11.1.1 Operators and expressions

Name: _____ Class: _____ Date: _____ **Total: 53 marks**

KEY WORDS expressions • arithmetic operators • precedence • data type • assignment

Objective

Learn to write and to work out **expressions** : the **arithmetic operators** (including `DIV` and `MOD`), the **comparison operators** and the **logical operators** `AND`, `OR` and `NOT`. Sheet 11.1 stored values in variables; this sheet calculates with them. Built-in functions come on sheet 11.1.2.

You must be able to:

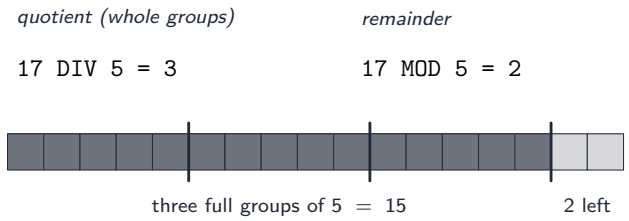
- write and work out expressions that use `+`, `-`, `*`, `/`, `DIV` and `MOD`
- apply the rules of **precedence** , and use brackets
- work out expressions that use comparison operators and `AND`, `OR`, `NOT`
- state the data type of the result of an expression
- use `DIV` and `MOD` to solve problems

1 Worked examples

Study these first. Each one shows a **method** that a question later on this sheet needs.

The arithmetic operators

operator	meaning	example	result
`+`	add	`7 + 2`	`9`
`-`	subtract	`7 - 2`	`5`
`*`	multiply	`7 * 2`	`14`
`/`	divide	`7 / 2`	`3.5` (always a `REAL`)
`DIV`	whole-number division	`7 DIV 2`	`3`
`MOD`	the remainder	`7 MOD 2`	`1`



- `17 DIV 5` asks "how many **whole** fives are in 17?" The answer is 3.
- `17 MOD 5` asks "what is **left over**?" The answer is 2, because $17 = 3 \times 5 + 2$.
- The result of `/` is always a `REAL`, even `8 / 2`, which is `4.0`. The results of `DIV` and `MOD` are `INTEGER`.

Precedence: which operator goes first?

1. brackets
2. `* / DIV MOD`
3. `+ -`

Operators on the same level are worked out from left to right.

Worked example: `3 + 4 * 2`. Multiply first: $4 \times 2 = 8$, then $3 + 8 =$ **11**. If you want the addition first, write brackets: `(3 + 4) * 2` is 14.

Worked example: `20 - 17 MOD 5 * 2`. `MOD` and `*` are on the same level, so go from left to right: `17 MOD 5` is 2, then $2 \times 2 = 4$, then $20 - 4 =$ **16**.

When an expression is long, write brackets even where they are not needed. They make the order clear to the reader.

What DIV and MOD are used for

job	expression	result
is `N` even?	`N MOD 2 = 0`	TRUE when nothing is left over
does `N` divide by 7?	`N MOD 7 = 0`	TRUE when the remainder is 0
the last digit of `N`	`N MOD 10`	347 gives 7
`N` without its last digit	`N DIV 10`	347 gives 34
whole hours in 135 minutes	`135 DIV 60`	2
minutes left over	`135 MOD 60`	15

Comparison and logical operators give a BOOLEAN

A comparison (`=`, `<>`, `<`, `>`, `<=`, `>=`) compares two values. Its result is `TRUE` or `FALSE`, so its data type is `BOOLEAN`.

`AND`, `OR` and `NOT` join or reverse Boolean values:

A	B	A AND B	A OR B	NOT A
FALSE	FALSE	FALSE	FALSE	TRUE
FALSE	TRUE	FALSE	TRUE	TRUE
TRUE	FALSE	FALSE	TRUE	FALSE
TRUE	TRUE	TRUE	TRUE	FALSE

In a mixed expression the order is: arithmetic first, then comparisons, then `NOT`, then `AND`, then `OR`.

Worked example: `Score` *is 40 and* `Bonus` *is 15. Work out* `Score + Bonus > 50 AND NOT (Score = 40)`.

1. Arithmetic: $40 + 15 = 55$.
2. Comparisons: `55 > 50` is TRUE, and `Score = 40` is TRUE.
3. `NOT TRUE` is FALSE.
4. `TRUE AND FALSE` is **FALSE**.

■ An expression must make sense

Both sides of an operator need a suitable data type. `"12" + 3` is an **error** (a string cannot be added to a number), and so is `TRUE + 1`. In the exam, write `ERROR`.

Past-paper practice: complete sheets 11.1, 11.1.1 and 11.1.2 before attempting past-paper set 11.1.2.

2 Practice

Now use the methods above. The first questions need one idea each.

2.1 Work out the value of each expression. [6]

(a) `23 DIV 4`

(b) `23 MOD 4`

(c) `9 / 2`

(d) `9 DIV 2`

(e) `30 MOD 6`

(f) `5 MOD 8`

`DIV` `MOD` `/`

2.2 Work out the value of each expression. Show the step you do first. [4]

(a) `3 + 4 * 2`

`+` `-` `*` `/` `MOD` `( )`

(b) `(3 + 4) * 2`

(c) `20 - 12 / 4`

(d) `2 + 15 MOD 4 * 3`

2.3 State the data type of the result of each expression. [4]

(a) `12 / 4`

(b) `12 DIV 4`

(c) `12 > 4`

(d) `12 MOD 5`

Result [ ]

2.4 `P` is TRUE, `Q` is FALSE and `N` is 10. Work out whether each expression is TRUE or FALSE. [4]

(a) `P AND Q`

(b) `P OR Q`

(c) `NOT Q AND N > 5`

(d) `N MOD 2 = 0 AND NOT P`

P	TRUE
Q	FALSE
N	10

2.5 Write an expression for each of these. [4]

(a) TRUE when the integer `Year` divides exactly by 4

Year [ ] Code [ ] Days [ ]

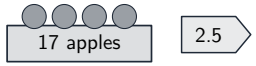
(b) the last digit of the integer `Code`

(c) the number of whole weeks in `Days` days

(d) the number of days that are left over after the whole weeks in (c)

3 Exam-style questions

3.1 The variables in a program have these values:



variable	value
Apples	17
BoxSize	5
Price	2.5
Ripe	TRUE

Evaluate each expression in the table. If an expression is not valid, write `ERROR`. [6]

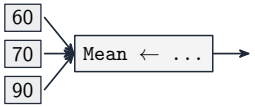
Expression	Evaluates to
`Apples DIV BoxSize`	
`Apples MOD BoxSize`	
`Apples * Price`	
`Apples / BoxSize > 3`	
`Ripe AND (Apples < 10)`	
`Ripe + Apples`	

3.2 Complete each expression so that it evaluates to the value shown. [4]

Expression	Evaluates to
`29 ..... 6`	`5`
`29 ..... 6`	`4`
`(4 + 6) ..... 4`	`2.5`
`NOT (8 ..... 3)`	`FALSE`

3.3 A student writes this statement to find the mean of three marks:

```
Mean ← Mark1 + Mark2 + Mark3 / 3
```



(a) The three marks are 60, 70 and 90. State the value that the statement stores in `Mean`. [1]

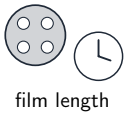
(b) Explain the error, and write the correct statement. [2]

(c) State the data type that `Mean` must have. Give a reason. [2]

3.4 A film lasts `DurationMinutes` minutes, where `DurationMinutes` is an integer. For example, a film of 135 minutes lasts 2 hours and 15 minutes.

(a) Write **two** assignment statements that store the whole number of hours in `Hours` and the minutes that are left over in `Minutes`. [2]

(b) Write pseudocode that inputs the length of a film in minutes, and outputs the length in this form: `2 hours 15 minutes`. Declare your variables. [5]



(c) State the values of `Hours` and `Minutes` when 59 is input. [1]

3.5 A shop sells eggs in boxes of 6. A farmer delivers `Eggs` eggs. An egg is wasted if it cannot go into a full box.

(a) Write an expression for the number of full boxes. [1]

(b) Write an expression that is TRUE when no egg is wasted. [2]

(c) A full box is sold for 1.80. Write a statement that stores the money from selling all the full boxes in the variable `Income`. [2]

(d) The farmer delivers 100 eggs. State the number of full boxes, the number of wasted eggs and the value of `Income`. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

(a) $23 = 5 \times 4 + 3$, so the whole-number part is **5**

(b) the remainder is **3**

(c) `/` gives a real number: **4.5**

(d) the whole-number part of 4.5 is **4**

(e) 6 goes into 30 exactly, so the remainder is **0**

(f) 8 goes into 5 zero times, so all 5 is left over: **5**

2.2

(a) multiply first: $4 \times 2 = 8$, then $3 + 8 =$ **11**

(b) brackets first: $3 + 4 = 7$, then $7 \times 2 =$ **14**

(c) divide first: $12/4 = 3$, then $20 - 3 =$ **17**

(d) `15 MOD 4` is 3, then $3 \times 3 = 9$, then $2 + 9 =$ **11**

2.3

(a) `/` always gives a **`REAL`**

(b) **`INTEGER`**

(c) a comparison gives a **`BOOLEAN`**

(d) **`INTEGER`**

2.4

(a) TRUE AND FALSE → **FALSE**

(b) TRUE OR FALSE → **TRUE**

(c) `NOT Q` is TRUE, and `10 > 5` is TRUE → **TRUE**

(d) `10 MOD 2 = 0` is TRUE, but `NOT P` is FALSE → **FALSE**

2.5

(a) `Year MOD 4 = 0`

(b) `Code MOD 10`

(c) `Days DIV 7`

(d) `Days MOD 7`

3.1

- `17 DIV 5` → **3**
- `17 MOD 5` → **2**
- $17 \times 2.5 =$ **42.5**
- $17/5 = 3.4$, and `3.4 > 3` → **TRUE**
- `17 < 10` is FALSE, and TRUE AND FALSE → **FALSE**
- a Boolean cannot be added to a number → **ERROR**

3.2

- $29 = 4 \times 6 + 5$, so the remainder 5 needs **`MOD`**
- the whole-number part 4 needs **`DIV`**
- $10/4 = 2.5$, so **`/`**
- the bracket must be TRUE for `NOT` to give FALSE: **`>`** (or `>=` or `<>`)

3.3

(a) the division is done first: $90/3 = 30$, then $60 + 70 + 30 =$ **160**

(b) `/` has a higher precedence than `+`, so only `Mark3` is divided by 3

- `Mean ← (Mark1 + Mark2 + Mark3) / 3`

(c) **`REAL`**

- `/` always gives a real number, and a mean is often not a whole number (for example 73.33)

3.4

(a) `Hours ← DurationMinutes DIV 60`

- `Minutes ← DurationMinutes MOD 60`

(b) three `INTEGER` variables declared

- `INPUT DurationMinutes`
- the two assignments from part (a)
- one `OUTPUT` with the values and the words in the correct order
- commas between the items, and the text in quotes

```
DECLARE DurationMinutes : INTEGER
DECLARE Hours : INTEGER
DECLARE Minutes : INTEGER
INPUT DurationMinutes
Hours ← DurationMinutes DIV 60
Minutes ← DurationMinutes MOD 60
OUTPUT Hours, " hours ", Minutes, " minutes"
```

(c) `59 DIV 60` is 0 and `59 MOD 60` is 59: **`Hours` = 0, `Minutes` = 59**

3.5

(a) `Eggs DIV 6`

(b) no egg is wasted when the remainder is 0

- `Eggs MOD 6 = 0`

(c) the number of full boxes multiplied by the price

- `Income ← (Eggs DIV 6) * 1.80`

(d) `100 DIV 6` = **16** full boxes

- `100 MOD 6` = **4** wasted eggs
- $16 \times 1.80 =$ **28.80**