CollegeBoard
Advanced Placement Program

AP® Physics C: Electricity and Magnetism
2007 Free-Response Questions

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AP Central is the official online home for the AP Program: apcentral.collegeboard.com.

TABLE OF INFORMATION FOR 2006 and 2007

CONSTANTS AND CONVERSION FACTORS

1 unified atomic mass unit,	$1\text{ u} = 1.66 \times 10^{-27}\text{ kg}$
	$= 931\text{ MeV}/c^2$
Proton mass,	$m_p = 1.67 \times 10^{-27}\text{ kg}$
Neutron mass,	$m_n = 1.67 \times 10^{-27}\text{ kg}$
Electron mass,	$m_e = 9.11 \times 10^{-31}\text{ kg}$
Electron charge magnitude,	$e = 1.60 \times 10^{-19}\text{ C}$
Avogadro's number,	$N_0 = 6.02 \times 10^{23}\text{ mol}^{-1}$
Universal gas constant,	$R = 8.31\text{ J/(mol}\cdot\text{K)}$
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}\text{ J/K}$
Speed of light,	$c = 3.00 \times 10^8\text{ m/s}$
Planck's constant,	$h = 6.63 \times 10^{-34}\text{ J}\cdot\text{s}$
	$= 4.14 \times 10^{-15}\text{ eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25}\text{ J}\cdot\text{m}$
	$= 1.24 \times 10^3\text{ eV}\cdot\text{nm}$
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2$
Coulomb's law constant,	$k = 1/4\pi\epsilon_0 = 9.0 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\text{ (T}\cdot\text{m)/A}$
Magnetic constant,	$k' = \mu_0/4\pi = 10^{-7}\text{ (T}\cdot\text{m)/A}$
Universal gravitational constant,	$G = 6.67 \times 10^{-11}\text{ m}^3/\text{kg}\cdot\text{s}^2$
Acceleration due to gravity at Earth's surface,	$g = 9.8\text{ m/s}^2$
1 atmosphere pressure,	$1\text{ atm} = 1.0 \times 10^5\text{ N/m}^2$
	$= 1.0 \times 10^5\text{ Pa}$
1 electron volt,	$1\text{ eV} = 1.60 \times 10^{-19}\text{ J}$

UNITS

Name	Symbol
meter	m
kilogram	kg
second	s
ampere	A
kelvin	K
mole	mol
hertz	Hz
newton	N
pascal	Pa
joule	J
watt	W
coulomb	C
volt	V
ohm	Ω
henry	H
farad	F
tesla	T
degree Celsius	°C
electron-volt	eV

PREFIXES

Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	1	0
30°	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$
37°	3/5	4/5	3/4
45°	$\sqrt{2}/2$	$\sqrt{2}/2$	1
53°	4/5	3/5	4/3
60°	$\sqrt{3}/2$	1/2	$\sqrt{3}$
90°	1	0	∞

The following conventions are used in this examination.

I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.
II. The direction of any electric current is the direction of flow of positive charge (conventional current).
III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.

MECHANICS

$v = v_0 + at$

$x = x_0 + v_0 t + \frac{1}{2}at^2$

$v^2 = v_0^{\,2} + 2a(x - x_0)$

$\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$

$\mathbf{F} = \frac{d\mathbf{p}}{dt}$

$\mathbf{J} = \int \mathbf{F}\,dt = \Delta\mathbf{p}$

$\mathbf{p} = m\mathbf{v}$

$F_{fric} \le \mu N$

$W = \int \mathbf{F} \cdot d\mathbf{r}$

$K = \frac{1}{2}mv^2$

$P = \frac{dW}{dt}$

$P = \mathbf{F} \cdot \mathbf{v}$

$\Delta U_g = mgh$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$

$I = \int r^2 dm = \sum mr^2$

$\mathbf{r}_{cm} = \sum m\mathbf{r} / \sum m$

$v = r\omega$

$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}$

$K = \frac{1}{2}I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
N = normal force
P = power
p = momentum
r = radius or distance
$\mathbf{r}$ = position vector
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration

$\mathbf{F}_s = -k\mathbf{x}$

$U_s = \frac{1}{2}kx^2$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$\mathbf{F}_G = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$

$U_G = -\frac{Gm_1m_2}{r}$

ELECTRICITY AND MAGNETISM

$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$

$\mathbf{E} = \frac{\mathbf{F}}{q}$

$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$

$E = -\frac{dV}{dr}$

$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$

$C = \frac{Q}{V}$

$C = \frac{\kappa\epsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$

$R = \frac{\rho\ell}{A}$

$\mathbf{E} = \rho\mathbf{J}$

$I = Nev_d A$

$V = IR$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = IV$

$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
ϕ_m = magnetic flux
κ = dielectric constant

$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$

$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\boldsymbol{\ell} \times \mathbf{r}}{r^3}$

$\mathbf{F} = \int I\,d\boldsymbol{\ell} \times \mathbf{B}$

$B_s = \mu_0 nI$

$\phi_m = \int \mathbf{B} \cdot d\mathbf{A}$

$\mathcal{E} = -\frac{d\phi_m}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2}LI^2$

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = \frac{1}{2}bh$

Circle
$A = \pi r^2$
$C = 2\pi r$

Parallelepiped
$V = \ell wh$

Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r\ell + 2\pi r^2$

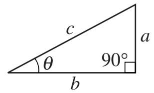
Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$

Right Triangle
$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

CALCULUS

$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \ne -1$

$\int e^x\,dx = e^x$

$\int \frac{dx}{x} = \ln|x|$

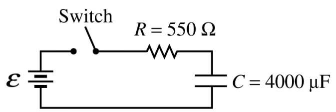
$\int \cos x\,dx = \sin x$

$\int \sin x\,dx = -\cos x$

PHYSICS C: ELECTRICITY AND MAGNETISM

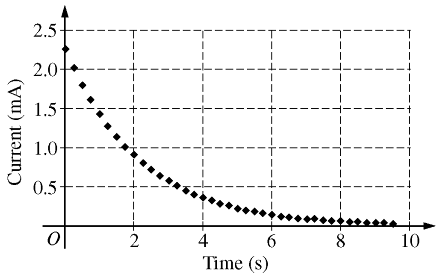
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.



E&M 1.

A student sets up the circuit above in the lab. The values of the resistance and capacitance are as shown, but the constant voltage $\mathcal{E}$ delivered by the ideal battery is unknown. At time $t = 0$, the capacitor is uncharged and the student closes the switch. The current as a function of time is measured using a computer system, and the following graph is obtained.



(a) Using the data above, calculate the battery voltage $\mathcal{E}$.

(b) Calculate the voltage across the capacitor at time $t = 4.0$ s.

(c) Calculate the charge on the capacitor at $t = 4.0$ s.

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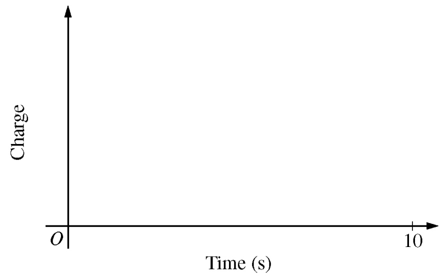
(d) On the axes below, sketch a graph of the charge on the capacitor as a function of time.



(e) Calculate the power being dissipated as heat in the resistor at $t = 4.0$ s.

(f) The capacitor is now discharged, its dielectric of constant $\kappa = 1$ is replaced by a dielectric of constant $\kappa = 3$, and the procedure is repeated. Is the amount of charge on one plate of the capacitor at $t = 4.0$ s now greater than, less than, or the same as before? Justify your answer.

_____Greater than   _____Less than   _____The same

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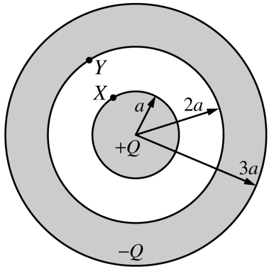

E&M 2.

In the figure above, a nonconducting solid sphere of radius a with charge $+Q$ uniformly distributed throughout its volume is concentric with a nonconducting spherical shell of inner radius $2a$ and outer radius $3a$ that has a charge $-Q$ uniformly distributed throughout its volume. Express all answers in terms of the given quantities and fundamental constants.

(a) Using Gauss's law, derive expressions for the magnitude of the electric field as a function of radius r in the following regions.

i. Within the solid sphere ($r < a$)

ii. Between the solid sphere and the spherical shell ($a < r < 2a$)

iii. Within the spherical shell ($2a < r < 3a$)

iv. Outside the spherical shell ($r > 3a$)

(b) What is the electric potential at the outer surface of the spherical shell ($r = 3a$)? Explain your reasoning.

(c) Derive an expression for the electric potential difference $V_X - V_Y$ between points X and Y shown in the figure.

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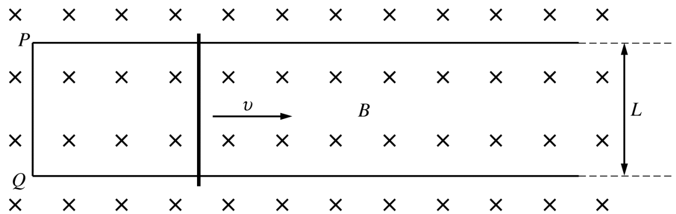

Top View

E&M 3.

In the diagram above, a nichrome wire of resistance per unit length λ is bent at points P and Q to form horizontal conducting rails that are a distance L apart. The wire is placed within a uniform magnetic field of magnitude B pointing into the page. A conducting rod of negligible resistance, which was aligned with end PQ at time $t = 0$, slides to the right with constant speed υ and negligible friction. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Indicate the direction of the current induced in the circuit.

______Clockwise   ______Counterclockwise

Justify your answer.

(b) Derive an expression for the magnitude of the induced current as a function of time t.

(c) Derive an expression for the magnitude of the magnetic force on the rod as a function of time.

(d) On the axes below, sketch a graph of the external force F_{ext} as a function of time that must be applied to the rod to keep it moving at constant speed while in the field. Label the values of any intercepts.

(e) The force pulling the rod is now removed. Indicate whether the speed of the rod increases, decreases, or remains the same.

______Increases   _____Decreases   ______Remains the same

Justify your answer.

END OF EXAM