

3.2.1 Truth tables and expressions from a logic circuit

Name: _____ Class: _____ Date: _____ Total: 44 marks

KEY WORDS logic circuit • truth table • logic expression
• working column • logic gate

Objective

Learn to **read a logic circuit** : write its **logic expression** , and complete its **truth table** with a **working column** for each gate. Sheet 3.2 taught the six gates one at a time; this sheet joins them into circuits with three inputs. Building a circuit yourself comes on sheet 3.2.2.

You must be able to:

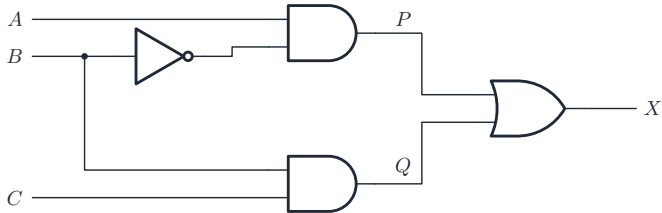
- construct a logic expression from a logic circuit
- construct a truth table from a logic circuit, using a working column for each gate
- construct a truth table from a logic expression
- work with **NAND**, **NOR** and **XOR** gates inside a larger circuit

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

■ From a circuit to an expression

Read the circuit from **left to right**, and give every gate's output a name as you go.



1. Write the expression for each gate on the left: $P = A \text{ AND NOT } B$ and $Q = B \text{ AND } C$.
2. Write the last gate using those names: $X = P \text{ OR } Q$.

3. Replace each name with its expression, **in brackets**: $X = (A \text{ AND NOT } B) \text{ OR } (B \text{ AND } C)$.
- The brackets show which gates come first. Without them the expression can be read in more than one way.
- The dot on the wire from B is a **junction**: B is one input that goes to two gates.

■ A three-input truth table, with working columns

Three inputs need $2^3 = 8$ rows (two inputs: 4; four inputs: 16). **Count up in binary** from 000 to 111, so no row is missed or written twice.

Add one **working column** for each gate, and fill one **whole column** at a time, from left to right across the circuit. Never work along a row.

A	B	C	$P = A \text{ AND NOT } B$	$Q = B \text{ AND } C$	$X = P \text{ OR } Q$
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	1	1
1	0	0	1	0	1
1	0	1	1	0	1
1	1	0	0	0	0
1	1	1	0	1	1

- To fill a column quickly, find the **few** rows that are special. For $P = A \text{ AND NOT } B$, the output is 1 only where $A = 1$ **and** $B = 0$: two rows. Every other row is 0.
- When the question prints a **working space**, use it. Correct working can still earn marks when the last column has a mistake.

■ NAND, NOR and XOR inside a circuit

NAND

NOR

XOR



- **NAND**: work out AND first, then **flip** every bit. The output is 0 **only** when both inputs are 1.
- **NOR**: work out OR first, then **flip** every bit. The output is 1 **only** when both inputs are 0.

- **XOR**: the output is 1 when the two inputs are **different**.

A	B	$A \text{ NAND } B$	$A \text{ NOR } B$	$A \text{ XOR } B$
0	0	1	1	0
0	1	1	0	1
1	0	1	0	1
1	1	0	0	0

■ From an expression straight to a truth table

With no circuit, the **brackets** give the order: one working column for each bracket, then combine the columns with the operator outside the brackets.

Worked example: $X = (A \text{ OR } B) \text{ AND NOT } (A \text{ AND } B)$.

A	B	$A \text{ OR } B$	$A \text{ AND } B$	$\text{NOT } (A \text{ AND } B)$	X
0	0	0	0	1	0
0	1	1	0	1	1
1	0	1	0	1	1
1	1	1	1	0	0

The last column is 1 only when the inputs are different. So this circuit of three gates does the same job as **one XOR gate**.

2 Practice

Now use the methods above. The first questions need one idea each.

2.1 State how many rows a truth table needs for a circuit with:

(a) two inputs

[1]

(b) three inputs

[1]

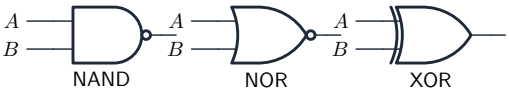
(c) four inputs

[1]



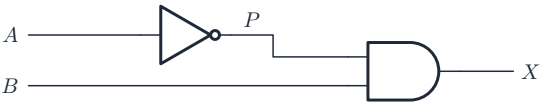
2.2 Give the output of each gate for the inputs shown.

[6]



Inputs	NAND	NOR	XOR
$A = 1, B = 0$			
$A = 1, B = 1$			

2.3 A logic circuit is shown. P is the output of the first gate.



(a) Write the logic expression for X .

[2]

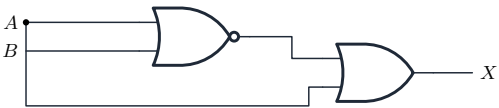
(b) Complete the truth table.

[2]

A	B	P	X
0	0		
0	1		
1	0		
1	1		

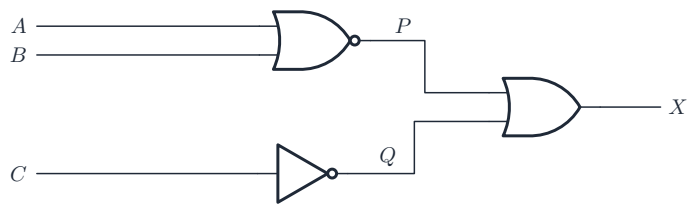
2.4 Complete the truth table for $X = (A \text{ NOR } B) \text{ OR } A$.

[3]



A	B	$A \text{ NOR } B$	X
0	0		
0	1		
1	0		
1	1		

2.5 A logic circuit is shown. P and Q are the outputs of the first two gates.



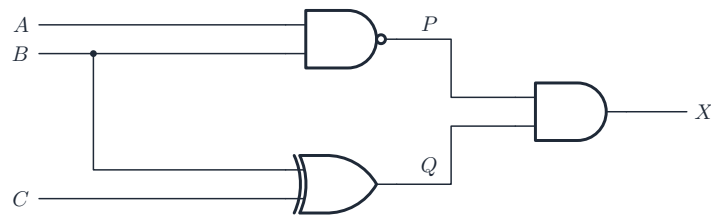
(a) Write the logic expression for X . [2]

(b) Complete the truth table. [4]

A	B	C	P	Q	X
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

3 Exam-style questions

3.1 A logic circuit is shown. P and Q are the outputs of the first two gates.



(a) Write the logic expression for X . [3]

(b) Complete the truth table. [4]

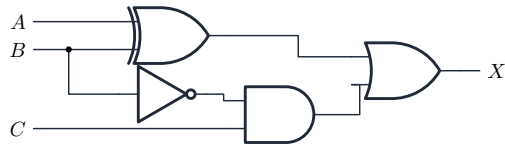
A	B	C	P	Q	X
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

(c) State the values of A , B and C for which $X = 1$. [1]

3.2 Complete the truth table for the logic expression:

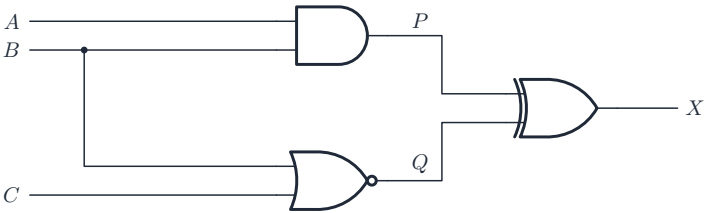
$$X = (A \text{ XOR } B) \text{ OR } (\text{NOT } B \text{ AND } C)$$

[4]



A	B	C	Working space	X
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

3.3 A logic circuit is shown. P and Q are the outputs of the first two gates.



(a) State how the diagram shows that input B goes to **two** gates.

[1]

(b) Write the logic expression for X .

[3]

(c) Complete the truth table.

[4]

A	B	C	P	Q	X
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

(d) A student says: "In this circuit, X is 1 whenever A is 1." Use your truth table to explain whether the student is correct.

[2]

Solutions

Each answer shows the steps, then the **result**.

2.1

(a) $2^2 = 4$ rows

(b) $2^3 = 8$ rows

(c) $2^4 = 16$ rows

2.2

- inputs 1, 0: AND is 0, so **NAND** = 1; OR is 1, so **NOR** = 0; the inputs are different, so **XOR** = 1
- inputs 1, 1: AND is 1, so **NAND** = 0; OR is 1, so **NOR** = 0; the inputs are the same, so **XOR** = 0

2.3

(a) the first gate gives $P = \text{NOT } A$

- $X = (\text{NOT } A) \text{ AND } B$

(b) P is the opposite of A : 1, 1, 0, 0

- X is 1 only where $P = 1$ and $B = 1$: **0, 1, 0, 0**

2.4

- $A \text{ NOR } B$ is 1 only when both inputs are 0: 1, 0, 0, 0
- OR this column with A (0, 0, 1, 1)
- $X = \mathbf{1, 0, 1, 1}$

2.5

(a) $P = A \text{ NOR } B$ and $Q = \text{NOT } C$

- $X = (A \text{ NOR } B) \text{ OR } (\text{NOT } C)$

(b) P column: 1, 1, 0, 0, 0, 0, 0

- Q column: 1, 0, 1, 0, 1, 0, 1, 0
- $X = P \text{ OR } Q$

A	B	C	P	Q	X
0	0	0	1	1	1
0	0	1	1	0	1
0	1	0	0	1	1
0	1	1	0	0	0
1	0	0	0	1	1
1	0	1	0	0	0
1	1	0	0	1	1
1	1	1	0	0	0

3.1

(a) $P = A \text{ NAND } B$

- $Q = B \text{ XOR } C$
- $X = (A \text{ NAND } B) \text{ AND } (B \text{ XOR } C)$

(b) P column: P is 0 only where A and B are both 1, the last two rows

- Q column: Q is 1 where B and C are different
- $X = P \text{ AND } Q$

A	B	C	P	Q	X
0	0	0	1	0	0
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	0	1	0
1	1	1	0	0	0

(c) $X = 1$ for $ABC = 001, 010$ and 101

3.2

- $X = \mathbf{0, 1, 1, 1, 1, 1, 0, 0}$
- the four marks are for the X column alone: eight rows correct earns 4, seven earns 3, six earns 2, five earns 1, four or fewer earns 0
- the working space is optional scratch, so a blank or unusual working space never costs a mark
- one reliable method: $A \text{ XOR } B$ is 0, 0, 1, 1, 1, 1, 0, 0, then $\text{NOT } B \text{ AND } C$ is 0, 1, 0, 0, 0, 1, 0, 0 (1 only where $B = 0$ and $C = 1$), then OR the two columns

3.3

(a) the wire from B **branches**, and a **junction dot** is drawn where it branches

(b) $P = A \text{ AND } B$

- $Q = B \text{ NOR } C$
- $X = (A \text{ AND } B) \text{ XOR } (B \text{ NOR } C)$

(c) P column: 1 only on the last two rows

- Q column: 1 only where $B = 0$ and $C = 0$, rows 000 and 100
- X is 1 where P and Q are different

A	B	C	P	Q	X
0	0	0	0	1	1
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	1	1
1	0	1	0	0	0
1	1	0	1	0	1
1	1	1	1	0	1

(d) the student is **not** correct

- on the row $ABC = 101$, A is 1 but X is 0; also X is 1 on the row 000, where A is 0