8 **(a)** It is given that λ is an eigenvalue of the non-singular square matrix $\mathbf{A}$, with corresponding eigenvector $\mathbf{e}$.

Show that $\mathbf{e}$ is an eigenvector of $\mathbf{A}^3$ with corresponding eigenvalue λ^3. [2]

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -1 & 3 & 4 \\ 0 & 1 & 0 \\ 0 & -2 & 5 \end{pmatrix}.$$

(b) Show that the eigenvalues of $\mathbf{A}$ are -1, 1 and 5. [2]

(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A} - 2\mathbf{I} = \mathbf{PDP}^{-1}$. [6]

(d) Use the characteristic equation of $\mathbf{A}$ to show that $(\mathbf{A} - 2\mathbf{I})^3 = a\mathbf{A}^2 + b\mathbf{A} + c\mathbf{I}$ where a, b and c are constants to be determined. [3]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.