6 **(a)** Starting from the definitions of tanh and sech in terms of exponentials, prove that

$$1-\tanh^2 u = \operatorname{sech}^2 u.$$ [3]

(b) Show that $\frac{\mathrm{d}}{\mathrm{d}t}(\operatorname{sech}^{-1} t) = -\frac{1}{t\sqrt{1-t^2}}$. [4]

It is given that

$$x = \tanh^{-1} t \quad \text{and} \quad y = t\operatorname{sech}^{-1} t, \quad \text{for } 0 < t < 1.$$

(c) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\sqrt{1-t^2} + \left(1-t^2\right)\operatorname{sech}^{-1} t$. [4]

(d) Find $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ in terms of t. [4]