Exercise walk-through

10.4.1

The circular queue, a linked list in arrays, and choosing the right ADT

A-Level Computer Science

You must be able to

- describe the key features of a stack, a queue and a linked list, and **justify** their use for a given situation
- describe how a queue, a stack and a linked list can be implemented using **arrays**
- use the pointers of a circular queue, including the wrap, and state the checks made before adding and removing
- add and delete a node in a linked list held in arrays, using the free list

1 The method

Method — The circular queue, and why it exists

A queue adds at the **rear** and removes from the **front**, so both pointers only ever move **forward**. In a plain linear queue the front pointer marches up the array and the cells behind it can never be used again: after a few hundred operations the queue reports itself full with most of the array empty.

A **circular queue** fixes that by wrapping a pointer back to the start when it passes the end:

$$\texttt{Rear} \leftarrow (\texttt{Rear MOD MaxSize}) + 1$$

With `MaxSize = 6`: if `Rear` is 5 then $(5 \bmod 6) + 1 = 6$, so the next item goes in cell 6; if `Rear` is 6 then $(6 \bmod 6) + 1 = \mathbf{1}$, and the pointer **wraps back** to cell 1. The same formula moves `Front`.

The two checks, and they are marks:

- before **adding** (enqueue), check the queue is **not full** – there is an unused element

Method — The circular queue, and why it exists

- before **removing** (dequeue), check the queue is **not empty** – there is at least one item
- Keep a **`NumberInQueue`** counter, and both checks are one comparison: full when it equals `MaxSize`, empty when it is 0. Without a counter, `Front = Rear` cannot tell full from empty, because both happen when the pointers meet. Say which convention you are using; the mark is for being explicit.

Method — The circular queue, and why it exists

A queue adds at the rear and removes from the front , so both pointers only ever move forward

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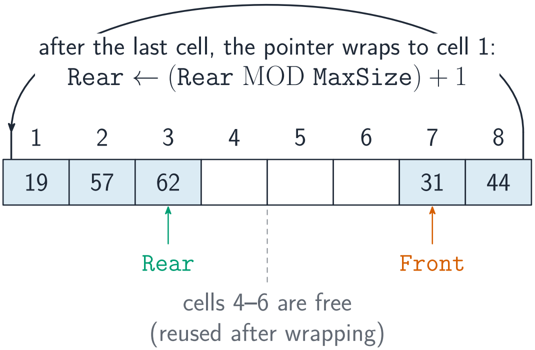


Method — A linked list held in arrays

Two parallel arrays: `Data[i]` holds a value and `Pointer[i]` holds the **index** of the next node, with 0 meaning "no next node". `Start` holds the index of the first node. The unused slots are not left loose – they are chained into a **free list** of their own, headed by `FreeListHead`, exactly as the data is chained. That gives the flexibility of a linked structure with the fixed allocation of an array.

Worked example. *Six slots. `Start = 2`, and the list is $2 \to 5 \to 1$ holding `Ann`, `Ben`, `Cal`, with `Pointer[1] = 0`. `FreeListHead = 3`, and the free list is $3 \to 4 \to 6$. Insert `Bea` between `Ann` and `Ben`.*

1. **Take a slot from the free list**: index 3. Move the head on – `FreeListHead <- Pointer[3]`, which is 4.
2. **Store the value**: `Data[3] <- "Bea"`.
3. **Point the new node at what the previous one pointed at**: `Pointer[3] <- Pointer[2]`, which is 5.

Method — A linked list held in arrays

4. **Point the previous node at the new one**: `Pointer[2] <- 3`.

The list is now $2 \to 3 \to 5 \to 1$ and the free list is $4 \to 6$.

- **Step 3 must come before step 4.** Setting `Pointer[2]` first destroys the address of `Ben`, and the rest of the list is lost.

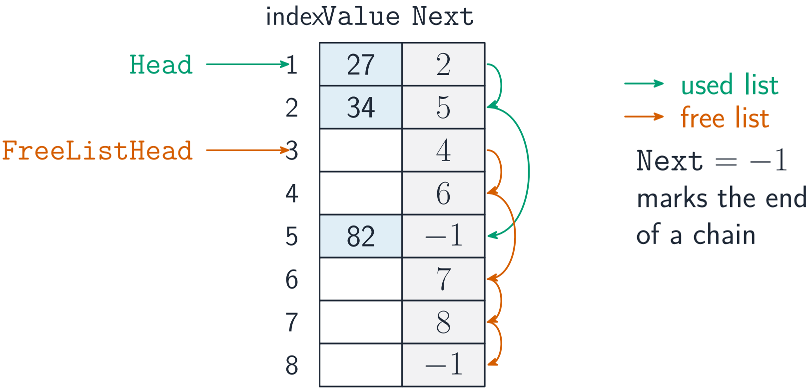
Deleting `Ben`, at index 5: find the node **before** it (index 3), then

- `Pointer[3] <- Pointer[5]`, which is 1, so the list bypasses it,
- return the slot: `Pointer[5] <- FreeListHead`, then `FreeListHead <- 5`.

The list is $2 \to 3 \to 1$ and the free list is $5 \to 4 \to 6$. Nothing is erased – a node is "deleted" by nothing pointing at it any more.

Method — A linked list held in arrays

Deleting Ben , at index 5: find the node before it (index 3), then The list is ... and the free list is ... .



Method — Choosing the right ADT

Situation	**ADT**	**Because**
undo in an editor; return addresses of nested calls; reversing something	**stack**	the **last** thing stored is the **first** needed back (LIFO)
a print spooler; keystrokes waiting; jobs served in turn	**queue**	the **first** thing stored is the **first** served (FIFO)
a list that is often inserted into or deleted from in the middle, and whose length is not known	**linked list**	inserting or deleting only changes **pointers**, with no shifting of the other items

Method — Choosing the right ADT

- A justification must name the **property**, not the structure: “a queue, because the first job submitted must print first” earns the mark; “a queue, because it is a queue of jobs” does not.
- Against a 1-D array, a linked list inserts and deletes without shifting anything and can grow until memory runs out; the costs are a pointer stored with every item and having to follow *n* pointers to reach the *n*th item, because there is no direct index.

2 Practice

Practice 2.1 ▪ 2 marks

State **one** key feature of a **stack** and **one** key feature of a **queue**.

Solution 2.1

Method: Choosing the right ADT — p.10

Worked steps

- a **stack** is **LIFO**: items are added and removed at the same end, the top, so the last item added is the first removed **B1**
- a **queue** is **FIFO**: items are added at the rear and removed from the front, so the first item added is the first removed **B1**

Practice 2.2 ▪ 2 marks

A circular queue is held in an array with `MaxSize = 6`. Give the statement that moves `Rear` on by one, and state the value it gives when `Rear` is 6.

Solution 2.2

Method: The circular queue, and why it exists — p.4

Worked steps

- `Rear <- (Rear MOD 6) + 1` B1
- when `Rear` is 6: $(6 \bmod 6) + 1 = \mathbf{1}$, so the pointer wraps back to cell 1 B1

Practice 2.3 ▪ 2 marks

State what is meant by the **free list** in a linked list held in arrays.

Solution 2.3

Method: A linked list held in arrays — p.7

Worked steps

- the free list is a chain of the array's **unused** slots, headed by its own pointer **B1**
- a slot is taken from it when a node is inserted and returned to it when a node is deleted, so the program always knows where there is room **B1**

Practice 2.4 ▪ 2 marks

State the check that must be made **before** adding an item to a queue, and the check that must be made **before** removing one.

Solution 2.4

Method: The circular queue, and why it exists — p.4

Worked steps

- before adding: check the queue is **not full** – that there is an unused element **B1**
- before removing: check the queue is **not empty** – that there is at least one item **B1**

Practice 2.5 ▪ 2 marks

Name the most suitable ADT for each situation.

Practice 2.5 (a)

storing the return addresses of nested procedure calls

Solution 2.5 (a)

Method: Choosing the right ADT — p.10

Worked steps

a **stack** B1

Practice 2.5 (b)

holding documents waiting to be printed

Solution 2.5 (b)

Worked steps

a **queue** B1

3 Exam-style

Exam-style 3.1 ▪ 5 marks

A circular queue is held in `Q[1:6]`, with `Front = 4`, `Rear = 6` and `NumberInQueue = 3`.

Exam-style 3.1 (a) ▪ 2 marks

An item is added. Give the new value of `Rear`, and state which cell the item is stored in.

Solution 3.1 (a)

Method: The circular queue, and why it exists — p.4

Worked steps

`Rear <- (6 MOD 6) + 1 = 1`, so `Rear` becomes **1** B1

- the item is stored in **cell 1**: the pointer has wrapped round to the start B1

Exam-style 3.1 (b) ▪ 2 marks

Two items are then removed. Give the new value of `Front` and of `NumberInQueue`.

Solution 3.1 (b)

Worked steps

`Front` moves on twice: $(4 \bmod 6) + 1 = 5$, then $(5 \bmod 6) + 1 = 6$, so `Front` is $\mathbf{6}$ **B1**

- `NumberInQueue` was 4 after the addition, so after two removals it is $\mathbf{2}$ **B1**

Exam-style 3.1 (c) ▪ 1 mark

State how the program knows the queue is **full**, given that it keeps `NumberInQueue`.

Solution 3.1 (c)

Worked steps

the queue is full when `NumberInQueue` equals `MaxSize`, that is 6 **B1**

Exam-style 3.2 ▪ 3 marks

Explain why a **linear** queue held in an array wastes space, and how a **circular** queue avoids it.

Solution 3.2

> Method: The circular queue, and why it exists — p.4

Worked steps

- in a linear queue both pointers only ever move **forward**, so once `Front` has passed a cell that cell can never be reused **B1**
- the queue therefore reports itself full when `Rear` reaches the end of the array, even though the cells before `Front` are empty **B1**
- a circular queue wraps a pointer back to cell 1 when it passes the last cell, using `(Pointer MOD MaxSize) + 1`, so every cell is reused and the queue is full only when it really holds `MaxSize` items **B1**

Exam-style 3.3 ▪ 4 marks

A linked list is held in `Data[1:6]` and `Pointer[1:6]`. `Start = 2`, the list is $2 \to 5 \to 1$, `Pointer[1] = 0`, and the free list is $3 \to 4 \to 6$ with `FreeListHead = 3`.
Describe the steps that insert a new value between the nodes at index 2 and index 5, giving the new value of every pointer that changes.

Solution 3.3

> Method: A linked list held in arrays — p.7

Worked steps

- take the slot at `FreeListHead`, index **3**, and move the head on: `FreeListHead <- Pointer[3]`, which is **4** **B1**
- store the value in `Data[3]` **B1**
- `Pointer[3] <- Pointer[2]`, which is **5**, so the new node points at what the previous node pointed at **B1**
- `Pointer[2] <- 3`, so the previous node now points at the new one **B1**
- the list is $2 \to 3 \to 5 \to 1$ and the free list is $4 \to 6$
- the order matters: setting `Pointer[2]` before `Pointer[3]` destroys the address of the rest of the list

Exam-style 3.4 ▪ 4 marks

Using the list as it is **after** the insertion in 3.3, describe the steps that **delete** the node at index 5, including what happens to its slot.

Solution 3.4

Method: A linked list held in arrays — p.7

Worked steps

- find the node **before** index 5, which is index 3, by following the pointers from `Start` B1
- `Pointer[3] <- Pointer[5]`, which is **1**, so the list bypasses the deleted node B1
- return the slot to the free list: `Pointer[5] <- FreeListHead`, which is 4 B1
- `FreeListHead <- 5`, so the free list is now $5 \to 4 \to 6$ and the list is $2 \to 3 \to 1$ B1
- the data in `Data[5]` is not erased; the node is deleted because nothing points at it any more

Exam-style 3.5 ▪ 3 marks

A hospital records patients waiting for a doctor. Patients are seen in the order they arrive, but a patient may leave before being seen, and the number waiting is not known in advance.

Name a suitable ADT and **justify** your choice.

Solution 3.5

Worked steps

- a **linked list** B1
- patients are seen in arrival order, and a linked list keeps them in that order by following the pointers from the head B1
- a patient may leave before being seen, and deleting a node from the middle of a linked list changes only pointers, with no shifting of the other items, which an array would need; and the list can grow as far as memory allows, which matters because the number waiting is not known in advance B1
- a plain queue is accepted **only** if the answer deals with removal from the middle; a queue removes from the front only, so a patient leaving early cannot be taken out of it