

Geometry

IGCSE Mathematics

This handout covers Topic 4, Geometry. Parts marked **(Extended)** are only tested on the Extended papers; everything else is for both levels. In the exam you must **give reasons** using the correct names below, not just the numbers.

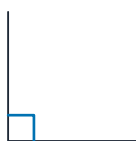
Lines and angles

A corner where two lines meet is a **vertex** 顶点. Two lines are **parallel** 平行 if they never meet, and **perpendicular** 垂直 if they meet at a right angle. Angles are named by their size:

Name	Size
acute angle 锐角	less than 90°
right angle 直角	exactly 90°
obtuse angle 钝角	between 90° and 180°
reflex angle 优角	between 180° and 360°



acute
 $< 90^\circ$



right
 $= 90^\circ$



obtuse
 $90^\circ - 180^\circ$



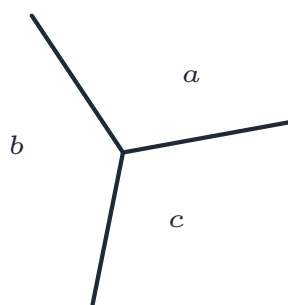
reflex
 $180^\circ - 360^\circ$

Angles by size: acute (under 90°), right (90° , shown by a square), obtuse ($90^\circ - 180^\circ$) and reflex ($180^\circ - 360^\circ$).

We name an angle with three letters, e.g. angle ABC is the angle at B .

Angle facts

Learn these basic facts. Each one is a valid reason in the exam.



$$a + b + c = 360^\circ \text{ (round a point)}$$

Angles around a point add up to 360 degrees



$$a + b = 180^\circ \text{ (straight line)}$$

Angles on a straight line add up to 180 degrees

- Angles at a point add up to 360° .
- Angles on a straight line add up to 180° .
- **Vertically opposite angles** 对顶角 (made by two crossing lines) are equal.

Worked example. Three angles on a straight line are x , 50° and 70° . Find x .

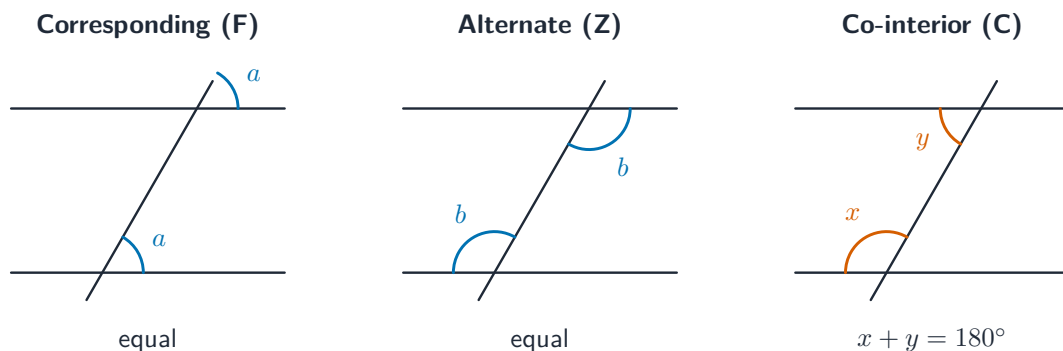
$$x + 50 + 70 = 180 \Rightarrow x = 60^\circ.$$

Angles in parallel lines

When a line crosses two parallel lines:

- **Corresponding angles** 同位角 (in matching positions, an "F" shape) are equal.
- **Alternate angles** 内错角 (opposite sides of the crossing line, a "Z" shape) are equal.
- **Co-interior angles** 同旁内角 (a "C" shape) add up to 180° ; we say they are **supplementary** 互补.

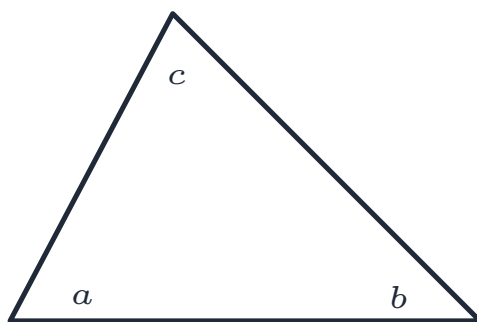
Worked example. A straight line crosses two parallel lines. One angle is 110° . The co-interior angle y satisfies $110 + y = 180$, so $y = 70^\circ$.



Corresponding (F) and alternate (Z) angles are equal; co-interior (C) angles add to 180° .

Triangles

A **triangle** 三角形 has three sides and angles that add up to 180° .



$$a + b + c = 180^\circ$$

The three angles of a triangle add up to 180 degrees

Type	Property
equilateral 等边	all sides equal, all angles 60°
isosceles 等腰	two sides equal, two angles equal
scalene 不等边	all sides and angles different
right-angled	has one right angle

Worked example. A triangle has angles x , $2x$ and 90° . Find x .

$$x + 2x + 90 = 180 \Rightarrow 3x = 90 \Rightarrow x = 30^\circ.$$

Quadrilaterals

A **quadrilateral** 四边形 has four sides and angles that add up to 360° .

Shape	Property
square 正方形	four equal sides, four right angles
rectangle 矩形	opposite sides equal, four right angles
parallelogram 平行四边形	opposite sides parallel and equal
rhombus 菱形	four equal sides, opposite sides parallel
kite 鸢形	two pairs of equal sides next to each other
trapezium 梯形	one pair of parallel sides

Polygons



A honeycomb tessellates the plane with regular hexagons.

Image: Vraj Acharya, CC BY-SA 4.0 (commons.wikimedia.org)

A **polygon** 多边形 is a shape with straight sides. A **regular polygon** 正多边形 has all sides and all angles equal.

Sides	Name
5	pentagon 五边形
6	hexagon 六边形
8	octagon 八边形
10	decagon 十边形

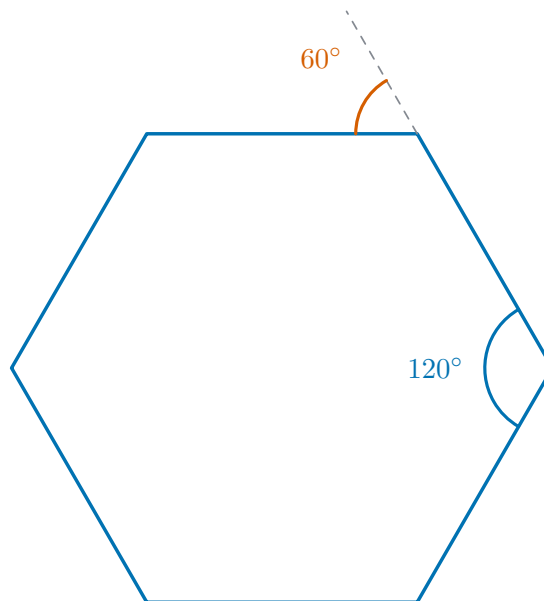
For a polygon with n sides:

$$\text{sum of interior angles} = (n - 2) \times 180^\circ, \quad \text{sum of exterior angles} = 360^\circ.$$

The **interior angle** 内角 and the **exterior angle** 外角 at each corner add up to 180° .

Worked example. Find each interior angle of a regular hexagon.

The exterior angle is $\frac{360^\circ}{6} = 60^\circ$, so each interior angle is $180^\circ - 60^\circ = 120^\circ$.



$$\text{interior} + \text{exterior} = 180^\circ$$

At each corner the interior and exterior angles add to 180° ; a regular hexagon has 60° exterior and 120° interior angles.

Symmetry



The Taj Mahal has a clear line of symmetry down its centre.

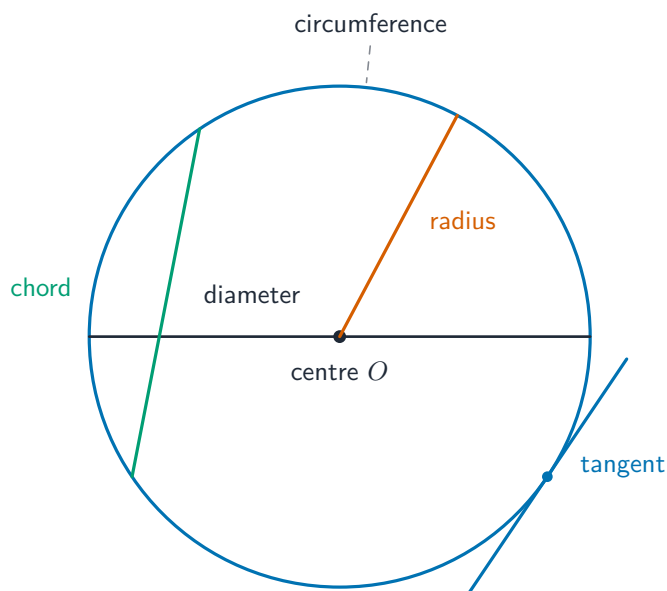
Image: Jonathan Freundlich, CC BY-SA 3.0 (commons.wikimedia.org)

- A shape has **line symmetry** 轴对称 if a mirror line splits it into two matching halves.
- A shape has **rotational symmetry** 旋转对称 if it fits onto itself as you turn it. The order is how many times it fits in one full turn.

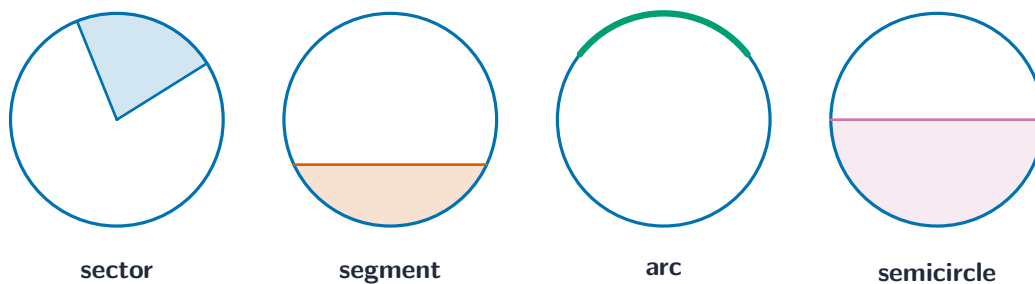
For solids (**Extended**), a flat slice that splits the solid into mirror halves is a **plane of symmetry** 对称面, and a line you can spin it around is an **axis of symmetry** 对称轴.

Circles: the parts

Term	Meaning
circle 圆	all points the same distance from a centre
centre 圆心	the middle point
radius 半径	centre to edge (plural <i>radii</i>)
diameter 直径	right across through the centre ($= 2 \times$ radius)
circumference 圆周	the distance all the way round
chord 弦	a straight line joining two points on the circle
arc 弧	part of the circumference
sector 扇形	a "pizza slice" between two radii
segment 弓形	the region cut off by a chord
semicircle 半圆	half a circle
tangent 切线	a line that touches the circle at one point



The main parts of a circle: a chord joins two points, while a tangent touches at just one.



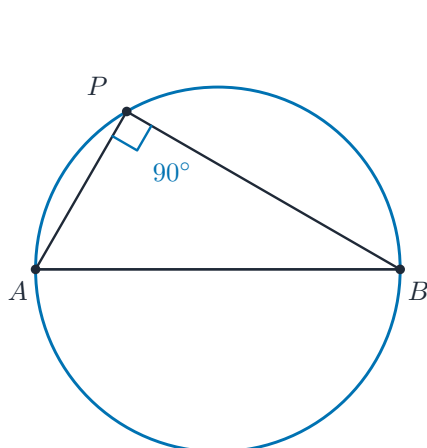
A sector is a slice between two radii, a segment is cut off by a chord, an arc is part of the circumference, and a semicircle is half the circle.

Circle theorems

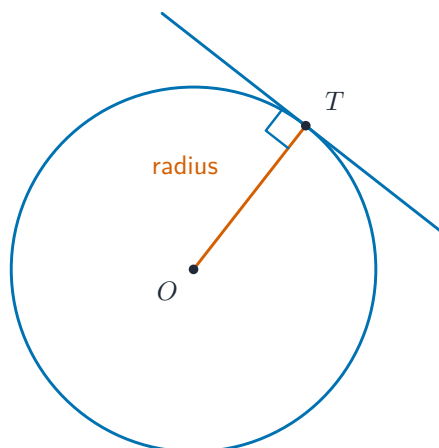
Use these to find unknown angles, always giving the reason.

For both levels:

- The angle in a **semicircle** is 90° .
- The angle between a **tangent** and a **radius** is 90° .



**angle in a
semicircle = 90°**



**tangent meets
radius = 90°**

Two theorems for both levels: the angle in a semicircle is 90° , and a tangent meets a radius at 90° .

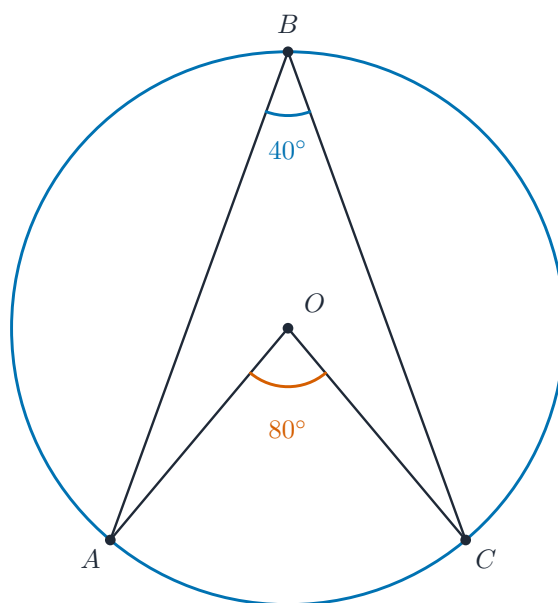
Extended (theorems I):

- The angle at the **centre** is twice the angle at the **circumference** (standing on the same arc).
- Angles in the same **segment** are equal.
- Opposite angles of a **cyclic quadrilateral** 圆内接四边形 add up to 180° .
- The **alternate segment theorem** 弦切角定理: the angle between a tangent and a chord equals the angle in the other segment.

Extended (theorems II): equal chords are the same distance from the centre; the perpendicular bisector of a chord passes through the centre; two tangents from the same outside point are equal in length.

Worked example. A, B, C are on a circle. The angle at the circumference ABC is 40° . Find the angle AOC at the centre O .

The angle at the centre is twice the angle at the circumference: $2 \times 40^\circ = 80^\circ$.



angle at centre = $2 \times$ angle at circumference

The angle at the centre (80°) is twice the angle at the circumference (40°) standing on the same arc.

Similar shapes

Two shapes are **similar** 相似 if one is an enlargement of the other: same angles, and all sides multiplied by the same **scale factor** 比例因子 k . (Shapes that are exactly the same size and shape are **congruent** 全等.)

For similar shapes and solids:

$$\frac{\text{area of } A}{\text{area of } B} = k^2, \quad \frac{\text{volume of } A}{\text{volume of } B} = k^3.$$

Worked example. Two similar solids have lengths in the ratio $2 : 3$. The smaller has volume 40 cm^3 . Find the volume of the larger.

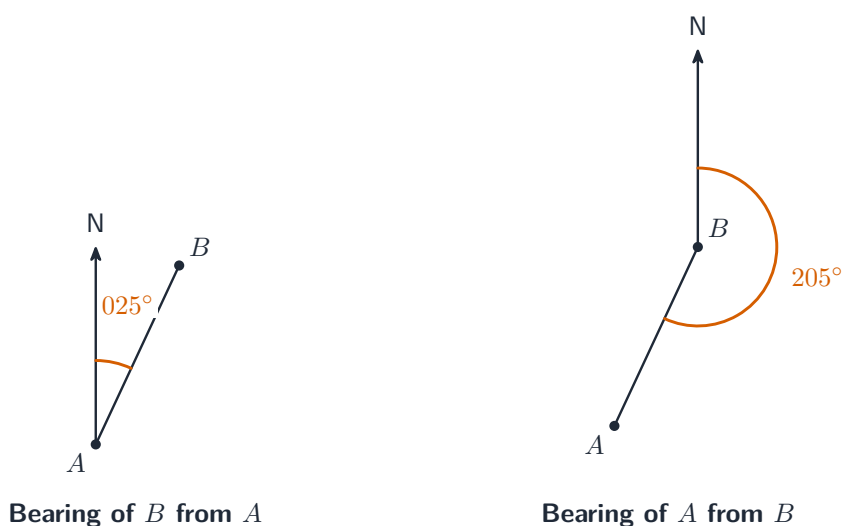
The volume ratio is $2^3 : 3^3 = 8 : 27$. So the larger volume is $40 \times \frac{27}{8} = 135 \text{ cm}^3$.

Bearings

A **bearing** 方位角 gives a direction as a three-figure angle, measured **clockwise** 顺时针 from north, from 000° to 360° . So due east is 090° and due south is 180° .

Worked example. The bearing of B from A is 025° . Find the bearing of A from B .

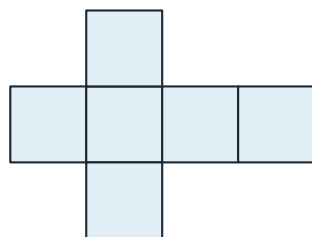
The return (back) bearing differs by 180° : $025^\circ + 180^\circ = 205^\circ$.



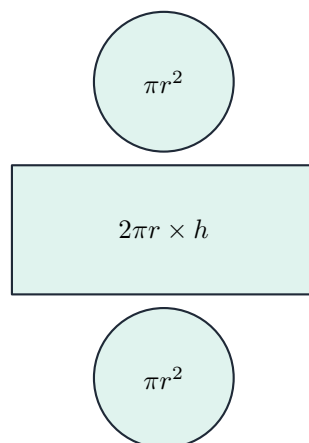
A bearing is measured clockwise from north as three figures; the back bearing differs by 180° .

Constructions, nets and solids

- To **construct** 作图 a triangle from three given sides, draw the base with a ruler, then use a pair of **compasses** 圆规 to mark each other side as an arc. Leave the construction arcs showing.
- A **net** 展开图 is a flat shape that folds up into a solid. You can use a net to work out surface areas.
- A **scale drawing** 比例图 shows a real object smaller (or larger) by a fixed scale, such as 1 cm to 5 m.



Cube net
6 equal square faces



Cylinder net
2 circles + a rectangle

Unfolding a solid gives its net. Add the areas of the pieces to get the surface area: a cylinder is $2\pi r^2 + 2\pi rh$.

Common **solids** 几何体 and their parts:

Solid	Note
cube 立方体 / cuboid 长方体	box shapes
prism 棱柱	the same shape all along its length
cylinder 圆柱	a circular prism
pyramid 棱锥 / cone 圆锥	come to a point
sphere 球 / hemisphere 半球	a ball / half a ball
frustum 平截头体	a cone or pyramid with the top cut off

A flat side of a solid is a **face** 面, two faces meet at an **edge** 棱, and the whole outside is its **surface** 表面.

Exam tips

- Angles on a straight line add to 180° , around a point to 360° , and in a triangle to 180° . Give a **reason** for every step of an angle question.
- Learn the circle theorems: the angle at the centre is *twice* the angle at the circumference; the angle in a semicircle is 90° ; angles in the same segment are equal; opposite angles of a cyclic quadrilateral add to 180° .
- The **exterior angles** of any polygon add to 360° , and each interior angle + its exterior angle = 180° .
- Bearings** are measured clockwise from north and always written with three figures (e.g. 072°).