

Exercise walk-through

1.1.2

Binary magnitudes, one's complement, subtraction and where BCD and hex are used

A-Level Computer Science

You must be able to

- show understanding of binary magnitudes and the difference between **binary prefixes** 二进制前缀 and **decimal prefixes**: kibi and kilo, mebi and mega, gibi and giga, tebi and tera
- use **one's complement** representation for binary numbers, and convert a value from one representation to another
- perform binary **subtraction** using positive and negative binary integers
- describe practical applications where **Binary Coded Decimal (BCD)** 二进制十进数 and **hexadecimal** 十六进制 are used

1

The method

Method — The two prefix families

A decimal prefix is a power of **10**; a binary prefix is a power of **2**. They are near enough to be confused and far enough apart to matter.

Decimal (SI)	Binary (memory)	In bytes
kilo (k) = 10^3	kibi (Ki) = 2^{10}	1024
mega (M) = 10^6	mebi (Mi) = 2^{20}	1 048 576
giga (G) = 10^9	gibi (Gi) = 2^{30}	1 073 741 824
tera (T) = 10^{12}	tebi (Ti) = 2^{40}	1 099 511 627 776

Method — The two prefix families

Worked example. *A drive is sold as “1 TB”. The operating system reports 931 GiB. Explain.*

The maker counts in decimal prefixes, so $1 \text{ TB} = 10^{12} = 1\,000\,000\,000\,000$ bytes. The operating system counts in binary prefixes, and one **gibibyte** is $2^{30} = 1\,073\,741\,824$ bytes, so

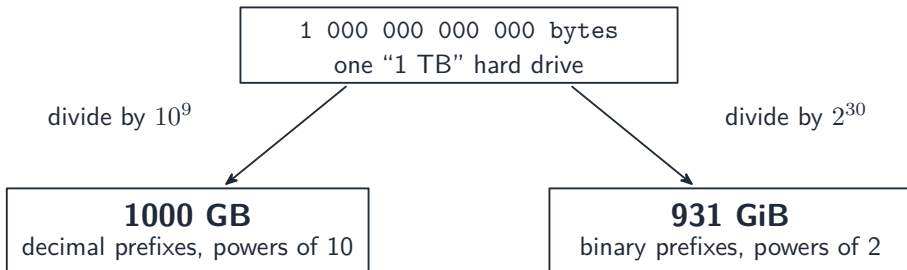
$$\frac{1\,000\,000\,000\,000}{1\,073\,741\,824} = \mathbf{931 \text{ GiB}}$$

Nothing is missing. The same bytes are being counted in a **larger unit**.

- Going the other way is multiplication: $4 \text{ GiB} = 4 \times 2^{30} = \mathbf{4\,294\,967\,296}$ bytes.
- Memory sizes are always powers of 2, because an address of n bits reaches exactly 2^n locations. That is why the binary family exists at all.

Method — The two prefix families

A decimal prefix is a power of 10 ; a binary prefix is a power of 2 .



The bytes never changed. Only the size of the unit did.

Method — One's complement, and why two's complement replaced it

One's complement: to write a negative number, **invert every bit** of the positive. There is no “add 1”.

	+30	−30
one's complement	00011110	11100001
two's complement	00011110	11100010

Reading a pattern **back** depends on which scheme you are told it is in. Take 11110100. The sign bit is 1, so it is negative either way.

Method — One's complement, and why two's complement replaced it

- as **one's** complement: invert to get $00001011 = 11$, so the value is -11 .
- as **two's** complement: invert to get 00001011 , add 1 to get $00001100 = 12$, so the value is -12 .

The drawback is **two zeros**: 00000000 is $+0$ and 11111111 is -0 . One bit pattern is wasted, a test for zero has to check both, and the adder needs an extra correction step. Two's complement has a **single** zero and lets one circuit do both addition and subtraction, which is why it is what computers use.

Method — Binary subtraction, two ways

Method 1, column subtraction. Work from the right. When the top bit is 0 and the bottom bit is 1, **borrow** 借位 from the left: the 0 becomes 2, and the column you borrowed from loses 1.

Method 2, the one the exam rewards: turn it into an addition. $A - B$ is $A + (-B)$, so take the **two's complement of B** and add. One circuit, no borrowing.

Worked example. *Work out $01011010 - 00110111$ (that is $90 - 55$).*

1 Two's complement of 00110111: invert to 11001000, add 1 to get 11001001.

2 Add:

$$01011010 + 11001001 = 1\ 00100011$$

1 **Discard the carry out of the eighth column.** The answer is $00100011 = 35$, and $90 - 55 = 35$.

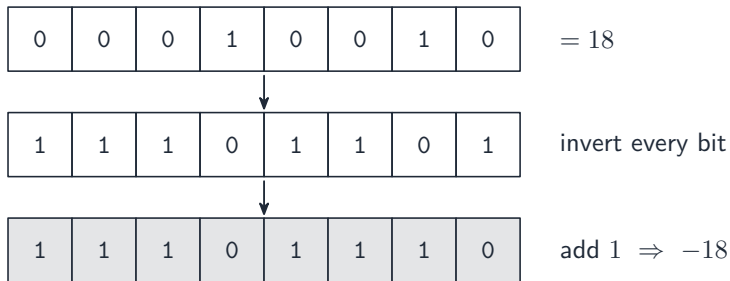
Method — Binary subtraction, two ways

Worked example. *Work out $00110010 - 01000110$ (that is $50 - 70$).*

- 1 Two's complement of 01000110 : invert to 10111001 , add 1 to get 10111010 .
 - 2 Add: $00110010 + 10111010 = 11101100$, with **no** carry out.
 - 3 The sign bit is 1, so the answer is negative. Read it back: invert to 00010011 , add 1 to get $00010100 = 20$. The answer is **-20**.
- A carry out of the top column is **discarded** in two's-complement subtraction; it is not an error and it is not overflow.
 - **Overflow** 溢出 is different: it is when the true answer will not fit in the bits available, and in signed arithmetic you spot it by the sign bit being wrong – two positives giving a negative, or two negatives giving a positive.

Method — Binary subtraction, two ways

Method 1, column subtraction.



The two's complement of a positive binary number: invert every bit, then add one

Method — Where BCD is used, and where hexadecimal is used

Both questions are “describe a practical application”, so each answer must name a use **and** link it to a property of the representation.

BCD stores each denary digit in its own 4 bits: 93 is 1001 0011, not binary 93 (01011101). Patterns 1010 to 1111 are unused.

- **Digital clocks, calculators and any device with a 7-segment display 七段显示器.** Each digit already has its own 4 bits, so it can be sent straight to its own display with no conversion of the whole number.
- **Currency and other decimal money amounts.** A value such as 0.10 is stored exactly, digit by digit; in ordinary binary a decimal fraction is a recurring value and has to be rounded, which accumulates error over many calculations.

Hexadecimal is not a different way of storing anything. One hex digit is exactly 4 bits, so a byte is exactly two hex digits – it is a shorter, more readable way for a **human** to write the binary that is already there.

Method — Where BCD is used, and where hexadecimal is used

- **Memory addresses** 内存地址 and memory dumps in low-level work, such as 0x7FFE.
- **Colour values** in HTML and CSS, such as #FF8800 – two hex digits per colour channel.
- **MAC addresses**, such as AC:DE:48:00:11:22.
- Also **error codes** and **Unicode code points**.

The reason is always the same: fewer characters than binary, fewer transcription mistakes, and conversion back to binary needs no arithmetic at all – just replace each digit with its nibble.

2

Practice

Practice 2.1 ■ 3 marks

State the number of bytes in each of these.

Practice 2.1 (a)

1 kibibyte (KiB)

Solution 2.1 (a)

Worked steps

$$2^{10} = \mathbf{1024} \text{ bytes} \quad \text{B1}$$

Practice 2.1 (b)

1 mebibyte (MiB)

Solution 2.1 (b)

Worked steps

$$2^{20} = 1\,048\,576 \text{ bytes} \quad \text{B1}$$

Practice 2.1 (c)

2 gibibytes (GiB)

Solution 2.1 (c)

Worked steps

$$2 \times 2^{30} = \mathbf{2\,147\,483\,648} \text{ bytes} \quad \text{B1}$$

Practice 2.2 ■ 2 marks

State the difference, in bytes, between one **kilobyte** (kB) and one **kibibyte** (KiB), and state which prefix family each belongs to.

Solution 2.2

Method: The two prefix families — p.4

Worked steps

- a kilobyte is $10^3 = 1000$ bytes and a kibibyte is $2^{10} = 1024$ bytes, so a kibibyte is **24 bytes** larger B1
- **kilo** is a decimal prefix, a power of 10; **kibi** is a binary prefix, a power of 2 B1

Practice 2.3 ■ 2 marks

The denary value $+45$ is stored in 8 bits.
Write it in **one's complement** and state what -45 is in one's complement.

Solution 2.3

Method: Where BCD is used, and where hexadecimal is used — p.12

Worked steps

- $+45 = 00101101$ **B1**
- one's complement is every bit inverted, so $-45 = 11010010$ **B1**

Practice 2.4 ■ 2 marks

The 8-bit pattern 11101100 is a negative number.
State its value (a) if it is **one's** complement, and (b) if it is **two's** complement.

Solution 2.4

Method: One's complement, and why two's complement replaced it — p.7

Worked steps

(a) one's complement: invert 11101100 to get 00010011 = 19, so the value is **-19**

B1

(b) two's complement: invert to 00010011, add 1 to get 00010100 = 20, so the value is **-20** **B1**

Practice 2.5 ■ 3 marks

Work out $01101001 - 00101101$ using two's complement. Show the two's complement you formed, the addition, and the final answer in binary.

Solution 2.5

Method: Binary subtraction, two ways — p.9

Worked steps

- two's complement of 00101101: invert to 11010010, add 1 to get 11010011
M1
- $01101001 + 11010011 = 1\ 00111100$ **M1**
- discard the carry: the answer is **00111100**, which is $60 -$ and $105 - 45 = 60$
A1

Practice 2.6 ■ 2 marks

State **one** practical application of BCD and **one** practical application of hexadecimal.

Solution 2.6

Method: Where BCD is used, and where hexadecimal is used — p.12

Worked steps

- BCD, any one: digital clocks, calculators or any device driving a 7-segment display; storing currency amounts **B1**
- hexadecimal, any one: memory addresses and memory dumps; colour values in HTML or CSS; MAC addresses; error codes; Unicode code points **B1**

3

Exam-style

Exam-style 3.1 ■ 4 marks

A memory module is described as 4 GiB and a hard drive is sold as 4 GB.

Exam-style 3.1 (a) ■ 2 marks

Calculate the number of bytes in each.

Solution 3.1 (a)

Method: The two prefix families — p.4

Worked steps

$$4 \text{ GiB} = 4 \times 2^{30} = \mathbf{4\,294\,967\,296} \text{ bytes} \quad \text{B1}$$

$$\blacksquare 4 \text{ GB} = 4 \times 10^9 = \mathbf{4\,000\,000\,000} \text{ bytes} \quad \text{B1}$$

Exam-style 3.1 (b) ■ 2 marks

Explain why the two figures are different even though both are written as “4 G”.

Solution 3.1 (b)

Worked steps

“GiB” uses a **binary** prefix, a power of 2, while “GB” uses a **decimal** prefix, a power of 10 **B1**

- a gibibyte is therefore larger than a gigabyte, so the same digit in front of it counts more bytes; memory is sized in powers of 2 because an n -bit address reaches 2^n locations, while drive makers quote the decimal figure **B1**

Exam-style 3.2 ■ 4 marks

A computer could store negative integers in one's complement or in two's complement.

Exam-style 3.2 (a) ■ 2 marks

Describe how a negative number is formed in **each** representation.

Solution 3.2 (a)

Worked steps

one's complement: invert every bit of the positive number B1

- **two's** complement: invert every bit of the positive number **and then add 1** B1

Exam-style 3.2 (b) ■ 2 marks

Give **two** reasons why two's complement is used in modern computers instead of one's complement.

Solution 3.2 (b)

Worked steps

any two: one's complement has **two** representations of zero (00000000 and 11111111), so a bit pattern is wasted and a test for zero must check both; two's complement has a single zero **B1**

- addition and subtraction can use the **same** circuit in two's complement, with no end-around carry correction, so the hardware is simpler and the arithmetic is faster **B1**

Exam-style 3.3 ■ 5 marks

Two 8-bit values are 00101110 and 01010001.

Exam-style 3.3 (a) ■ 3 marks

Work out $00101110 - 01010001$ using two's complement.

Solution 3.3 (a)

Method: Binary subtraction, two ways — p.9

Worked steps

two's complement of 01010001: invert to 10101110, add 1 to get 10101111 M1

■ $00101110 + 10101111$ M1

■ $= 11011101$, with no carry out of the eighth column A1

Exam-style 3.3 (b) ■ 2 marks

State the denary value of your answer, and explain how you read a negative two's-complement result.

Solution 3.3 (b)

Worked steps

the sign bit is 1, so the value is negative: invert 11011101 to 00100010, add 1 to get 00100011 = 35, so the answer is **-35** B1

■ and $46 - 81 = -35$, which checks B1

Exam-style 3.4 ■ 3 marks

A programmer writing low-level code reads a memory dump printed in hexadecimal rather than in binary.
Explain **two** reasons why hexadecimal is used, and state **one** other practical application of hexadecimal.

Solution 3.4

Method: Where BCD is used, and where hexadecimal is used — p.12

Worked steps

- any two reasons: one hex digit is exactly 4 bits, so a byte is two characters instead of eight – far shorter to write and to read **B1**
- fewer characters means fewer mistakes when a human copies or reads a long value, and converting back to binary needs no arithmetic, just replacing each digit with its nibble **B1**
- one other application: colour values in HTML or CSS; MAC addresses; error codes; Unicode code points **B1**
- a reason that only says “hex is shorter” without saying **why** (4 bits per digit) does not earn the second mark

Exam-style 3.5 ■ 3 marks

A supermarket till stores each price using BCD rather than ordinary binary.
Justify the use of BCD in this application.

Solution 3.5

Method: Where BCD is used, and where hexadecimal is used — p.12

Worked steps

- in BCD each **denary digit** is stored in its own 4 bits, so the digits of a price are kept separately **B1**
- a decimal fraction such as 0.10 is stored **exactly**, whereas converting it to ordinary binary gives a recurring value that must be rounded, and the error grows across many prices **B1**
- each digit can also be sent straight to its own position on the display, and prices can be added digit by digit, with no conversion of the whole number **B1**