

Astronomy and cosmology

A-Level Physics

Luminosity and radiant flux intensity

The **luminosity** 光度 L of a star is the **total power** 功率 of radiation it gives out — the **energy** 能量 radiated per second in all directions. Unit: watt (W).

At distance d , this power has spread over a sphere of area $4\pi d^2$. The **radiant flux intensity** 辐射通量密度 F (power per unit area) at distance d is

$$F = \frac{L}{4\pi d^2}.$$

Worked example. The Sun's luminosity is $L = 3.8 \times 10^{26}$ W. Find the radiant flux intensity at the Earth, a distance $d = 1.5 \times 10^{11}$ m away.

$$F = \frac{L}{4\pi d^2} = \frac{3.8 \times 10^{26}}{4\pi(1.5 \times 10^{11})^2} \approx 1.4 \times 10^3 \text{ W m}^{-2}.$$

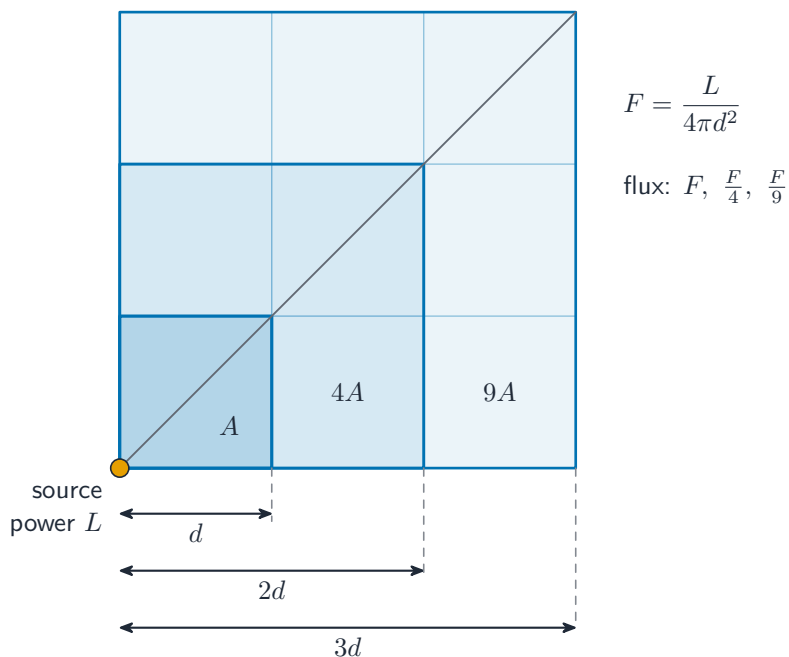
Unit: W m^{-2} . This is the **inverse-square law** 平方反比定律 for flux: doubling the distance cuts the flux to a quarter. A telescope measures F ; if L is known, the distance follows:

$$d = \sqrt{\frac{L}{4\pi F}}.$$



The four units of ESO's Very Large Telescope in Chile, used to measure the flux from distant stars

Image: G.Gillet/ESO, CC BY 3.0 (commons.wikimedia.org)



The same power spreads over a larger area as distance grows, so flux falls as $1/d^2$

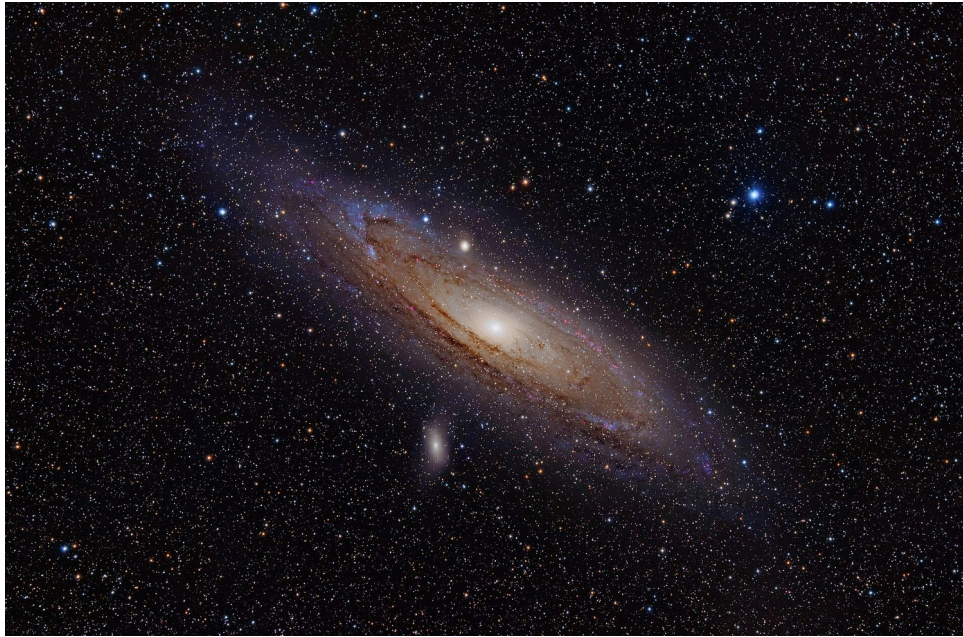
Standard candles

A **standard candle** 标准烛光 is an object whose **luminosity is known** from its type. Once you find one in a distant galaxy and measure the flux F from it, you get its distance from $d = \sqrt{L/(4\pi F)}$.

Examples:

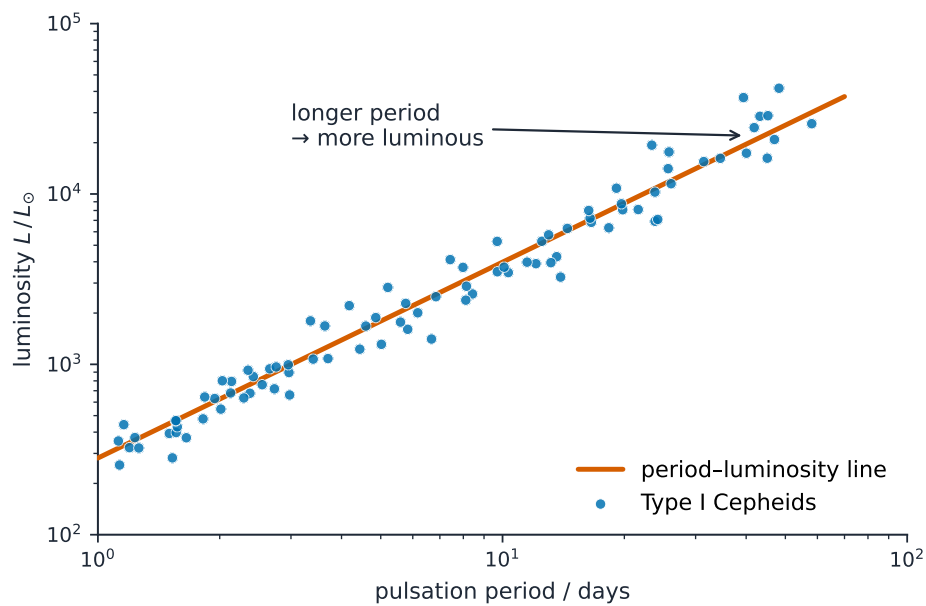
- **Cepheid variables** 造父变星 (pulsating stars) —the pulsation period is tightly linked to the luminosity, so the period gives L .
- **Type Ia supernovae** 超新星—a white dwarf reaching a critical mass and exploding always has about the same peak luminosity.

A standard candle gives L without first knowing the distance, so it reaches galaxies far beyond **parallax** 视差.



*The Andromeda Galaxy, our nearest large galaxy, about 2.5 million light-years away —
Cepheids in it are standard candles*

Image: Adam Evans, CC BY 2.0 (commons.wikimedia.org)



For Cepheid variables the pulsation period sets the luminosity, making them standard candles

Stellar surface temperature

Wien's displacement law

A hot body gives out a continuous (**blackbody** 黑体) spectrum with a peak at a **wavelength** 波长 $\lambda_{\max}$ set by its **temperature** 温度. **Wien's displacement law** 维恩位移定律:

$$\lambda_{\max}T = \text{constant}, \quad b \approx 2.90 \times 10^{-3} \text{ m K}.$$

Worked example. A star's blackbody spectrum peaks at $\lambda_{\max} = 500 \text{ nm}$. Find its surface temperature. ($b = 2.90 \times 10^{-3} \text{ m K}$.)

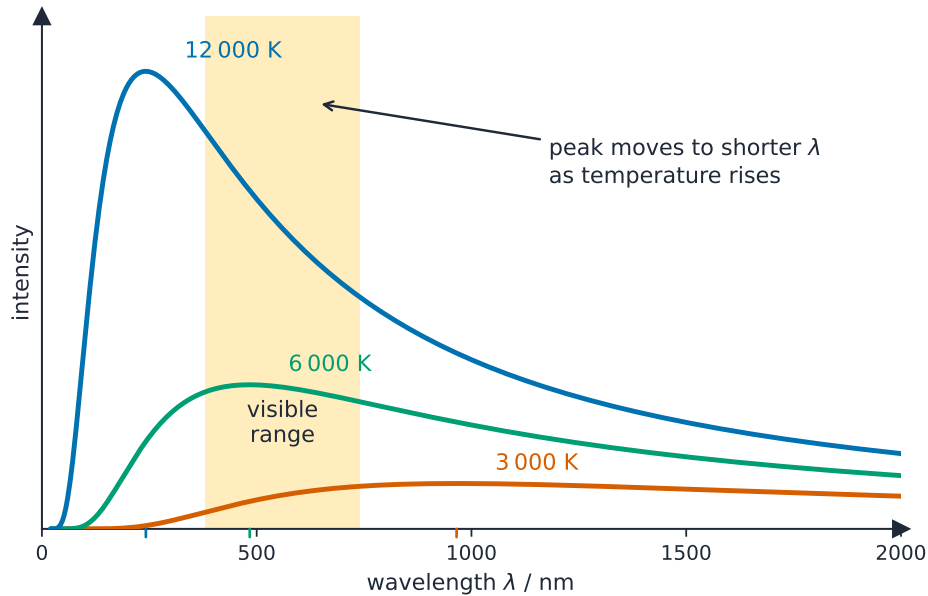
$$T = \frac{b}{\lambda_{\max}} = \frac{2.90 \times 10^{-3}}{500 \times 10^{-9}} \approx 5800 \text{ K}.$$

Hotter stars peak at **shorter** wavelengths: a cool red star ($\sim 3000 \text{ K}$) peaks in the infrared; the Sun ($\sim 5800 \text{ K}$) peaks near 500 nm ; a hot blue-white star ($\sim 20,000 \text{ K}$) peaks in the ultraviolet. Measuring $\lambda_{\max}$ gives the surface temperature.



The Pillars of Creation in the Eagle Nebula —clouds of gas and dust lit by hot, newly formed stars

Image: NASA, ESA, and the Hubble Heritage Team (STScI/AURA), Public domain (commons.wikimedia.org)



A hotter black body radiates more, and its peak wavelength shifts towards the blue (Wien's law)

Stefan–Boltzmann law

A star, treated as a blackbody sphere of radius r and surface temperature T , has luminosity

$$L = 4\pi\sigma r^2 T^4,$$

where $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ is the **Stefan–Boltzmann constant** 斯特藩-玻尔兹曼常量 (the **Stefan–Boltzmann law** 斯特藩-玻尔兹曼定律). Two strong dependences:

- $L \propto r^2$ —twice the radius, four times the luminosity (same T).
- $L \propto T^4$ —twice the temperature, sixteen times the luminosity (same r).

Worked example. A star has radius $r = 7.0 \times 10^8 \text{ m}$ and surface temperature $T = 5800 \text{ K}$. Find its luminosity. ($\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$.)

$$L = 4\pi\sigma r^2 T^4 = 4\pi(5.67 \times 10^{-8})(7.0 \times 10^8)^2(5800)^4 \approx 3.9 \times 10^{26} \text{ W}.$$

Estimating a star's radius

Combine the two laws:

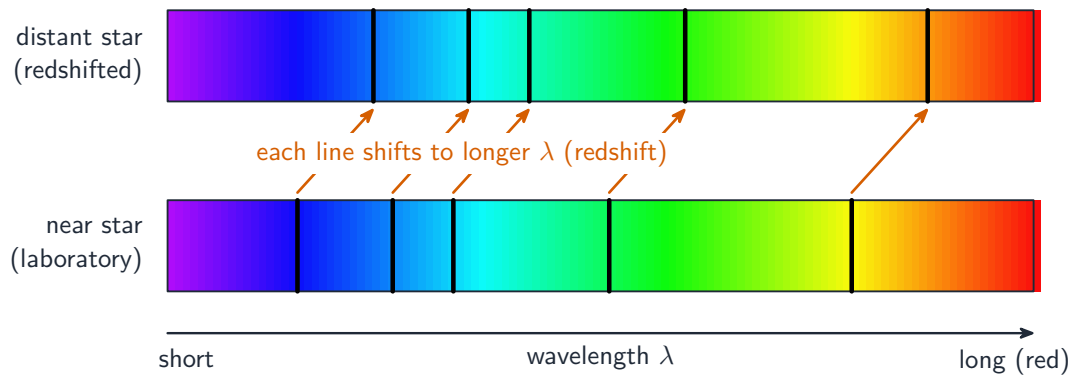
1. measure $\lambda_{\text{max}} \rightarrow$ get T from Wien's law.
2. find L (e.g. from flux F and distance d : $L = 4\pi d^2 F$).
3. solve the Stefan–Boltzmann law for r : $r = \sqrt{L/(4\pi\sigma T^4)}$.

This is how astronomers estimate radii of stars they cannot see as a disc.

Redshift, Hubble's law and the Big Bang

Cosmological redshift

The **spectral lines** 谱线 of light from distant galaxies are seen at **longer wavelengths** than their known laboratory values —the whole spectrum is stretched towards the red. This is **redshift** 红移.



The hydrogen absorption lines of a distant star are shifted to longer wavelengths —a redshift

Reading it as a Doppler shift, the galaxy is moving **away**. For $v \ll c$:

$$\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c},$$

where $\Delta\lambda = \lambda_{\text{observed}} - \lambda_{\text{emitted}}$ and v is the speed of **recession** 退行. Example: light emitted at 4.62×10^{-7} m but seen at 4.91×10^{-7} m gives $\Delta\lambda = 0.29 \times 10^{-7}$ m and

$$v \approx \frac{\Delta\lambda}{\lambda_{\text{em}}} c \approx 1.9 \times 10^7 \text{ m s}^{-1}.$$

Why redshift means an expanding Universe

Almost every distant galaxy is redshifted (a few near ones are **blueshifted** 蓝移 by local motion). So galaxies are, on average, moving apart —not just from us but from each other. The Universe is **expanding**, with the space between galaxies stretching. More distant galaxies are redshifted more.



The Hubble Ultra Deep Field —almost every point of light is a whole galaxy, most of them redshifted and receding

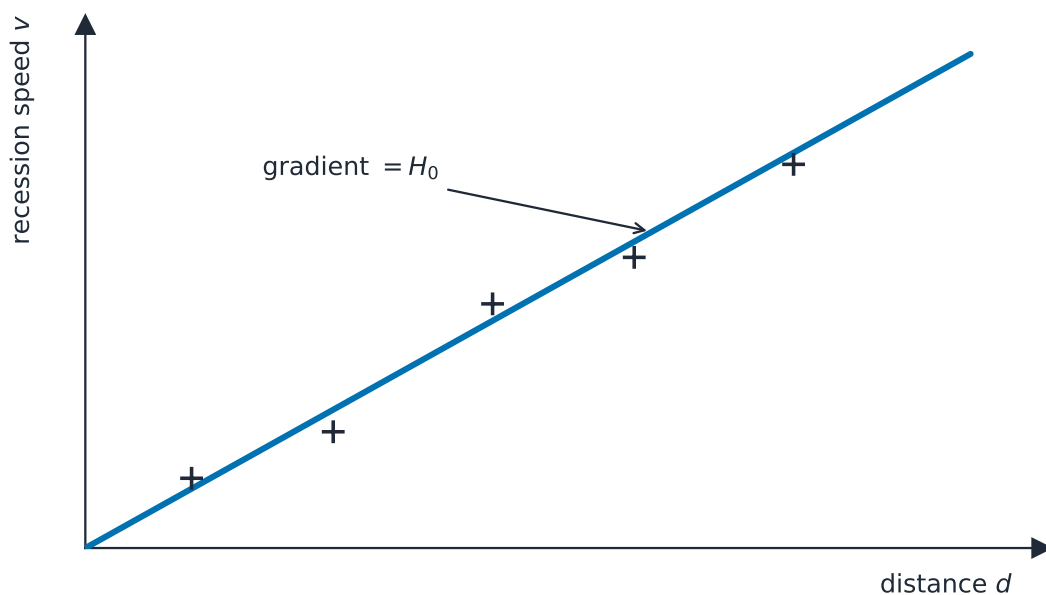
Image: NASA and the European Space Agency. Edited by Noodle snacks, Public domain (commons.wikimedia.org)

Hubble's law

The link between recession speed v and distance d is **Hubble's law** 哈勃定律:

$$v \approx H_0 \cdot d,$$

where H_0 is the **Hubble constant** 哈勃常数 ($\approx 2.3 \times 10^{-18} \text{ s}^{-1}$). Always use SI units.
Example: a galaxy receding at $1.9 \times 10^7 \text{ m s}^{-1}$ is at $d = v/H_0 \approx 8.3 \times 10^{24} \text{ m}$.



Hubble's law: a galaxy's recession speed is proportional to its distance, $v = H_0 d$

From Hubble's law to the Big Bang

Hubble's law means the galaxies were once together. Running the expansion backwards, all distances shrink to zero at $t = -1/H_0$ —the Universe was once a tiny, hugely dense, hot point. This is the **Big Bang** 大爆炸. The age of the Universe (for steady expansion) is about

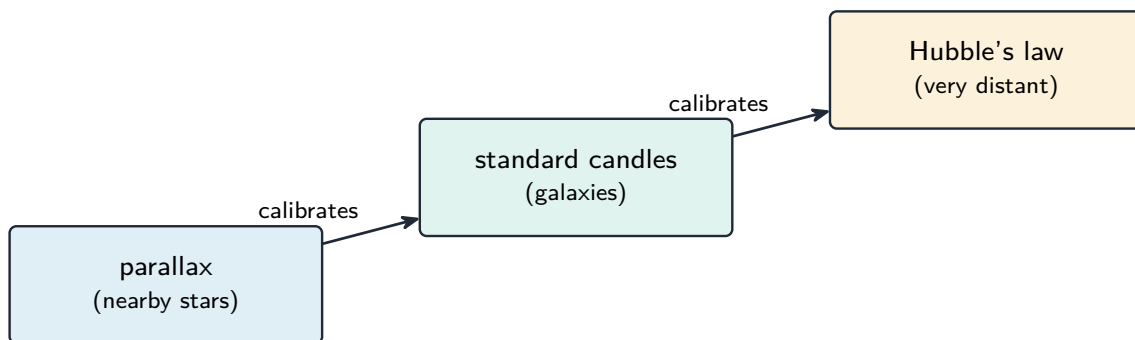
$$T_{\text{age}} \approx \frac{1}{H_0} \approx 4.3 \times 10^{17} \text{ s} \approx 14 \text{ billion years.}$$

The expansion, the redshift of galaxies, the **cosmic microwave background** 宇宙微波背景, and the hydrogen/helium abundances are the main evidence for the Big Bang.

Distance ladder

Astronomers combine methods, each calibrated by the one below:

1. **parallax** —for nearby stars.
2. **standard candles** (Cepheids, Type Ia supernovae) —for galaxies.
3. **Hubble's law** ($d = v/H_0$, with v from redshift) —for very distant galaxies.



each rung is calibrated by the one below, reaching ever further

The distance ladder: each method is calibrated by the one below and reaches further out

Exam tips

- $F = L/4\pi d^2$ (inverse-square): a standard candle has known luminosity L , so the distance follows from the measured flux F .
- **Wien's law** ($\lambda_{\text{max}} \propto 1/T$) gives surface temperature; **Stefan's law** $L = 4\pi r^2 \sigma T^4$.
- **Redshift** $z = \Delta\lambda/\lambda \approx v/c$; Hubble's law $v = H_0 d$ leads to the age of the universe (evidence for the Big Bang).