plain OR loses the first mark

Exam-style 3.5 ▪ 3 marks

A student is asked to draw the circuit for $X = A \text{ AND } (B \text{ OR } C)$, but wires it as $(A \text{ AND } B) \text{ OR } C$.
Explain, naming **one** row of the truth table, how the two circuits behave differently.

Solution 3.5

Method: From an expression to a circuit — p.4

Worked steps

- the brackets change which gate comes first: A AND $(B$ OR $C)$ ORs B and C **before** the AND, while $(A$ AND $B)$ OR C ANDs first and lets C reach the output on its own **B1**
- so in the second circuit $C = 1$ forces $X = 1$ whatever A is, which the first circuit does not do **B1**
- any row with $A = 0$ and $C = 1$ shows it, for example $A = 0$, $B = 0$, $C = 1$: the correct circuit gives $X = 0$ and the student's gives $X = 1$ **B1**
- $A = 0$, $B = 1$, $C = 1$ is also accepted; a row where the two agree proves nothing