PRECALCULUS

SECTION I, Part A

Time—1 hour and 20 minutes

28 Questions

NO CALCULATOR IS ALLOWED FOR THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

In this exam:

1. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
2. Angle measures for trigonometric functions are assumed to be in radians.

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A

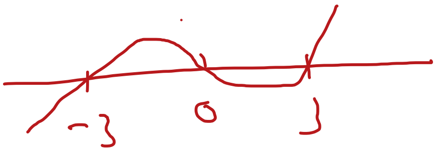
1. The polynomial function f is given by $f(x) = x(x^2 - 9)$. What are all intervals on which $f(x) < 0$?

(A) $(-\infty, -3)$ and $(0, 3)$

(B) $(-\infty, -3)$ and $(3, \infty)$

(C) $(-3, 0)$ and $(3, \infty)$

(D) $(-3, 3)$



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2. If $\log_8 c = 4$, which of the following must be true?

D

(A) $4^8 = c$

(B) $4^c = 8$

(C) $8^c = 4$

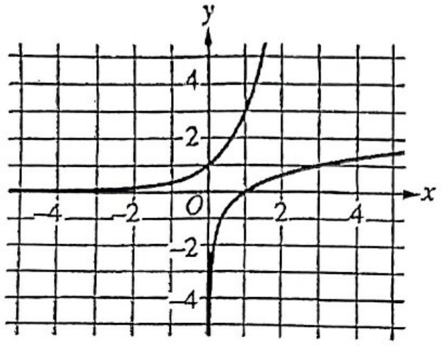
(D) $8^4 = c$

$c = 8^4$

A

3. The functions f and g are inverse functions such that $g(x) = f^{-1}(x)$. Which of the following could be the graphs of $y = f(x)$ and $y = g(x)$ in the same xy-plane?

(A)



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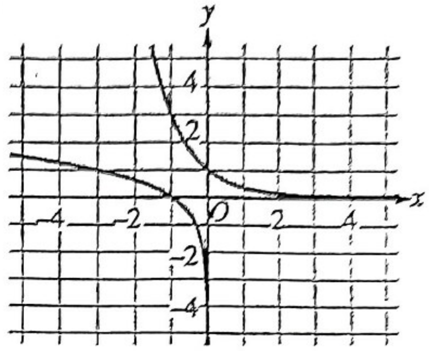

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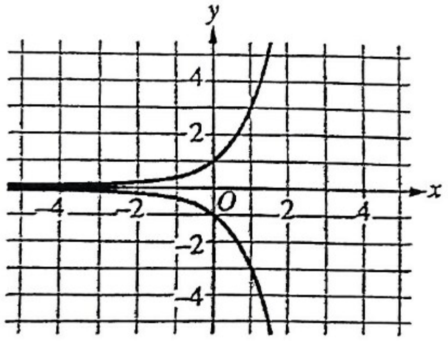

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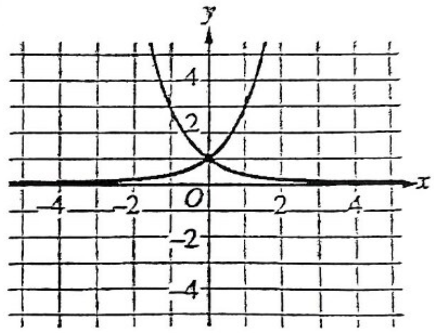

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4. The number of cells in a culture is known to double every 5 hours. There are 37 cells at time $t = 0$ hours. Which of the following is an appropriate model for $N(t)$, the number of cells in the culture at time t hours?

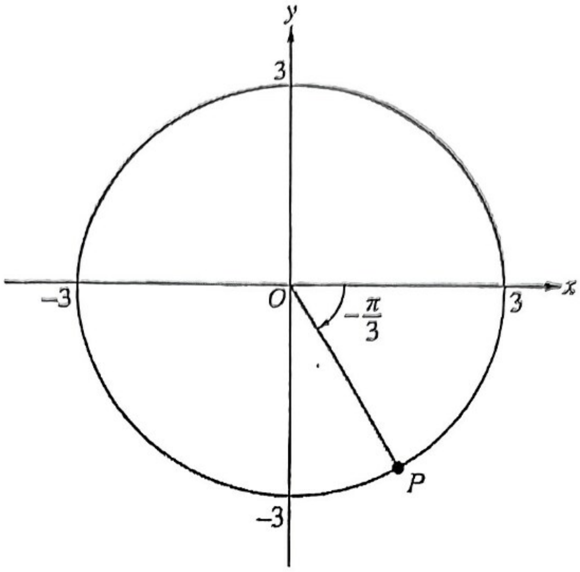
C

(A) $N(t) = 37 + 2^{(t/5)}$

(B) $N(t) = 37 + 2^{(5t)}$

(C) $N(t) = 37 \cdot 2^{(t/5)}$

(D) $N(t) = 37 \cdot 2^{(5t)}$



5. The figure shows a circle of radius 3 centered at the origin in the xy-plane and an angle of measure $-\frac{\pi}{3}$ in standard position. What are the coordinates of point P on the terminal ray that intersects this circle of radius 3 ?

B

(A) $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ (B) $\left(\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$ (C) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ (D) $\left(\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$

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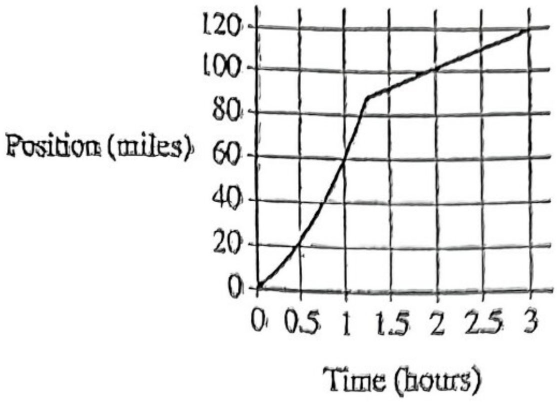

A

6. A car is traveling on a straight road. A graph of the car's position, in miles, from its starting point at time $t = 0$ hours on a particular day is shown. Which of the following is true about the car's position for $0 \le t \le 3$ hours?

(A) The rate of change of the car's position is increasing until about time $t = 1.25$, when the position begins to increase at a constant rate.

(B) The rate of change of the car's position is increasing until about time $t = 1.25$; after this time, the position is constant.

(C) The rate of change of the car's position is increasing at an increasing rate during the entire time interval.

(D) The rate of change of the car's position is increasing at a constant rate during the entire time interval.

x	6	7	8	9
$k(x)$	-3	-1	$-\frac{1}{3}$	$-\frac{1}{9}$

7. Values for an exponential function k are given in the table at selected values of x. Based on this information which of the following describes k ?

D

(A) k is always decreasing, and the graph of k is always concave up.

(B) k is always decreasing, and the graph of k is always concave down.

(C) k is always increasing, and the graph of k is always concave up.

(D) k is always increasing, and the graph of k is always concave down.

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x	-2	-1	0	1	2
$f(x)$	-4	-2	-1	-1	4
$g(x)$	-2	4	2	-4	3

8. The functions f and g both have a domain of all real numbers. The table gives values for $f(x)$ and $g(x)$ at selected values of x. Which of the following statements gives a solution to the equation $f(g(x)) = 4$ with correct reasoning?

B

(A) $x = 0$, because $f(0) = -1$ and $g(-1) = 4$.

(B) $x = 0$, because $g(0) = 2$ and $f(2) = 4$.

(C) $x = 1$, because $f(1) = -1$ and $g(-1) = 4$.

(D) $x = 1$, because $g(1) = -4$ and $f(1) = -1$.

9. The function g is graphed in the xy-plane and has a minimum at the point $(-2, 3)$. The function h results from two additive transformations of the function g and is graphed in the same xy-plane. If $h(x) = g(x+1) - 3$, which of the following points must be the location of a minimum of h ?

(A) $(-5, 2)$

(B) $(-3, 0)$

C

(C) $(-1, 0)$

(D) $(-1, 6)$

10. Let f and g be polynomial functions given by $f(x) = 3x^4 + 2x^2 - 5x + 1$ and $g(x) = x^2 + x + 1$. What is the remainder when f is divided by g ?

C

(A) $-4x$

(B) 3

(C) $-4x - 1$

(D) $6x + 9$

$3x^2 - 3x + 2$

$x^2+x+1 \,\big)\, 3x^4 + 0x^3 + 2x^2 - 5x + 1$

$3x^4 + 3x^3 + 3x^2$

$-3x^3 - x^2 - 5x$

$-3x^3 - 3x^2 - 3x$

$2x^2 - 2x + 1$

$2x^2 + 2x + 2$

$-4x - 1$

A

11. The temperature of a hot object that is cooling can be modeled by the function h. Time t is measured in minutes since the object started cooling, and $h(t)$ is measured in degrees Celsius (°C). The function h is an exponential decay function.

It is known that the temperature of the object decreases 10°C from time $t = 4$ to $t = 7$ minutes. It is also known that after time $t = 7$ minutes, the temperature of the object decreases an additional 10°C. Which of the following statements is true about the amount of time the object will require to cool that additional 10°C ?

(A) The object will require more than 3 minutes to cool that additional 10°C because as t increases, the rates of change of h are negative and increasing toward 0.

(B) The object will require more than 3 minutes to cool that additional 10°C because as t increases, the rates of change of h are negative and decreasing without bound.

(C) The object will require fewer than 3 minutes to cool that additional 10°C because as t increases, the rates of change of h are negative and increasing toward 0.

(D) The object will require fewer than 3 minutes to cool that additional 10°C because as t increases, the rates of change of h are negative and decreasing without bound.

C

12. A cubic polynomial function p has a non-real zero of $x = -3 + i$. The graph of $y = p(x)$ has an x-intercept of $(1, 0)$. Which of the following could be an expression for $p(x)$?

(A) $(x-1)(x^2 - 6x - 10)$

(B) $(x-1)(x^2 - 6x + 10)$

(C) $(x-1)(x^2 + 6x + 10)$

(D) $(x+1)(x^2 + 6x - 10)$

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13. Let r be the function given by $r(x) = \dfrac{2(x-3)(x+1)(x-1)}{(x+1)(x-1)^2(x+2)}$. Which of the following is true about the location of holes in the graph of r in the xy-plane?

B

(A) The graph of r has exactly one hole, which is at $x = -1$.
(B) The graph of r has exactly two holes, which are at $x = -1$ and $x = 1$.
(C) The graph of r has exactly two holes, which are at $x = -2$ and $x = 1$.
(D) The graph of r has exactly two holes, which are at $x = 1$ and $x = 3$.

x	0	4	8	12	16	20	24
$g(x)$	35	30	35	40	35	30	35

14. The table provides values for a periodic function g at selected values of x. The function g can be defined by $g(x) = 5\cos(B(x + C)) + 35$, where B and C are nonzero constants. Which of the following are possible values for B and C ?

C

(A) $B = \frac{\pi}{4}$ and $C = 4$

(B) $B = \frac{\pi}{4}$ and $C = 8$

(C) $B = \frac{\pi}{8}$ and $C = 4$

(D) $B = \frac{\pi}{8}$ and $C = 8$

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15. The polynomial function p is given by $p(x) = ax^n + 2x^3 - 4x^2 - 8$, where $a > 0$ and n is an odd integer greater than 3. Which of the following describes the end behavior of p ?

B

(A) As the input values of p increase without bound, the output values can be described by $\lim_{x \to \infty} p(x) = -\infty$.

(B) As the input values of p increase without bound, the output values increase without bound.

(C) As the input values of p decrease without bound, the output values can be described by $\lim_{x \to -\infty} p(x) = \infty$.

(D) As the input values of p decrease without bound, the output values get arbitrarily close to -8 and stay arbitrarily close to -8.

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16. Let f be the rational function defined by $f(x)=\dfrac{-3(x+1)^2(x-4)}{(x+2)}$. Which of the following represents all intervals where $f(x)<0$?

B

(A) $(4, \infty)$ only

(B) $(-\infty, -2)$ and $(4, \infty)$

(C) $(-2, -1)$ and $(-1, 4)$

(D) $(-2, -1)$ and $(4, \infty)$

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17. The function f is given by $f(x)=2+\log_2 x$, and the function g is given by $g(x)=2^{(x-2)}$. Which of the following statements provides correct reasoning for why functions f and g are inverse functions of each other?

B

(A) Because both functions have the same domain and the same range

(B) Because $2+\log_2(2^{(x-2)})=x$ and $2^{((2+\log_2 x)-2)}=x$

(C) Because f involves a logarithmic function with base 2, and g involves an exponential function with base 2

(D) Because the reflection of the graph of f over the y-axis produces the graph of g, and the reflection of the graph of g over the y-axis produces the graph of f

18. The population of rabbits in a national park is modeled by the function P given by $P(t) = 500 + 100\cos\left(\frac{\pi}{3}t\right)$, where t is measured in years for $0 \leq t \leq 15$. Based on this model, for $0 \leq t \leq 15$, at what time t is the first time that the rabbit population equals 400 ?

D

(A) $t = 15$
(B) $t = 4.5$
(C) $t = \pi$
(D) $t = 3$

GO ON TO THE NEXT PAGE.

x	$f(x)$
-1	$-\frac{\pi}{2}$
$-\frac{\sqrt{2}}{2}$	$-\frac{\pi}{4}$
0	0
$\frac{\sqrt{2}}{2}$	$\frac{\pi}{4}$
1	$\frac{\pi}{2}$

19. The table gives values for a function f at selected values of x. Which of the following could be an expression for $f(x)$?

C

(A) $\frac{1}{\sin x}$ (B) $\frac{1}{\cos x}$ (C) $\sin^{-1}x$ (D) $\cos^{-1}x$

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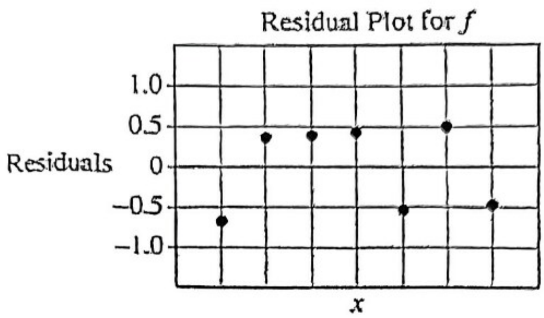

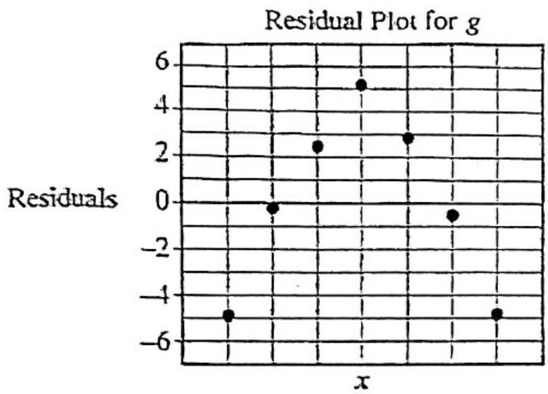

D

20. Two regression models f and g are constructed from a data set. The residual plots for both models are shown. Which of the following statements is true?

(A) Model f is justified as appropriate for the data set because the residual plot does not show a pattern.

(B) Model f is justified as appropriate for the data set because the residual plot does show a pattern.

(C) Model g is justified as appropriate for the data set because the residual plot does not show a pattern.

(D) Model g is justified as appropriate for the data set because the residual plot does show a pattern.

C

21. Let $f(x) = \ln x$ and $g(x) = 3\ln(x+2)$. In the xy-plane, which of the following statements about the vertical asymptotes of the graphs of f and g is true?

(A) The vertical asymptote of the graph of g is shifted 3 units to the left of the vertical asymptote of the graph of f.

(B) The vertical asymptote of the graph of g is shifted 3 units to the right of the vertical asymptote of the graph of f.

(C) The vertical asymptote of the graph of g is shifted 2 units to the left of the vertical asymptote of the graph of f.

(D) The vertical asymptote of the graph of g is shifted 2 units to the right of the vertical asymptote of the graph of f.

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x	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
$p(x)$	-12	-4.375	0	1.875	2	1.125	0	-0.625	0	2.625	8

22. The table provides values for a cubic polynomial function p at selected values of x. What are all intervals of x for which $p(x) \geq 0$?

B

(A) $[-2, \infty)$

(B) $[-2, 0]$ and $[1, \infty)$ only

(C) $[-2, 0]$ and $[1, 2]$ only

(D) $[-1.5, -0.5]$ and $[1.5, 2]$ only

GO ON TO THE NEXT PAGE.

23. Which of the following is an equivalent form of the expression $\sin x \cos(-3x) + \sin(-3x) \cos x$?

(A) $\cos(4x)$
(B) $\sin(4x)$
(C) $\cos(-2x)$
(D) $\sin(-2x)$

D

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24. A computer program can find a specific name in an alphabetical list of names. The function h models the number of steps the program goes through to find the name in a list with a total of x names. The values of x change proportionately as values of $h(x)$ increase in equal-length intervals. Which of the following could be an expression for $h(x)$?

(A) $3 + 2x^2$
(B) $3e^{2x}$
(C) $3 \log_2 x$
(D) $3 \sin(2x)$

B

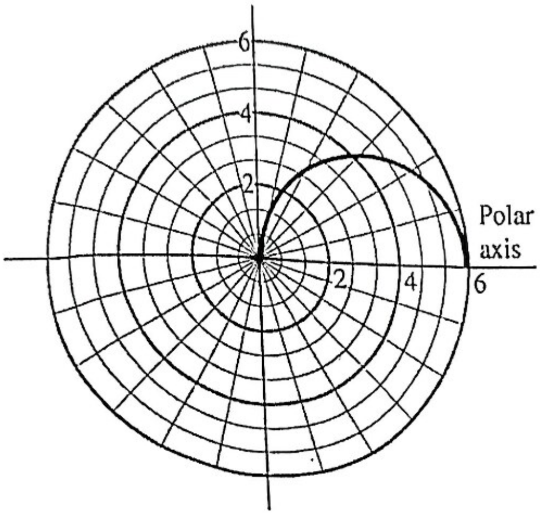
GO ON TO THE NEXT PAGE.

25. Consider the graph of the polar function $r = f(\theta)$, where $f(\theta) = -6\cos\theta$, in the polar coordinate system.

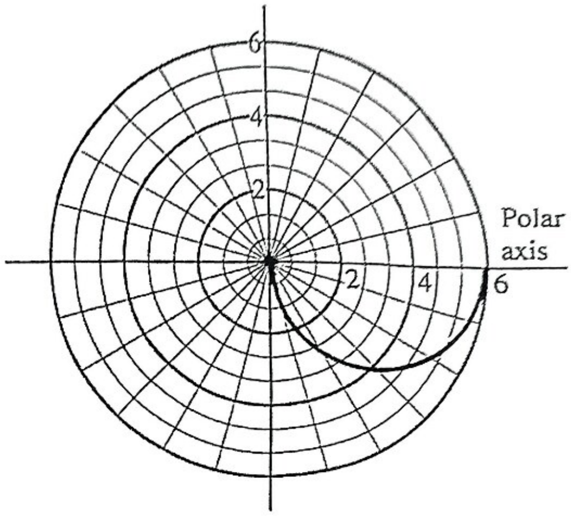
Which of the following is the graph of $r = f(\theta)$ for $0 \le \theta \le \frac{\pi}{2}$?

A

(A)



(B)



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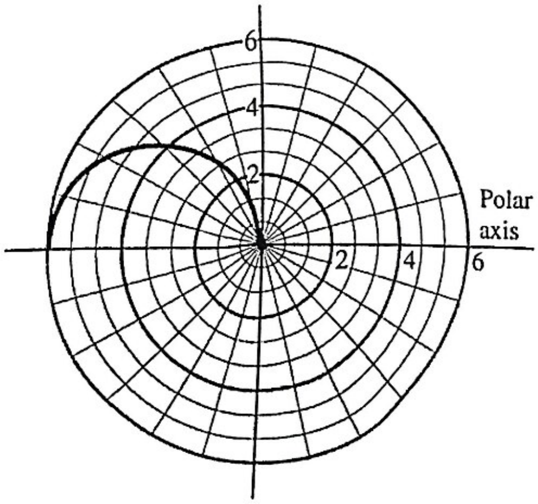

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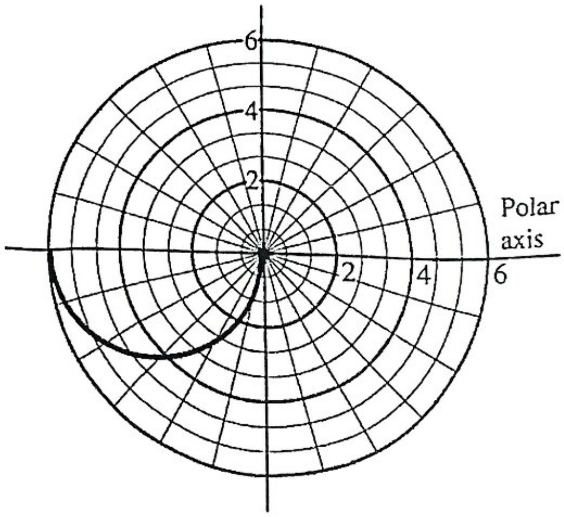

26. The function k is given by $k(x) = 2\sec x$ on the interval $[0, 2\pi]$. Which of the following describes the asymptotes of the graph of k in the xy-plane?

(A) The graph of k has vertical asymptotes at $x = 0$, $x = \pi$, and $x = 2\pi$ because the graph of $y = \sin x$ has x-intercepts at $x = 0$, $x = \pi$, and $x = 2\pi$.

B

(B) The graph of k has vertical asymptotes at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$ because the graph of $y = \cos x$ has x-intercepts at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

(C) The graph of k has vertical asymptotes at $x = 0$, $x = \pi$, and $x = 2\pi$ because the graph of $y = \cos x$ has x-intercepts at $x = 0$, $x = \pi$, and $x = 2\pi$.

(D) The graph of k has vertical asymptotes at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$ because the graph of $y = \sin x$ has x-intercepts at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

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27. The function k is a transformation of a logarithmic function with base 2. The domain of k is all real numbers less than 2. In the xy-plane, the graph of k has a y-intercept of $(0, 1)$ and a vertical asymptote at $x = 2$. Which of the following could be an expression for $k(x)$?

C

(A) $\log_2(x - 2)$

(B) $\log_2(x + 2)$

(C) $\log_2(2 - x)$

(D) $\log_2(2 - x) + 1$

GO ON TO THE NEXT PAGE.

28. The polar function $r = f(\theta)$, where $f(\theta) = 2 + \sin\theta$, is graphed in the polar coordinate system for $\frac{\pi}{2} \leq \theta \leq \pi$. Which of the following statements is true for the polar function as θ increases from $\frac{\pi}{2}$ to π ?

(A) The distance between the point with polar coordinates $(f(\theta), \theta)$ and the origin is increasing.

(B) The distance between the point with polar coordinates $(f(\theta), \theta)$ and the origin is decreasing.

(C) The distance between the point with polar coordinates $(f(\theta), \theta)$ and the origin is increasing, then decreasing.

D

(D) The distance between the point with polar coordinates $(f(\theta), \theta)$ and the origin is decreasing, then increasing.

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

PRECALCULUS

SECTION I, Part B

Time—40 minutes

12 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

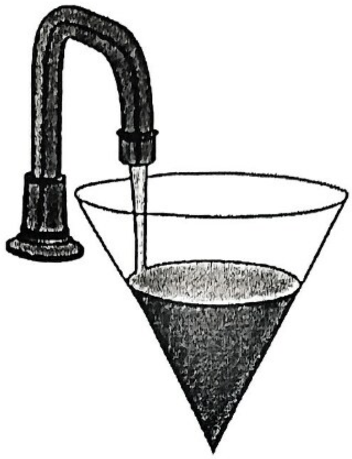
Directions: Solve each of the following problems, using the available space for scratch work. After examining the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

BE SURE YOU FILL IN THE CIRCLES ON THE ANSWER SHEET THAT CORRESPOND TO QUESTIONS NUMBERED 76–87.

YOU MAY NOT RETURN TO QUESTIONS NUMBERED 1–28.

In this exam:

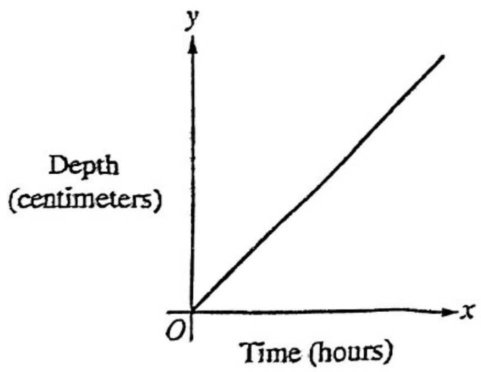
1. The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
3. Angle measures for trigonometric functions are assumed to be in radians. Make sure your calculator is in radian mode.



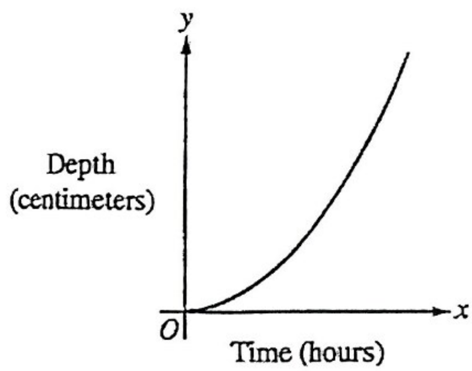
75. The figure shows a container in the shape of a circular cone. The container is being filled with water at a constant rate until the container is completely full. Which of the following could be the graph that models the dependent variable of depth of the water, in centimeters, as a function of the independent variable of time, in hours?

D

(A)



(B)



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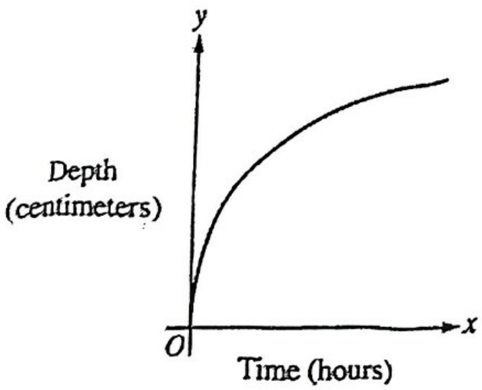

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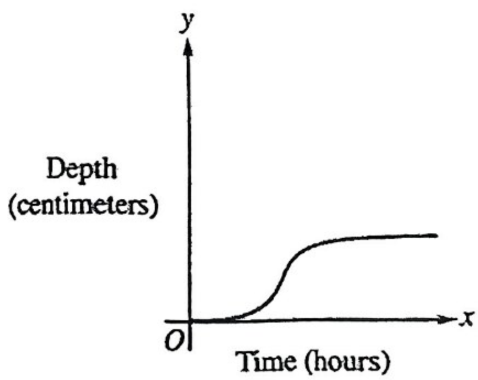

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77. The first four terms of a geometric sequence are given by $g_1 = \frac{1}{5}$, $g_2 = 1$, $g_3 = 5$, and $g_4 = 25$. Which of the following could be an expression for the general term of the sequence, g_n ?

A

(A) $g_n = \frac{1}{5}(5)^{(n-1)}$

(B) $g_n = 5\left(\frac{1}{5}\right)^{(n-1)}$

(C) $g_n = \frac{1}{5} + \frac{4}{5}n$

(D) $g_n = \frac{1}{5} + \frac{4}{5}(n-1)$

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78. The rate, in liters per hour, at which water is being pumped from a well is modeled by the function r given by $r(t) = (t+2)\cos\left(\frac{t}{3}\right) + 14$, where t is measured in hours since midnight ($t = 0$). What is the absolute maximum rate, in liters per hour, at which water is being pumped from the well over the 24-hour period for $0 \le t \le 24$?

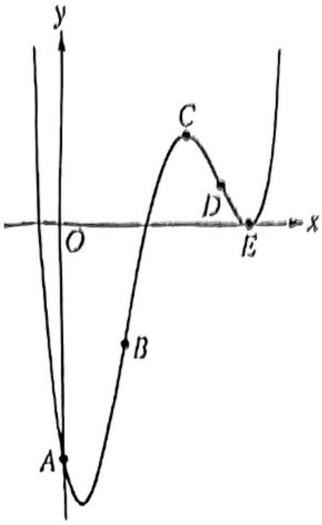
A

(A) 52.206

(B) 35.061

(C) 17.145

(D) 10.217



Graph of f

The figure shows the graph of a polynomial function f. Consider the five points labeled A, B, C, D, and E. These points indicate locations of intercepts and extrema for f and points of inflection of the graph of f. Which of the following statements is true?

B

(A) A is the location of a relative minimum, C is the location of a relative maximum, and D is a point of inflection of the graph.

(B) E is the location of a relative minimum, C is the location of a relative maximum, and B is a point of inflection of the graph.

(C) E is the location of a relative minimum, D is the location of a relative maximum, and B is a point of inflection of the graph.

(D) E is the location of a relative minimum, C is the location of a relative maximum, and A is a point of inflection of the graph.

x	$g(x)$
1.5	5
2.1	1
5.7	15
8.5	3
10.6	−4
14.3	2

80. The table presents values for a function g at selected values of x. A quartic regression is used to model the data. What is the value of $g(6.5)$ predicted by the quartic function model?

C

(A) 3.913
(B) 5.466
(C) 10.900
(D) 12.208

$$\frac{15-3}{2.7} \quad 3.5$$

$$y = \frac{12}{-2.7}(x-3) + 1.5$$

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81. The rational functions f and g are defined by $f(x) = \dfrac{30x}{x^2+10}$ and $g(x) = \dfrac{2x-1}{x^3+15}$. If the function h is defined by $h(x) = f(x) - g(x)$, what are all real zeros of h ?

C

(A) 0 and 0.5
(B) −1.901 and 3.230
(C) −2.395 and −0.023
(D) −4.566 and −0.067

$$\frac{30x}{x^2+10} - \frac{2x-1}{x^3+15} = 0$$

GO ON TO THE NEXT PAGE.

2. During a 24-hour period, the temperature, in degrees Celsius (°C), outside of a building can be modeled by a sinusoidal function f. The two consecutive lowest temperatures of 4°C during this time interval are recorded at times $t = 0$ and $t = 24$ hours. The highest temperature of 28°C during this time interval is recorded at time $t = 12$ hours. If $y = f(t)$, what is an equation for the midline of the graph of f in the coordinate plane?

(A) $y = \frac{2\pi}{24}$

(B) $y = 8$

(C) $y = 12$

(D) $y = 16$

? ? ?

GO ON TO THE NEXT PAGE.

83. The decreasing function f has the property that for all real numbers a and b in the domain of f, if $a < b$, then $f(a) > f(b)$. In the xy-plane, the graph of f is always concave up. Of the following, which is a possible table of values for f ?

D

(A)

x	$f(x)$
-1	7
0	6
1	4
2	2

(B)

x	$f(x)$
1	7.05
2	5.35
3	2.65
4	-1.05

(C)

x	$f(x)$
2	6.39
3	7.70
4	9.29
5	11.19

(D)

x	$f(x)$
7	4.34
8	1.87
9	0.80
10	0.35

B B B B B B B

4. At time $t = 0$, a bank account that earns 4% annual interest contains \$100. The amount of money, in dollars, in the account can be modeled by the exponential function v given by $v(t) = 100(1.04)^t$, with time t measured in years. How many years would it take for the value of the account to increase from \$200 to \$300 ? Assume that no money was taken out of the account, and no additional money besides interest was added to the account.

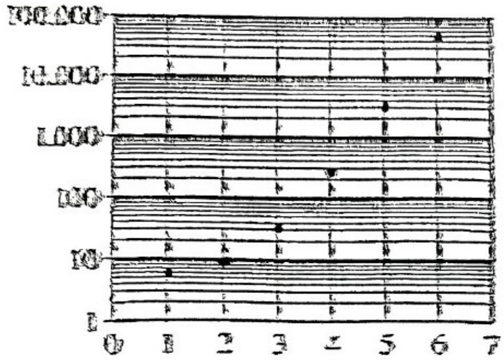
A

(A) 10.338
(B) 14.006
(C) 17.673
(D) 28.011

GO ON TO THE NEXT PAGE.

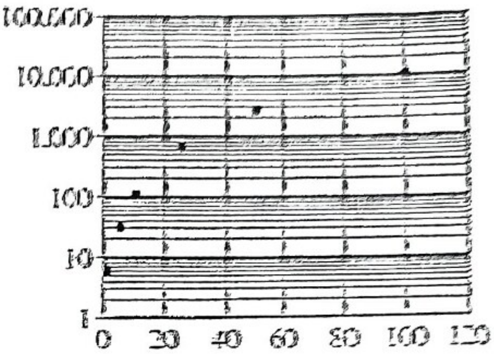
35. A data set is modeled by an exponential function of the form $g(x) = ab^x$, where $a > 0$ and $b > 1$. A semi-log plot of the data set is constructed. If g is an appropriate model, which of the following could be a semi-log plot of the data?

(A)

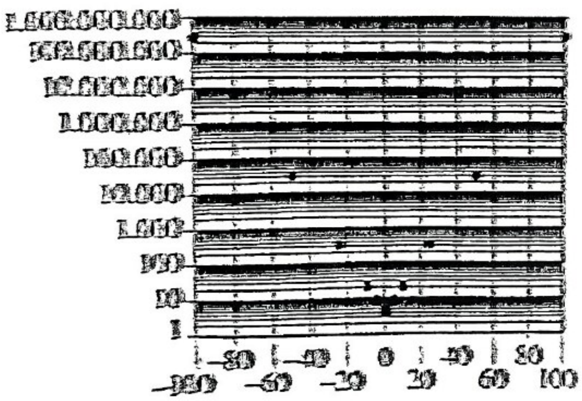


A

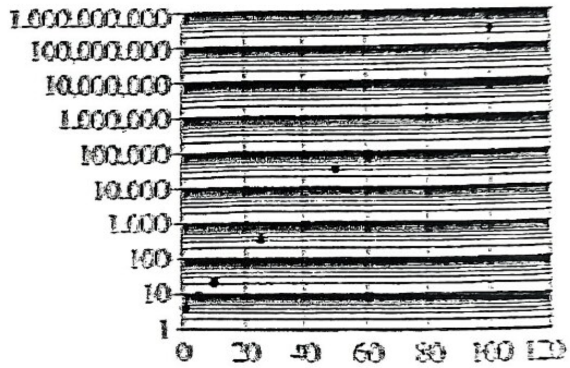
(B)



(C)



(D)



? ? ?

The function g is given by $g(x) = (x^2 - 3)\sin(x - 1)$. The function k is given by $k(x) = \cos x$. On the closed interval $[0, 2\pi]$, what are the x-coordinates of the points of intersection of the graphs of g and k in the xy-plane?

B

(A) $x = -0.510$, $x = -0.110$, and $x = 0.772$
(B) $x = 0.689$, $x = 1.681$, and $x = 4.177$
(C) $x = 1$, $x = 1.732$, and $x = 4.142$
(D) $x = 1.379$, $x = 3.274$, and $x = 6.058$

$(x^2-3)\sin(x-1) = \cos x$

87. Let $r = f(\theta)$ be a polar function with $f(\theta) = -4\sin\theta\cos(2\theta)$. Which of the following is an estimate for $r = f(1.5)$, using the average rate of change of r with respect to θ over the interval $\left[\frac{\pi}{6}, \frac{2\pi}{3}\right]$?

C

(A) -0.643

(B) 0.366

(C) 0.698

(D) 1.609

x	1	2	3	4	5	6
$f(x)$	4	6	9	13.5	20.25	30.375

1. The domain of f consists of the six real numbers 1, 2, 3, 4, 5, and 6. The table defines the function f for these values. The function g is given by $g(x) = \dfrac{12(x-2)}{(x+2)(x-5)}$.

(A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(6)$ as a decimal approximation, or indicate that it is not defined.

(ii) Find the value of $f^{-1}(4)$, or indicate that it is not defined.

(B) (i) Find all values of x, as decimal approximations, for which $g(x) = 5$, or indicate that there are no such values.

(ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

(C) (i) Use the table of values of $f(x)$ to determine if f is best modeled by a linear, quadratic, exponential, or logarithmic function.

(ii) Give a reason for your answer based on the relationship between the change in the output values of f and the change in the input values of f.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

2. Each spring a particular species of fish migrates up a river. "Migrates" refers to moving from one region to another according to the seasons. In 2015 ($t = 0$), there were [illegible] fish that migrated up the river. In 20[illegible] ($t = 6$), there were [illegible] fish that migrated up the river.

For $t \geq 0$, the number of fish that migrated up the river can be modeled by the function F given by $F(t) = ab^t$, where $F(t)$ is the number of fish that migrated up the river in year t, and t is the number of years since 2015.

(A) (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $F(t)$.

(ii) Find the values for a and b as decimal approximations.

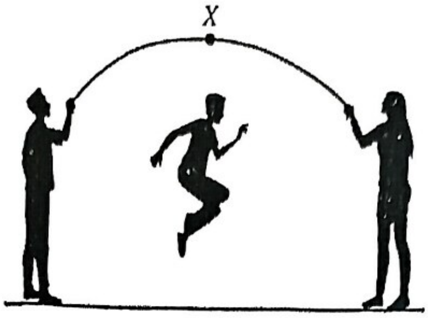
(B) (i) Use the given data to find the average rate of change of the number of fish that migrated up the river, in fish per year, from $t = 0$ to $t = 6$ years. Express your answer as a decimal approximation. Show the computations that lead to your answer.

(ii) Use the average rate of change found in (i) to estimate the number of fish that migrated up the river in 2017 ($t = 2$). Show the work that leads to your answer.

(iii) Let A_t represent the estimate of the number of fish that migrated up the river using the average rate of change found in (i). For A_2 found in (ii), it can be shown that $A_2 > F(2)$. Explain why, in general, $A_t > F(t)$ for all t, where $0 < t < 6$.

(C) Conservation efforts that guarantee an increase in the number of fish that migrate up the river are implemented when the number of fish that migrate up the river drops below [illegible]. Explain how this information can be used to determine the domain limitations for the model F.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.



Note: Figure not drawn to scale.

3. The figure shows two people turning a jump rope at a constant rate as a third person attempts to jump over the rope as it passes below their feet. Point X is located in the middle of the jump rope and is touching the ground at time $t = 0$ seconds. The next time that point X touches the ground is at time $t = 1$ second. The maximum height of X above the ground is 78 inches. As the rope turns, the height of X above the ground periodically increases and decreases.

The sinusoidal function h models the height of point X above the ground, in inches, as a function of time t, in seconds.

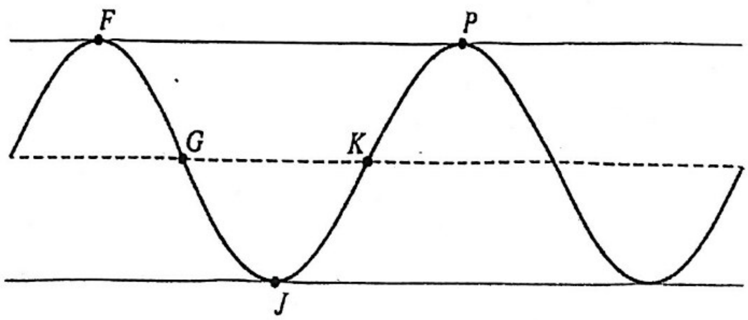
(A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.

Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.

$F(\frac{1}{2}, 78)$

$G(1, 39)$

$J(\frac{3}{2}, 0)$



$K(2, \cancel{8}39)$

$P(\frac{5}{2}, 78)$

(B) The function h can be written in the form $h(t) = a\cos(b(t + c)) + d$. Find values of constants a, b, c, and d.

$h(t) = a\cos(b(t+c)) + d$

$a = 39$

$\frac{2\pi}{b} = 2 \quad b = \pi$

$c = \frac{-1}{2}$

$d = 39$

$h(t) = 39\cos[\pi(t - \frac{1}{2})] + 39$

$\frac{\pi}{2} + bc) = 0$

$\pi c = \frac{?}{?}$

$c = c$

(C) Refer to the graph of h in part (A). The t-coordinate of G is t_1, and the t-coordinate of J is t_2.

(i) On the interval (t_1, t_2), which of the following is true about h ?

b

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

(ii) Describe how the rate of change of h is changing on the interval (t_1, t_2).

Because the graph of h is concave up on the interval (t_1, t_2) the rate of change

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

of h is increase on the interval (t_1, t_2)

4. Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x+3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = \arcsin(2x-1)$$
$$h(x) = 2^{(3x+1)}.$$

(i) Solve $g(x) = \frac{\pi}{6}$ for values of x in the domain of g.

(ii) Solve $h(x) = 7$ for values of x in the domain of h.

$(i)\ g(x) = \arcsin(2x-1) = \frac{\pi}{6}$

$2x-1 = \frac{1}{2} \quad x = \frac{3}{4}$

$(ii)\ h(x) = 2^{3x+1} = 7 \quad 3x+1 = \log_2 7$

$3x = \log_2 7 - 1$

$x = \frac{1}{3}(\log_2 7 - 1)$

(B) The functions j and k are given by

$$j(x) = \ln(x+2) - \ln 5 + 3\ln x$$
$$k(x) = \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)\cos x.$$

(i) Rewrite $j(x)$ as a single natural logarithm without negative exponents in any part of the expression. Your result should be of the form ln(expression).

$j(x) = \ln(x+2) - \ln 5 + 3\ln x = \ln \frac{(x+2)}{5} \cdot x^3$

$= \ln \frac{(x+2)x^3}{5}$

(ii) Rewrite $k(x)$ as an expression in which $\sin x$ appears once and no other trigonometric functions are involved.

$k(x) = \left(\frac{\sin^2 x + \cos^2 x}{\cos x \cdot \sin x}\right) \cdot \cos x$

$= \left(\frac{1}{\cos x \cdot \sin x}\right) \cdot \cos x$

$= \frac{1}{\sin x}$

(C) The function m is given by

$$m(x) = 2\cos^2 x - \cos x.$$

Find all values in the domain of m that yield an output value of 1.

$2\cos^2 x - \cos x = 1$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

$\cos x = -\frac{1}{2} \qquad \cos x = 1$

$x = 2k\pi + \frac{2\pi}{3},\ x = 2k\pi + \frac{4}{3}\pi, \quad x = 2k\pi$

Answer QUESTION 1 part (b) on this page.

x	1	2	3	4	5	6
$f(x)$	4	6	9	13.5	20.25	30.375

Response for question 1(b)

(i)

(i) $h(6) = g(f(6)) = g(30.375) = 0.414$

(ii)

(ii) $f^{-1}(4) = 1$

(i) $g(x) = 5 = \frac{12(x-2)}{(x+2)(x-5)}$

$x = 6.234$, $x = -0.834$

Answer QUESTION 1 parts (B) and (C) on this page.

Response for question 1(B)

(i)

$$g(x) = 5 = \frac{12(x-2)}{(x+2)(x-5)}$$

$x = 6.234$, $x = 0.834$

(ii)

As x increase without bound the output values of g not increase without bound therefore $\lim_{x\to\infty} g(x) = 0$

Response for question 1(C)

(i)

f is beset modeled by a exponential function

(ii)

The input values increase in equal length intervals as output values change proportionately.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

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Answer QUESTION 2 part (A) on this page.

Response for question 2(A)

(i)

$t=0 \quad F(t)=4500 \quad , \quad t=6 \quad F(t)=900$

$4500 = ab^0$

$900 = ab^6$

(ii)

$a = 4500 \qquad b = \left(\frac{1}{5}\right)^{\frac{1}{6}}$

Answer QUESTION 2 parts (B) and (C) on this page.

Response for question 2(B)

(i)

$$\frac{F(6)-F(0)}{6-0} = \frac{900-4500}{6} = -600$$

(ii)

The average rate of change is $r = -600$

using $(6, F(6))$

$y = 900 + r(x-6)$ $\quad y = 900 + (-600)\cdot(7-6) = 300$ fish

(iii)

The estimate $A(t)$ is the y-coordinate of a point on secant line at passes through $(0, F(0))$ and $(6, F(6))$. Because the graph of $F(t)$ is concave down for $t \geq 0$, this line is below the graph of F for $0 < t < 6$..

in general $A_t > F(t)$ for all t.

Response for question 2(C)

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.