1 Mark for Review

If $y = 1 + x - x^2 - x^3$, then $\frac{dy}{dx} =$

A $1 - 2x - 3x^2$

B $1 - x - x^2$

C $1 - \frac{1}{2}x - \frac{1}{3}x^2$

D $x + \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4$

2 Mark for Review

$\lim\limits_{x \to \infty} \frac{(6x+5)(-2x+3)}{(3x+1)(x+3)} =$

A -4

B -3

C 2

D 5

3 Mark for Review

Let f be the function defined by $f(x) = \sqrt{4x+1}$. Which of the following is the slope of the line tangent to the graph of f at $x = 6$?

A $\frac{1}{10}$

B $\frac{1}{5}$

C $\frac{2}{5}$

D $\frac{4}{5}$

Selected values of f and g and their first derivatives, f' and g', are shown in the table.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	3	2	1	4
3	2	4	3	5

4 Mark for Review

If $h(x) = g(f(x))$, what is the value of $h'(2)$?

(A) 4

(B) 5

(C) 10

(D) 14

5 Mark for Review

$\int \frac{1}{e^x} dx =$

(A) $-e^{-x} + C$

(B) $e^{-x} + C$

(C) $-\ln(e^x) + C$

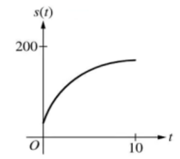
(D) $\ln(e^x) + C$

The position of a car traveling on a straight road at time t is given by the function $s(t)$, where t is measured in seconds and $s(t)$ is measured in meters. For $0 \le t \le 10$, the car has positive velocity and negative acceleration.

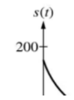
6 Mark for Review

Which of the following could be the graph of $s(t)$?

(A)



(B)



7 Mark for Review

Let *f* be the function defined by $f(x) = x^4 - 4x^3 + 8$. On what interval is *f* decreasing?

(A) $(-\infty, 3]$

(B) $[0, 2]$ only

(C) $[0, 3]$ only

(D) $[3, \infty)$

8 Mark for Review

$\lim_{x \to 0} \frac{2x - \sin x}{2 \sin x}$ is

(A) $-\frac{3}{2}$

(B) $\frac{1}{2}$

(C) 1

(D) nonexistent

9 Mark for Review

A particle moves along the x-axis with initial position $x = 0$ and initial velocity $v = 0$ at time $t = 0$. If the acceleration of the particle at any time $t \geq 0$ is $24t + 4$, then the position of the particle at any time $t > 0$ is

(A) $2t^3 + 4t^2$

(B) $4t^3 + 2t^2$

(C) $12t^3 + 4t^2$

(D) $12t^2 + 4t$

10 Mark for Review

Let h be the function defined by $h(x) = \begin{cases} \frac{3+x}{x^2+1} & \text{for } x < 2 \\ x^2 & \text{for } x \geq 2 \end{cases}$. Which of the following statements is true?

(A) h is continuous at $x = 2$.

(B) h has a removable discontinuity at $x = 2$.

(C) h has a jump discontinuity at $x = 2$.

(D) h has a discontinuity due to a vertical asymptote at $x = 2$.

11 Mark for Review

Consider the curve defined by $x^2 - 2xy - y^2 + 2 = 0$, where $\frac{dy}{dx} = \frac{x-y}{x+y}$. What are all points (x, y) at which the curve has a vertical tangent line?

(A) $(1, 1)$ and $(-1, -1)$

(B) $(1, -1)$ and $(-1, 1)$

(C) $(2, -2)$ and $(-2, 2)$

(D) There are no such points.

12 Mark for Review

If $f(x) = 5\arctan\left(e^{-3x}\right)$, then $f'(x) =$

(A) $\frac{-15e^{-3x}}{1+e^{-6x}}$

(B) $\frac{-15e^{-3x}}{1+x^2}$

(C) $\frac{5e^{-3x}}{1+e^{-6x}}$

(D) $\frac{5e^{-3x}}{1+x^2}$

13 Mark for Review

$\int(\sec^2 x + \frac{1}{x^2})dx =$

(A) $\tan x + \ln(x^2) + C$

(B) $\tan x - \frac{1}{x} + C$

(C) $\sec x \tan x + \ln(x^2) + C$

(D) $\sec x \tan x - \frac{1}{x} + C$

14 Mark for Review

Let f be a continuous function. Which of the following is a right Riemann sum approximation for $\int_2^6 f(x)dx$?

(A) $\sum_{k=1}^{8} f(k) \cdot \frac{1}{2}$

(B) $\sum_{k=1}^{8} f(2+k) \cdot \frac{1}{2}$

(C) $\sum_{k=1}^{8} f\left(\frac{k}{2}\right) \cdot \frac{1}{2}$

(D) $\sum_{k=1}^{8} f\left(2+\frac{k}{2}\right) \cdot \frac{1}{2}$

15 Mark for Review

What is the area of the region bounded by the y-axis, the graph of $y = \sqrt{x}$, and the horizontal line $y = 4$?

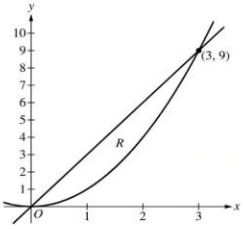
(A) $\frac{8}{3}$

(B) $\frac{32}{3}$

(C) $\frac{64}{3}$

(D) $\frac{128}{3}$

Let R be the region bounded by the graphs of $y = 3x$ and $y = x^2$, as shown in the figure.



16 Mark for Review

Which of the following integrals gives the volume of the solid formed by revolving R about the x-axis?

(A) $\pi \int_0^3 (3x - x^2)\,dx$

(B) $\pi \int_0^3 (3x - x^2)^2\,dx$

(C) $\pi \int_0^3 \left((3x)^2 - (x^2)^2\right)dx$

(D) $\pi \int_0^3 \left((x^2)^2 - (3x)^2\right)dx$

The first derivative, f', of a function f has the properties shown in the table.

x	$x < 2$	2	$2 < x < 4$	4	$x > 4$
$f'(x)$	Negative	0	Negative	0	Positive

17 Mark for Review

Which of the following statements about f must be true?

(A) f has a local minimum at $x = 4$ and no other local extrema.

(B) The graph of f has a point of inflection at $x = 4$ and no other points of inflection.

(C) f has a local minimum at $x = 2$ and a local maximum at $x = 4$.

(D) The graph of f has points of inflection at $x = 2$ and $x = 4$.

18 Mark for Review

If $y = f(x)$ is a solution to the separable differential equation $\frac{dy}{dx} = \frac{e^x}{2y}$, where $f(0) = 2$, what is the value of $f(1)$?

(A) $\sqrt{e+1}$

(B) $\sqrt{e+3}$

(C) $e+1$

(D) $\sqrt{e}+3$

19 Mark for Review

The function f is defined by $f(x) = \frac{x^2+8x+k}{x^2-25}$, where k is a constant. If $\lim_{x \to -5} f(x)$ exists, $\lim_{x \to 5^-} f(x) = -\infty$, and $\lim_{x \to 5^+} f(x) = \infty$, what is the value of k?

(A) -65

(B) 15

(C) 16

(D) k can be any real number.

20 Mark for Review

The function f is given by $f(x) = axe^{bx}$, where a and b are nonzero constants and $f(1) = 100$. For which of the following values of a and b does f have a local extremum at $x = 2$?

(A) $a = 2$ and $b = \ln 50$

(B) $a = 100\sqrt{e}$ and $b = -\frac{1}{2}$

(C) $a = 100e$ and $b = -1$

(D) $a = 100e^{e/4}$ and $b = -\frac{e}{4}$

21 Mark for Review

The radius and height of a right circular cylinder are each increasing at a rate of $\frac{1}{3}$ centimeter per second. At what rate, in cubic centimeters per second, is the volume of the cylinder increasing when the radius is 3 centimeters and the height is 10 centimeters? (The volume V of a right circular cylinder with radius r and height h is $V = \pi r^2 h$.)

(A) $\frac{2\pi}{3}$

(B) $\frac{20\pi}{3}$

(C) 23π

(D) 69π

22 Mark for Review

When substitution of variables is used, $\int_{-3}^{2} \frac{x}{\sqrt{x^2+1}} dx$ is equivalent to which of the following?

(A) $\frac{1}{2} \int_{-3}^{2} \frac{1}{\sqrt{u}} du$

(B) $\int_{-3}^{2} \frac{1}{\sqrt{u}} du$

(C) $\frac{1}{2} \int_{5}^{10} \frac{1}{\sqrt{u}} du$

(D) $\frac{1}{2} \int_{10}^{5} \frac{1}{\sqrt{u}} du$

23 Mark for Review

Let f be the function defined by $f(x) = 3e^{2x}$. What is the approximation of $f(0.1)$ found by using the line tangent to the graph of f at $x = 0$?

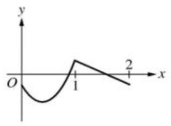
(A) 3.6

(B) 3.3

(C) 3

(D) 2.4

The graph of $y = f(x)$ on the closed interval $[0, 2]$ is shown.

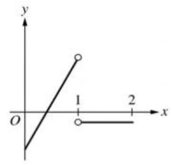


Graph of f

24 Mark for Review

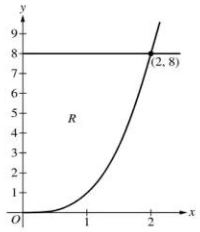
Which of the following could be the graph of $y = f'(x)$?

(A)



(B)

Let R be the region in the first quadrant bounded by the y-axis, the graph of $y = x^3$, and the horizontal line $y = 8$, as shown in the figure.



The region R is the base of a solid. For this solid, each cross section perpendicular to the y-axis is a square.

25 Mark for Review

What is the volume of the solid?

Ⓐ 4

Ⓑ 12

Ⓒ $\frac{128}{7}$

Ⓓ $\frac{96}{5}$

26 Mark for Review

$\int \frac{x+3}{x+1}\,dx =$

Ⓐ $x + \ln|x+1| + C$

Ⓑ $x + 2\ln|x+1| + C$

Ⓒ $\frac{x^2}{2} + 3x + \ln|x+1| + C$

Ⓓ $x^2 + 3x + \ln|x+1| + C$

27 Mark for Review

If $g(x) = \ln(1+2x)$, which of the following limits is equal to $g'(3)$?

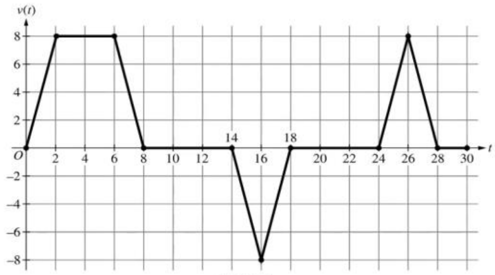
Ⓐ $\lim\limits_{x\to 3} \frac{\ln(1+2x)-\ln(7)}{x-3}$

Ⓑ $\lim\limits_{x\to 3} \frac{\ln(1+2x)-\ln(3)}{x-3}$

Ⓒ $\lim\limits_{h\to 0} \frac{\ln(1+2h)-\ln(7)}{h}$

Ⓓ $\lim\limits_{h\to 0} \frac{\ln(1+2h)-\ln(3)}{h}$

An elevator in a building moves vertically up and down from floor to floor. Floors in the building are 16 feet apart. The function v models the elevator's velocity at time t seconds, where $v(t)$ is measured in feet per second. The graph of v is shown for $0 \leq t \leq 30$.



Graph of v

28 Mark for Review

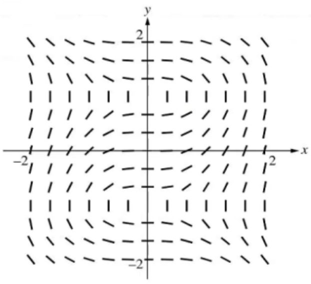
If the elevator is on the fourth floor at time $t = 0$, on what floor is the elevator at time $t = 30$?

Ⓐ Third

Ⓑ Fifth

Ⓒ Seventh

Ⓓ Ninth



29 Mark for Review

The figure shown is a slope field for which of the following differential equations?

Ⓐ $\frac{dy}{dx} = \frac{x}{1-y}$

Ⓑ $\frac{dy}{dx} = \frac{x^2}{1-y}$

Ⓒ $\frac{dy}{dx} = \frac{x}{1-y^2}$

Ⓓ $\frac{dy}{dx} = \frac{x^2}{1-y^2}$

30 Mark for Review

If a is a positive real number, what is $\int_{-a}^{a} x|x|\,dx$?

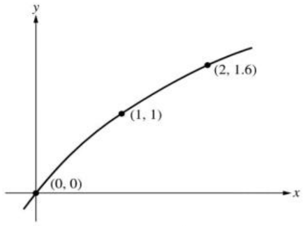
Ⓐ $-\frac{2a^3}{3}$

Ⓑ 0

Ⓒ $\frac{2a^3}{3}$

Ⓓ a^3

The graph of the function f shown is concave down on the open interval $(0, 2)$.



Graph of f

31 Mark for Review

Which of the following statements about $f'(1)$ is true?

(A) $0 < f'(1) < 0.6$

(B) $0.6 < f'(1) < 1$

(C) $f'(1) = 1$

(D) $f'(1) > 1$

32 Mark for Review

The first derivative of the function f is defined by $f'(x) = 2 - \left(\sqrt{x+9}\right) \cos x$ for $-7 < x < 7$. The graph of f has how many points of inflection on the open interval $-7 < x < 7$?

(A) One

(B) Three

(C) Four

(D) Five

33 Mark for Review

A particle moves along the x-axis so that at any time $t \geq 0$ its position is given by $x(t) = 4\ln(t + 0.5) + \frac{1}{2}e^{0.3t}$. What is the velocity of the particle at time $t = 6$?

(A) 1.523

(B) 2.131

(C) 10.512

(D) 34.469

During a race, the amount of gasoline consumed by a race car is given by the function V, where $V(t)$ is measured in gallons and t is the number of minutes since the start of the race. Values of $V(t)$ for selected values of t are given in the table.

t (minutes)	0	10	20	26	30
$V(t)$ (gallons)	0	8	11	15	17

34 Mark for Review

Using the average rate of change of $V(t)$ over the smallest interval of time in the table containing $t = 23$, what is the approximation for the instantaneous rate at which gasoline is consumed, in gallons per minute, at time $t = 23$ minutes?

(A) $\frac{1}{2}$

(B) $\frac{17}{30}$

(C) $\frac{2}{3}$

(D) $\frac{13}{3}$

35 Mark for Review

The rate at which a new product sells can be modeled by $S'(t) = \frac{3000}{1+3e^{-0.2t}}$, where $S(t)$ is the number of units sold and t is the time in weeks since the product went on sale. To the nearest integer, what was the average rate at which the product sold, in units per week, during the first 8 weeks of sales?

(A) 140

(B) 1289

(C) 1868

(D) 10,309

The velocity, in units per second, of a particle moving along the x-axis is given by $v(t) = \frac{t^3-9t+6}{3+\sin t}$, where t is the time in seconds. The particle is 4 units to the right of the origin at time $t = 0$.

36 Mark for Review

What is the position of the particle at time $t = 2$?

(A) 0.388 units to the left of the origin

(B) 0.977 units to the right of the origin

(C) 1.666 units to the right of the origin

(D) 3.612 units to the right of the origin

37 Mark for Review

The temperature, W, of a cup of hot water cools with respect to time at a rate that is directly proportional to the difference between the temperature of the water in degrees Fahrenheit ($^\circ\mathrm{F}$) and the room temperature of $68^\circ\mathrm{F}$. Which of the following is a differential equation that models this situation, where k is a constant?

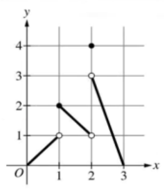
(A) $\frac{dW}{dt} = \frac{k}{t-68}$

(B) $\frac{dW}{dt} = k(t-68)$

(C) $\frac{dW}{dt} = \frac{k}{W-68}$

(D) $\frac{dW}{dt} = k(W-68)$

The graph of a function f is shown.



Graph of f

38 Mark for Review

What is $\lim_{x\to 2^+} f(x) + \lim_{x\to 1^-} f(x)$?

(A) 2

(B) 3

(C) 4

(D) 6

39 Mark for Review

The differentiable function $N(t)$ models the number of members of a health club, where t is the number of months after the club opened. Which of the following is the best interpretation of the statement $N'(4) = 13$?

(A) 4 months after the club opened, there were 13 more members of the club than when the club opened.

(B) 4 months after the club opened, the number of members of the club was increasing by 13 members per month.

(C) 4 months after the club opened, the rate of change of the number of members of the club was increasing by 13 members per month per month.

(D) During the first 4 months after the club opened, the number of members of the club was increasing an average of 13 members each month.

A function f is continuous for all x. The graph of $y = f(x)$ has a local maximum at $(-3, 5)$ and a local minimum at $(4, -2)$.

40 Mark for Review

Which of the following statements must be true?

(A) The graph of f has horizontal tangents at $x = -3$ and $x = 4$.

(B) The graph of f has a point of inflection at some x between $x = -3$ and $x = 4$.

(C) The graph of f has a slope of -1 at some x between $x = -3$ and $x = 4$.

(D) The graph of f has an x-intercept at some x between $x = -3$ and $x = 4$.

41 Mark for Review

The function f is continuous for $1 \leq x \leq 6$ with $\int_6^1 f(x)dx = -5$ and $\int_4^6 f(x)dx = -3$. What is the value of $\int_1^4 (5f(x) + 4)dx$?

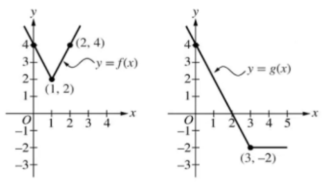
(A) -28

(B) 2

(C) 44

(D) 52

The graphs of the functions f and g consist of line segments, as shown.



42 Mark for Review

If h is the function given by $h(x) = f(g(2x))$, then $h'(1)$ is

(A) -8

(B) -4

(C) 4

(D) 8

Selected values of an increasing function f and its derivative f' are given in the table.

x	$f(x)$	$f'(x)$
2	3	4
3	4	5
4	6	3

43 Mark for Review

What is the value of $(f^{-1})'(4)$?

(A) $\frac{1}{6}$

(B) $\frac{1}{5}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

44 Mark for Review

The function f is given by $f(x) = x + 3\sin x - 4\cos x$. What is the difference between the two values of x that satisfy the conclusion of the Mean Value Theorem applied to f on the closed interval $[0, 6]$?

(A) 2.168

(B) 2.739

(C) 3.096

(D) 3.930

45 Mark for Review

The function f is defined by $f(x) = \int_1^x t\ln(t)e^{-t}dt$ for $x \geq 1$. What is the slope of the line tangent to the graph of f at the point where $x = 5$?

(A) -0.037

(B) -0.001

(C) 0.054

(D) 0.514

2748. An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function *C* defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species *t* weeks after the species appears. It can be shown that $C'(t) = \frac{38}{25+t^2}$.

(Note: Your calculator should be in radian mode.)

A. Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.

B. Find the time *t* when the instantaneous rate of change of *C* equals the average rate of change of *C* over the time interval $0 \leq t \leq 4$. Show the setup for your calculations.

C. Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

D. At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function *A*, defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x)dx$, models the number of acres affected by the species over the time interval $4 \leq t \leq 36$. At what time *t*, for $4 \leq t \leq 36$, does *A* attain its maximum value? Justify your answer.

2749.

A. Find the area of *R*. Show the setup for your calculations.

B. Region *R* is the base of a solid. For this solid, at each *x* the cross section perpendicular to the *x*-axis is a rectangle with height *x* and base in region *R*. Find the volume of the solid. Show the setup for your calculations.

C. Write, but do not evaluate, an integral expression for the volume of the solid generated when the region *R* is rotated about the horizontal line $y = -2$.

D. It can be shown that $g'(x) = 1 + \pi\cos(\pi x)$. Find the value of *x*, for $0 < x < 1$, at which the line tangent to the graph of *f* is parallel to the line tangent to the graph of *g*.

The shaded region *R* is bounded by the graphs of the functions *f* and *g*, where $f(x) = x^2 - 2x$ and $g(x) = x + \sin(\pi x)$, as shown in the figure.

- The graph shows two curves labeled y equals f open parentheses x close parentheses and y equals g open parenthesis x close parenthesis.
- Each graph is as follows:
- y equals f open parentheses x close parentheses:

- The curve begins to the left of the y axis, above the x axis, and moves downward and to the right.
- It crosses the origin, and reaches a minimum at the point with coordinates 1 comma negative 1, and then moves upward and to the right.
- It crosses the x axis at 2 and passes through the point 3 comma 3 before it ends.

- y equals g open parenthesis x close parenthesis:
  - The curve begins to the left of the y axis, below the x axis.
  - It moves downward and to the right and reaches a relative minimum at the point with approximate coordinates negative 0.6 comma negative 1.6, then moves upward and to the right.
  - It intersects the function y equals f open parentheses x close parentheses at the origin, and reaches a relative maximum at the point with approximate coordinates 0.6 comma 1.6.
  - It then moves downward and to the right and reaches a relative minimum at the point with approximate coordinates 1.4 comma 0.4.
  - The curve then moves upward and to the right and reaches a relative maximum at the point with approximate coordinates 2.6 comma 3.6.
  - The curve then moves downward and to the right, intersects the function y equals f open parentheses x close parentheses at the point with coordinates 3 comma 3, and reaches a relative minimum at the point with approximate coordinates 3.4 comma 2.4.
  - It then moves upward and to the right before ending.
- The region bounded from below by the function y equals f open parentheses x close parentheses and bounded from above by the function y equals g open parentheses x close parentheses is labeled R.

(Note: Your calculator should be in radian mode.)

2750.

A. Approximate $R'(1)$ using the average rate of change of *R* over the interval $0 \leq t \leq 2$. Show the work that leads to your answer. Indicate units of measure.

B. Must there be a value *c*, for $0 < c < 10$, such that $R(c) = 155$? Justify your answer.

C. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{10} R(t)dt$. Show the work that leads to your answer.

D. A teacher also starts reading at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the teacher reads is modeled by the function *W* defined by $W(t) = -\frac{3}{10}t^2 + 8t + 100$, where $W(t)$ is measured in words per minute. Based on the model, how many words has the teacher read by the end of the 10 minutes? Show the work that leads to your answer.

A student starts reading a book at time $t = 0$ minutes and continues reading for the next 10 minutes. The rate at which the student reads is modeled by the differentiable function *R*, where $R(t)$ is measured in words per minute. Selected values of $R(t)$ are given in the table shown.

***t* (minutes)**	0	2	8	10
$R(t)$ **(words per minute)**	90	100	150	162

2751.

A. Find $g'(8)$. Give a reason for your answer.

B. Find all values of *x* in the open interval $-6 < x < 12$ at which the graph of *g* has a point of inflection. Give a reason for your answer.

C. Find $g(12)$ and $g(0)$. Label your answers.

D. Find the value of *x* at which *g* attains an absolute minimum on the closed interval $-6 \leq x \leq 12$. Justify your answer.

The continuous function *f* is defined on the closed interval $-6 \leq x \leq 12$. The graph of *f*, consisting of two semicircles and one line segment, is shown in the figure.

- The graph consists of two semicircles and one line segment. Four closed circles are indicated on the graph.
- The first semicircle opens upward. It begins at a closed circle at the point with coordinates negative 6 comma 0, reaches a minimum at the point with coordinates negative 3 comma negative 3, and ends at a closed circle at the origin.
- The second semicircle begins where the first semicircle ends, and opens downward. It reaches a maximum at the point with coordinates 3 comma 3, and ends at a closed circle at the point with coordinates 6 comma 0.
- The line segment begins where the second semicircle ends, and slants upward and to the right until it ends at a closed circle at the point with coordinates 12 comma 3.

Let g be the function defined by $g(x) = \int_6^x f(t)dt$.

2752. Two particles, *H* and *J*, are moving along the *x*-axis. For $0 \le t \le 5$, the position of particle *H* at time *t* is given by $x_H(t) = e^{t^2-4t}$ and the velocity of particle *J* at time *t* is given by $v_J(t) = 2t\left(t^2 - 1\right)^3$.

A. Find the velocity of particle *H* at time $t = 1$. Show the work that leads to your answer.

B. During what open intervals of time *t*, for $0 < t < 5$, are particles *H* and *J* moving in opposite directions? Give a reason for your answer.

C. It can be shown that $v_J{}'(2) > 0$. Is the speed of particle *J* increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.

D. Particle *J* is at position $x = 7$ at time $t = 0$. Find the position of particle *J* at time $t = 2$. Show the work that leads to your answer.

2753. Consider the curve *G* defined by the equation $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$.

A. Show that $\frac{dy}{dx} = \frac{-x}{2(3y^2-2y-1)}$.

B. There is a point *P* on the curve *G* near $(2, -1)$ with *x*-coordinate 1.6. Use the line tangent to the curve at $(2, -1)$ to approximate the *y*-coordinate of point *P*.

C. For $x > 0$ and $y > 0$, there is a point *S* on the curve *G* at which the line tangent to the curve at that point is vertical. Find the *y*-coordinate of point *S*. Show the work that leads to your answer.

D. A particle moves along the curve *H* defined by the equation $2xy + \ln y = 8$. At the instant when the particle is at the point $(4, 1)$, $\frac{dx}{dt} = 3$. Find $\frac{dy}{dt}$ at that instant. Show the work that leads to your answer.

2025 微积分 AB 简答题答案：

2748. Invasive Species Problem

A.

Average number of acres affected from $t = 0$ to $t = 4$:

$$\text{Average} = \frac{1}{4}\int_0^4 7.6\arctan(0.2t)\,dt$$

Using the given derivative $C'(t)$, the integral evaluates to:

$$\text{Average} = \frac{7.6\arctan(0.8)}{4} \approx 1.9\arctan(0.8) \approx 1.9 \times 0.6747 \approx 1.28\text{ acres}$$

B.

Find t where $C'(t) = \text{Average rate of change}$:

$$\frac{38}{25+t^2} = \frac{7.6\arctan(0.8)}{4}$$

Solve numerically: $t \approx 2.24$ weeks.

C.

End behavior of the rate of change:

$$\lim_{t\to\infty} C'(t) = \lim_{t\to\infty}\frac{38}{25+t^2} = 0$$

D.

Maximize $A(t) = C(t) - \int_4^t 0.1\ln(x)\,dx$:

Find critical points by setting $A'(t) = C'(t) - 0.1\ln(t) = 0$.

Solve numerically for t in $[4, 36]$: $t \approx 10.6$ weeks.

2749. Area and Volume Problem

A.

Area of region R:

$$\text{Area} = \int_0^3 [g(x) - f(x)]\, dx = \int_0^3 [x + \sin(\pi x) - (x^2 - 2x)]\, dx = \int_0^3 [-x^2 + 3x + \sin(\pi x)]\, d$$

Evaluate:

$$\text{Area} = \left[-\frac{x^3}{3} + \frac{3x^2}{2} - \frac{\cos(\pi x)}{\pi}\right]_0^3 \approx 4.5 \text{ square units}$$

B.

Volume of the solid:

$$\text{Volume} = \int_0^3 x \cdot [g(x) - f(x)]\, dx = \int_0^3 x(-x^2 + 3x + \sin(\pi x))\, dx$$

Evaluate numerically: ≈ 6.75 cubic units.

C.

Integral for rotation about $y = -2$:

$$\text{Volume} = \pi \int_0^3 [(g(x) + 2)^2 - (f(x) + 2)^2]\, dx$$

D.

Find x where $f'(x) = g'(x)$:

$$2x - 2 = 1 + \pi \cos(\pi x)$$

Solve numerically: $x \approx 0.5$.

2750. Reading Rate Problem

A.

Approximate $R'(1)$:

$$R'(1) \approx \frac{R(2) - R(0)}{2} = \frac{100 - 90}{2} = 5 \text{ words/min}^2$$

B.

By IVT, since $R(t)$ is differentiable and $R(0) = 90 < 155 < R(10) = 162$, there exists c such that $R(c) = 155$.

C.

Trapezoidal sum for $\int_0^{10} R(t)\,dt$:

$$\text{Approximation} = \frac{2}{2}(90 + 100) + \frac{6}{2}(100 + 150) + \frac{2}{2}(150 + 162) = 190 + 750 + 312 = 1252$$

D.

Total words read by the teacher:

$$\int_0^{10} W(t)\,dt = \int_0^{10} \left(-\frac{3}{10}t^2 + 8t + 100\right) dt = \left[-\frac{t^3}{10} + 4t^2 + 100t\right]_0^{10} = 1300 \text{ words}$$

2751. Function Analysis Problem

A.

$g'(8) = f(8)$. From the graph, $f(8) = 1.5$ (linear part slope).

B.

Points of inflection occur where $g''(x) = f'(x)$ changes sign:
At $x = 3$ and $x = 6$.

C.

$g(12) = \int_0^{12} f(t)\,dt$:
Calculate areas under semicircles and line segment:

$$g(12) = 9\pi + 18 \approx 46.27$$

$g(0) = 0$.

D.

Absolute minimum of g occurs at $x = -6$ (most negative area).

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$g(0) = 0$.

D.
Absolute minimum of g occurs at $x = -6$ (most negative area).

2752. Particle Motion Problem

A.
Velocity of particle H at $t = 1$:

$$v_H(t) = \frac{d}{dt}e^{t^2-4t} = (2t-4)e^{t^2-4t}$$

At $t = 1$: $v_H(1) = -2e^{-3}$.

B.
Opposite directions when $v_H(t) \cdot v_J(t) < 0$:
Solve $(2t-4)e^{t^2-4t} \cdot 2t(t^2-1)^3 < 0$.
Intervals: $(1, 2)$.

C.
At $t = 2$, $v_J'(2) > 0$ and $v_J(2) = 0$.
Speed is increasing.

D.
Position of particle J at $t = 2$:

$$x_J(2) = 7 + \int_0^2 2t(t^2-1)^3\,dt = 7 + \left[\frac{(t^2-1)^4}{4}\right]_0^2 = 7 + \frac{81}{4} - \frac{1}{4} = 27$$

2753. Curve Analysis Problem

A.

Implicit differentiation:

$$3y^2\frac{dy}{dx} - 2y\frac{dy}{dx} - \frac{dy}{dx} + \frac{x}{2} = 0 \implies \frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$$

B.

Tangent line approximation at $(2, -1)$:

$$y \approx -1 + \left.\frac{dy}{dx}\right|_{(2,-1)}(1.6 - 2) = -1 + \frac{-2}{2(3(-1)^2 - 2(-1) - 1)}(-0.4) \approx -0.9$$

C.

Vertical tangent when denominator is zero:

$$3y^2 - 2y - 1 = 0 \implies y = 1 \text{ or } y = -\frac{1}{3}$$

For $y > 0$, $y = 1$.

D.

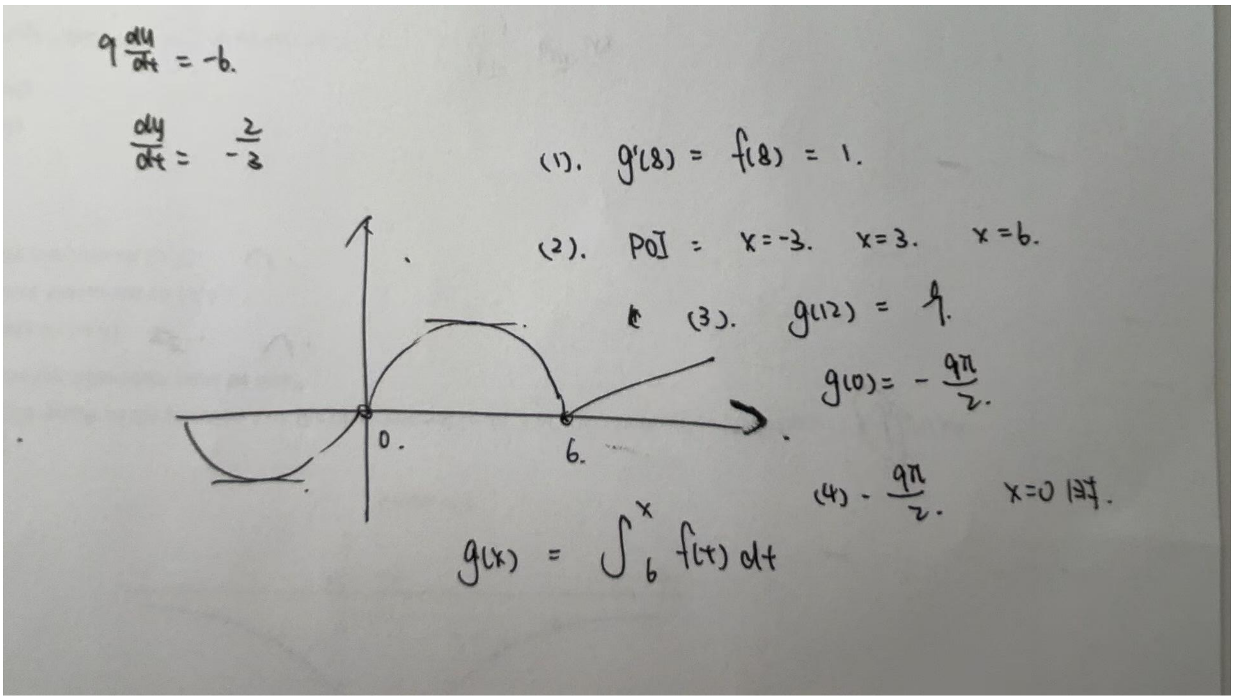
For $2xy + \ln y = 8$, at $(4, 1)$:

$$2y\frac{dx}{dt} + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0 \implies \frac{dy}{dt} = -6$$

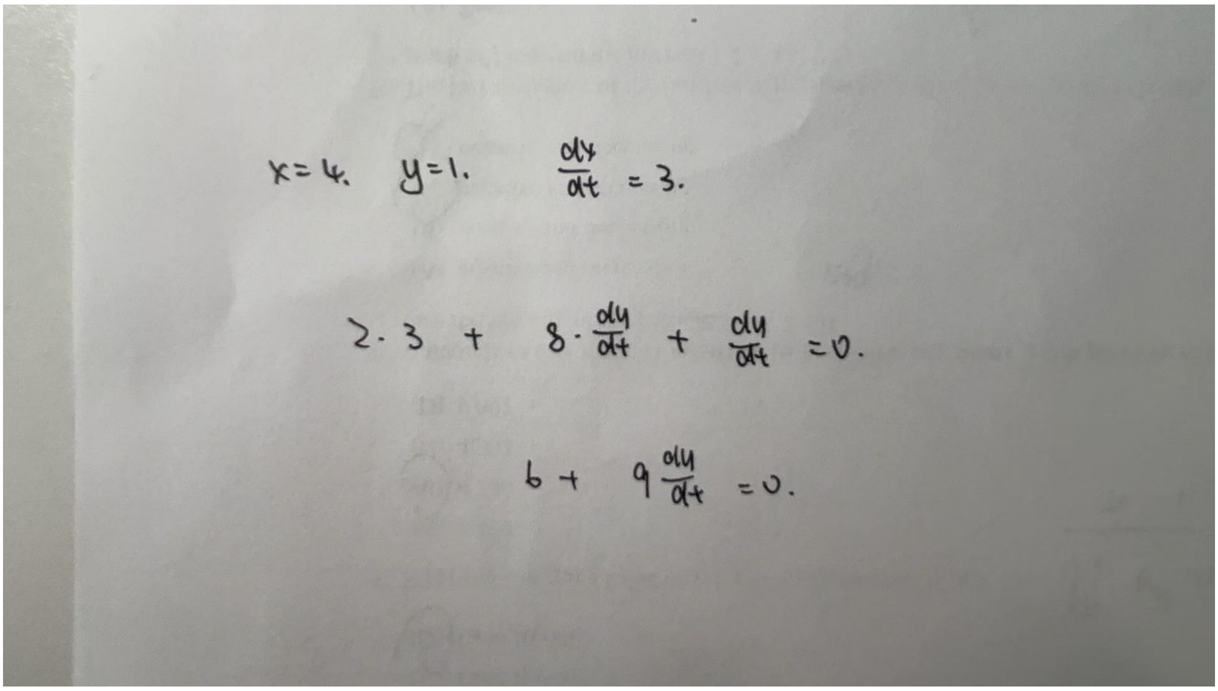
At $\frac{dx}{dt} = 3$, $\frac{dy}{dt} = -6$.

答案更正：

Frq 非计算器第二题纠正



2753 这道题的答案 D 也是错的



11 Mark for Review

Consider the curve defined by $x^2 - 2xy - y^2 + 2 = 0$, where $\frac{dy}{dx} = \frac{x-y}{x+y}$. What are all points (x, y) at which the curve has a vertical tangent line?

- (A) $(1, 1)$ and $(-1, -1)$
- (B) $(1, -1)$ and $(-1, 1)$
- (C) $(2, -2)$ and $(-2, 2)$
- (D) There are no such points.

这道题的答案是 D