

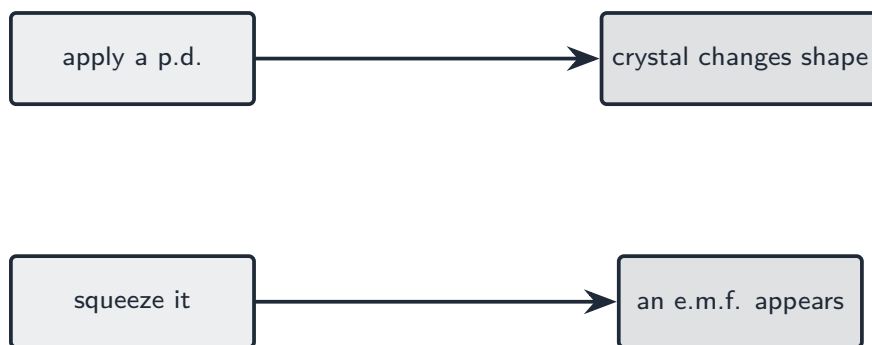
# Medical physics

## A-Level Physics

### Ultrasound

#### Piezo-electric effect

A **piezo-electric** 压电 crystal changes shape a little when a p.d. is put across it, and the reverse: it makes an **electromotive force** 电动势 (e.m.f.) across itself when its shape is changed. Quartz and PZT are common examples. Two linked effects:



*Applying a p.d. deforms the crystal; squeezing it makes a p.d.*

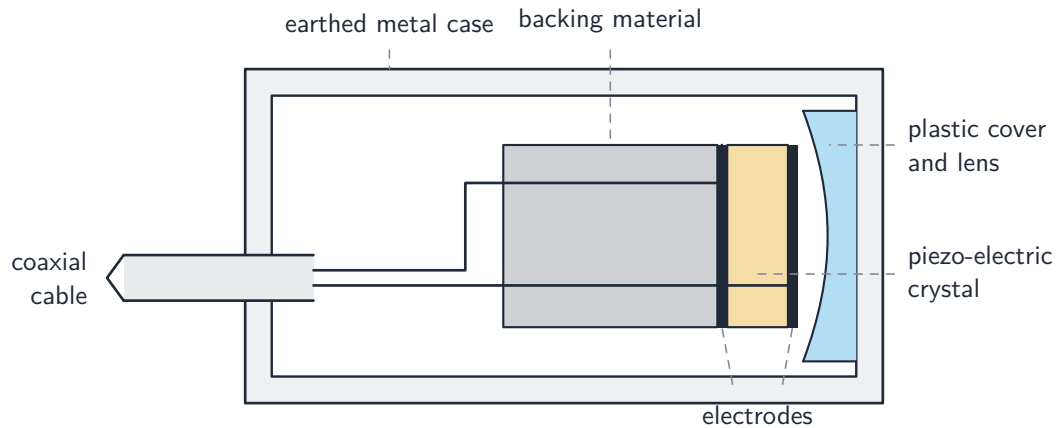
- **apply a p.d.** → the crystal changes shape (used to make vibrations).
- **change the shape** (a wave squeezes it) → an e.m.f. appears (used to detect vibrations).

#### Piezo-electric transducer

A **transducer** 换能器 uses this effect to both **make** and **detect ultrasound** 超声波.

- an alternating p.d. (a few MHz) makes the crystal vibrate at the same frequency, sending out **longitudinal** 纵波 waves above 20 kHz (1–10 MHz for medical imaging).
- the same crystal then detects: returning ultrasound makes it vibrate and produce an e.m.f.

So one transducer is both emitter and detector, switching between sending and listening.



*A piezo-electric transducer both sends and detects ultrasound using a vibrating crystal*

## Pulse-echo imaging

To see inside the body (**pulse-echo** 脉冲回波 imaging):

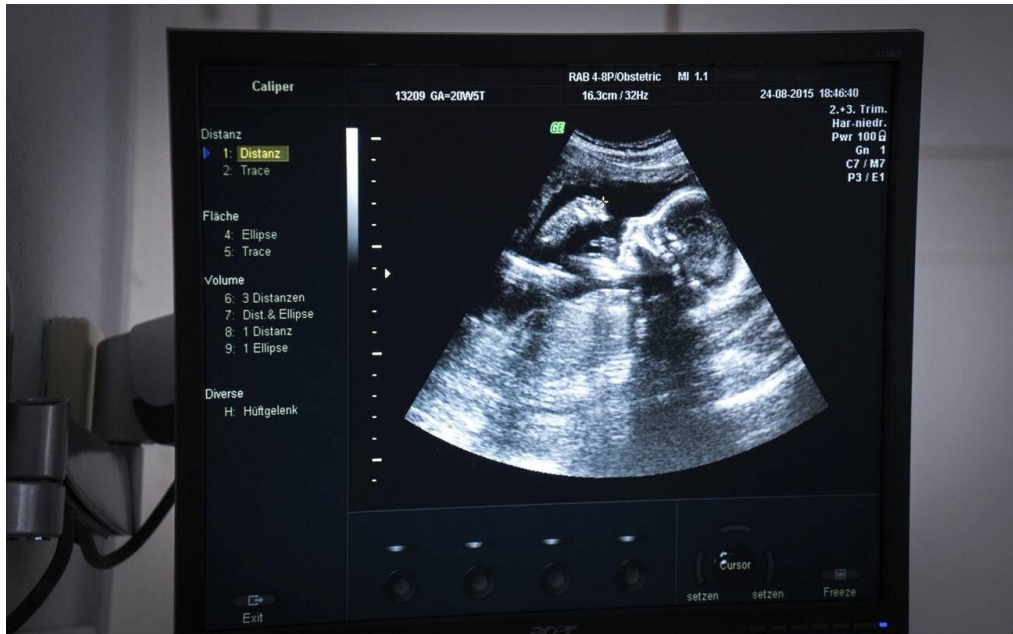
1. the transducer sends a **short pulse** into the body.
2. at each tissue boundary, part of the pulse is **reflected** and part goes on.
3. the transducer detects each reflected pulse.
4. the **time delay** gives the depth:  $d = ct/2$  (there and back). The echo's amplitude gives the strength of the reflection.

**Worked example.** An ultrasound pulse returns to the transducer  $60 \mu\text{s}$  after it was sent. The speed of sound in the tissue is  $1500 \text{ m s}^{-1}$ . Find the depth of the reflecting boundary.

The pulse travels there and back, so  $d = \frac{ct}{2}$ :

$$d = \frac{1500 \times 60 \times 10^{-6}}{2} = 0.045 \text{ m } (= 4.5 \text{ cm}).$$

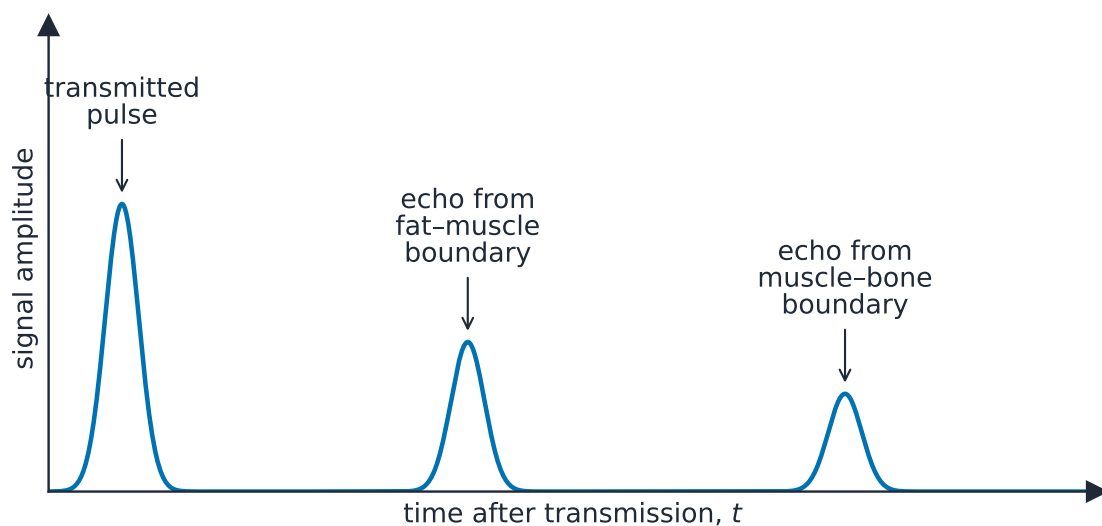
1. sweeping across the body builds a 2-D image.



*A real ultrasound image —the fan shape comes from the transducer sweeping across the body. Every bright speck is an echo from a boundary between tissues, and its depth was worked out from the echo's time delay, exactly as in the worked example above*

Image: Goleisureintl, CC BY 4.0 (commons.wikimedia.org)

A **coupling gel** 耦合剂 is put between the transducer and the skin to push out the air; without it almost all the ultrasound would reflect at the skin–air boundary and never enter the body.



*An A-scan shows the transmitted pulse and the echoes from each tissue boundary*

## Specific acoustic impedance

The **specific acoustic impedance** 声阻抗 of a medium is

$$Z = \rho c,$$

where  $\rho$  is the **density** 密度 and  $c$  the speed of sound. Unit:  $\text{kg m}^{-2} \text{s}^{-1}$ . Bone has large  $Z$ ; air has small  $Z$ ; soft tissue is in between.

**Worked example.** Find the specific acoustic impedance of soft tissue. (Density  $1060 \text{ kg m}^{-3}$ , speed of sound  $1540 \text{ m s}^{-1}$ .)

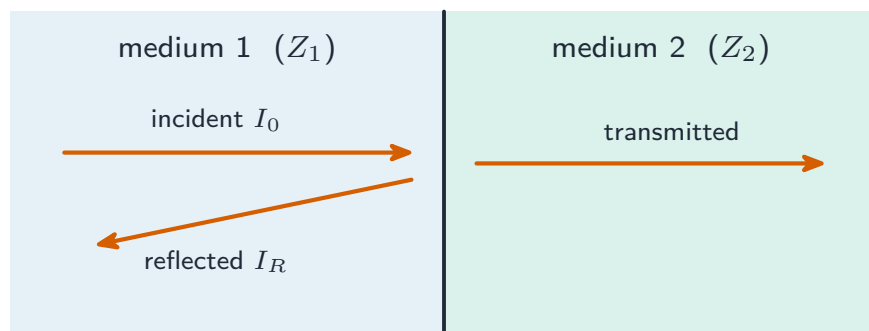
$$Z = \rho c = 1060 \times 1540 \approx 1.6 \times 10^6 \text{ kg m}^{-2} \text{s}^{-1}.$$

## Reflection at a boundary

At a boundary between media of impedance  $Z_1$  and  $Z_2$ , the **intensity reflection coefficient** 强度反射系数 (fraction reflected) is

$$\frac{I_R}{I_0} = \left( \frac{Z_1 - Z_2}{Z_1 + Z_2} \right)^2.$$

- very different impedances: almost all is reflected (skin/air —hence the gel).
- very similar impedances: almost nothing is reflected, so the boundary cannot be seen.
- best for imaging: different enough to give an echo, but not so different that nothing passes on.



$$\frac{I_R}{I_0} = \left( \frac{Z_1 - Z_2}{Z_1 + Z_2} \right)^2 \text{ — a bigger impedance difference gives a bigger echo}$$

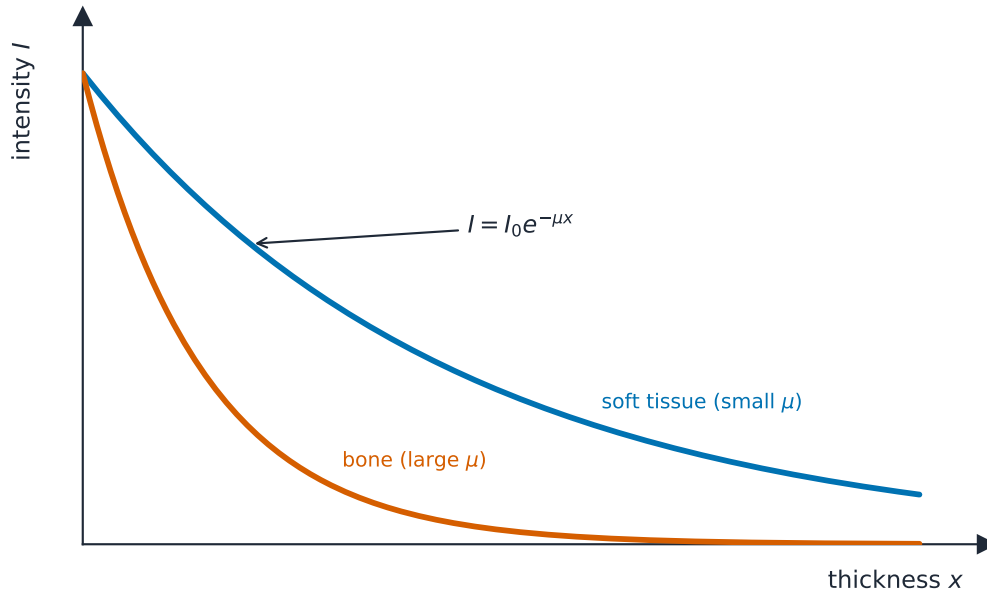
*At a boundary, part of the pulse reflects and part transmits —a bigger impedance difference gives a bigger echo*

## Attenuation

As ultrasound goes through tissue, its intensity falls with distance:

$$I = I_0 e^{-\mu x},$$

where  $\mu$  is the **attenuation coefficient** 衰减系数 (unit  $\text{m}^{-1}$ ). The same form applies to X-rays.



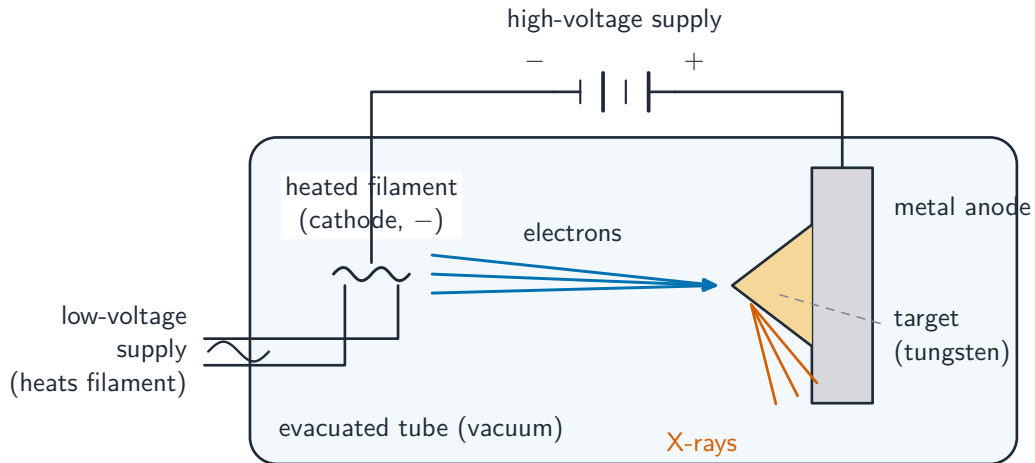
*Intensity falls exponentially with thickness; a larger attenuation coefficient (bone) falls off faster*

## X-rays

### Production

X-rays come from an **X-ray tube** X 射线管:

1. a heated **cathode** 阴极 emits electrons by **thermionic emission** 热电子发射.
2. a high p.d. (tens to hundreds of kV) accelerates the electrons across a **vacuum** 真空 to a metal **target** 靶 (the **anode** 阳极, often **tungsten** 钨).
3. the electrons hit the target and slow sharply. Most of their **kinetic energy** 动能 becomes heat; a small part is emitted as X-ray **photons** 光子 (**Bremsstrahlung** 轫致辐射, "braking radiation"). Some electrons knock out inner electrons of the metal atoms, and the refilling emits **characteristic** 特征 X-ray lines.



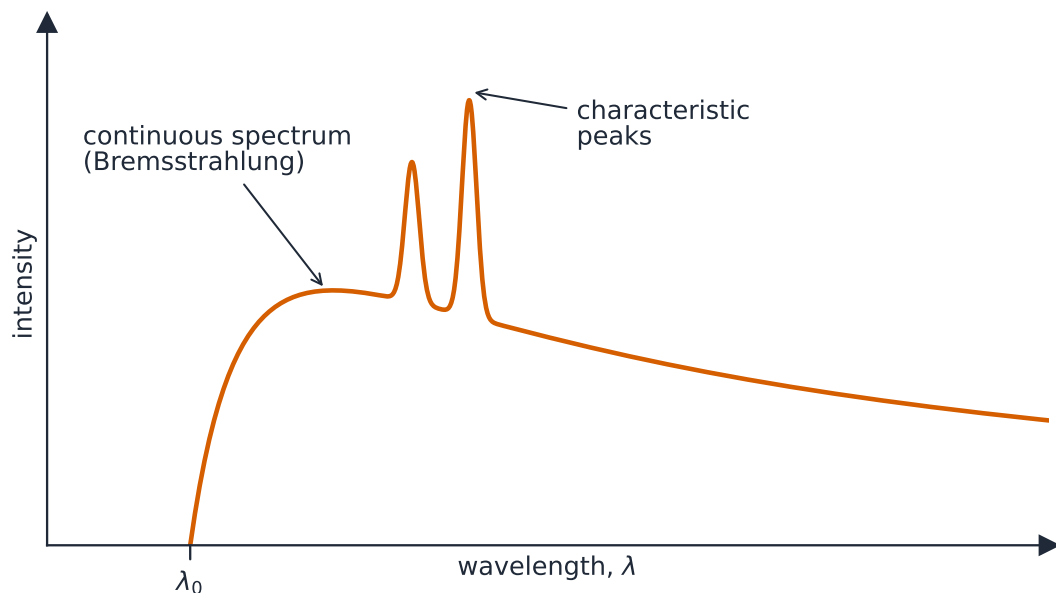
*In an X-ray tube electrons from the heated cathode are accelerated onto a metal target anode*

## Minimum wavelength

The most energy one X-ray photon can have is the full kinetic energy of one accelerated electron, lost in a single event. For accelerating p.d.  $V$ , the KE is  $eV$ , so

$$hf_{\max} = eV, \quad \lambda_{\min} = \frac{hc}{eV}.$$

This is the short-wavelength cut-off. The continuous Bremsstrahlung spectrum tails off above  $\lambda_{\min}$ , with sharp characteristic peaks set by the target metal.

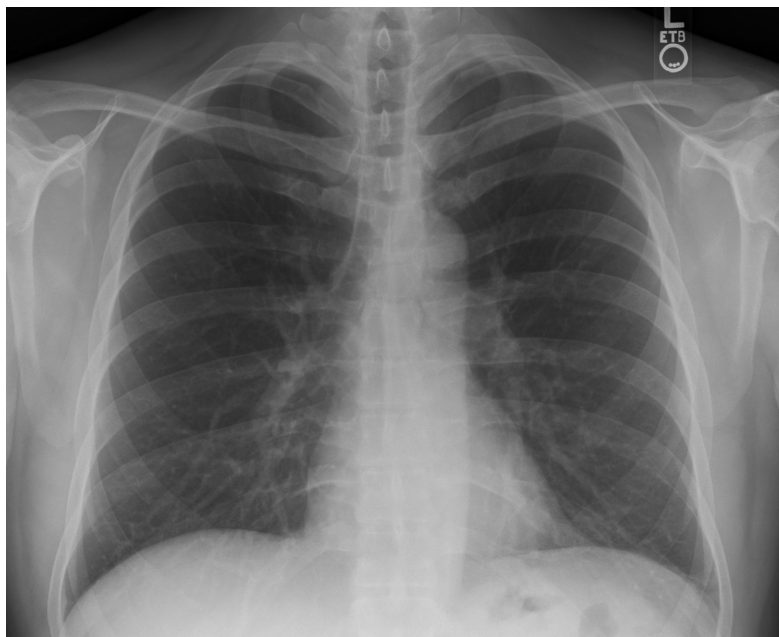


*A typical X-ray spectrum — a continuous Bremsstrahlung curve cut off at  $\lambda_0$ , with sharp characteristic peaks*

## Imaging with X-rays

X-rays pass through the patient onto a detector. Tissues that **attenuate** 衰减 more (bone, high  $Z$ ) cast a stronger shadow and look lighter; tissues that attenuate less (soft tissue, lung) look darker.

The **contrast** 对比度 is the difference in attenuation between tissues. A **contrast medium** 造影剂 (e.g. a barium meal) can be given to make soft tissues stand out.



*A real chest X-ray: dense bone absorbs more X-rays and looks white; the air-filled lungs let X-rays through and look dark*

Image: Stillwaterising, CC0 (commons.wikimedia.org)

## Attenuation law

$$I = I_0 e^{-\mu x}.$$

Higher-energy X-rays penetrate further (smaller  $\mu$ ); bone has a much larger  $\mu$  than soft tissue. To find the thickness for a given fraction, take logs:  $x = \frac{1}{\mu} \ln \frac{I_0}{I}$ . The **half-value thickness** 半值厚度  $x_{1/2} = \ln 2 / \mu$  halves the intensity (like half-life in decay).

**Worked example.** X-rays pass through 3.0 cm of tissue with attenuation coefficient  $\mu = 40 \text{ m}^{-1}$ . Find the fraction of the intensity that gets through.

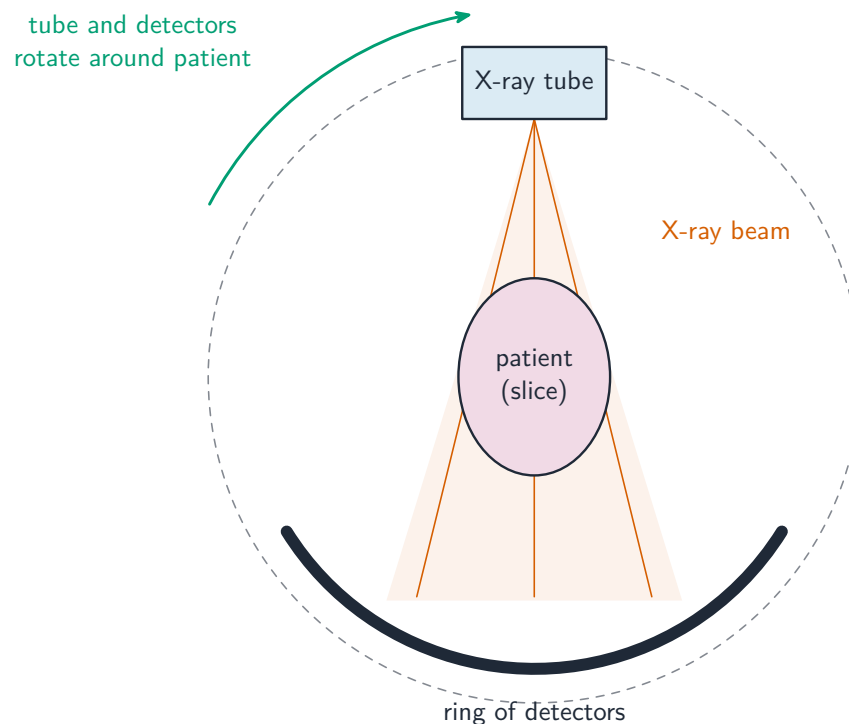
$$\frac{I}{I_0} = e^{-\mu x} = e^{-40 \times 0.030} = e^{-1.2} \approx 0.30.$$

## Computed tomography (CT)

A **computed tomography** 计算机断层扫描 (CT) scan builds a 3-D image:

1. the tube and detectors rotate around the patient, taking many images of one thin slice from **different angles**.
2. a computer combines these into a 2-D cross-section of the slice.
3. the patient is moved along, and the next slice is imaged.
4. the slices are stacked into a 3-D image.

CT shows far more than a single X-ray, because overlapping soft tissues are separated by the many-angle reconstruction.



*In a CT scan the X-ray tube and detectors rotate around the patient to image a slice from many angles*

## PET scanning

### Tracer

A **tracer** 示踪剂 is a substance with radioactive nuclei put into the body. It is taken up more by the tissue being studied (e.g. a tumour takes up more glucose-tagged tracer due to its high **metabolism** 代谢). Its decay is detected from outside.

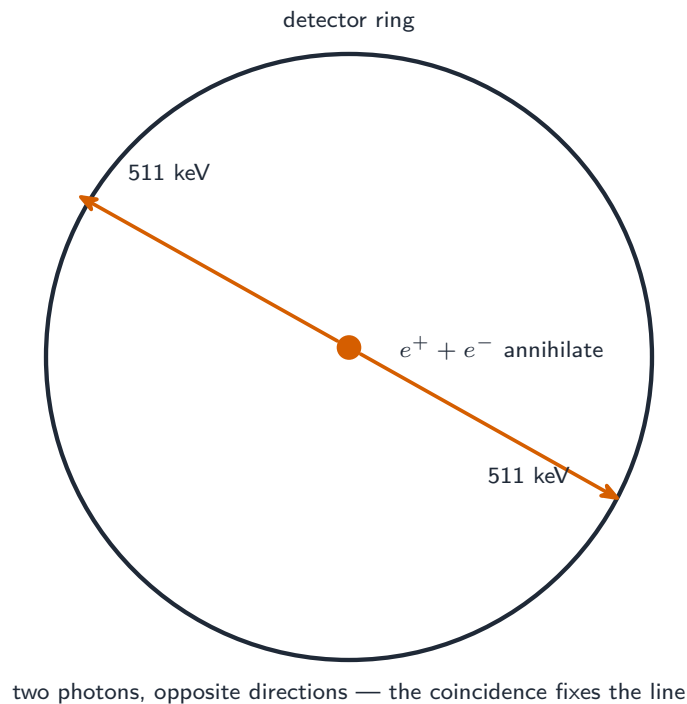
In **positron emission tomography** 正电子发射断层扫描 (PET), the tracer is a  $\beta^+$  emitter —it gives out a **positron** 正电子. A common one is fluorine-18 on a glucose analogue (FDG).



## Annihilation

When a particle meets its **antiparticle** 反粒子 they **annihilate** 湮灭: their mass turns into electromagnetic energy. In PET:

- a positron travels a few mm before meeting an **electron** 电子.
- they annihilate. Energy and **momentum** 动量 are conserved.
- since the total momentum is about zero, two photons are produced going in **opposite directions**, each 511 keV ( $= m_e c^2$ ).



*PET: annihilation gives two 511 keV photons in opposite directions; a coincidence fixes the line*

## Energy of the annihilation photons

By energy conservation, the total photon energy equals the pair's rest **energy** 能量:

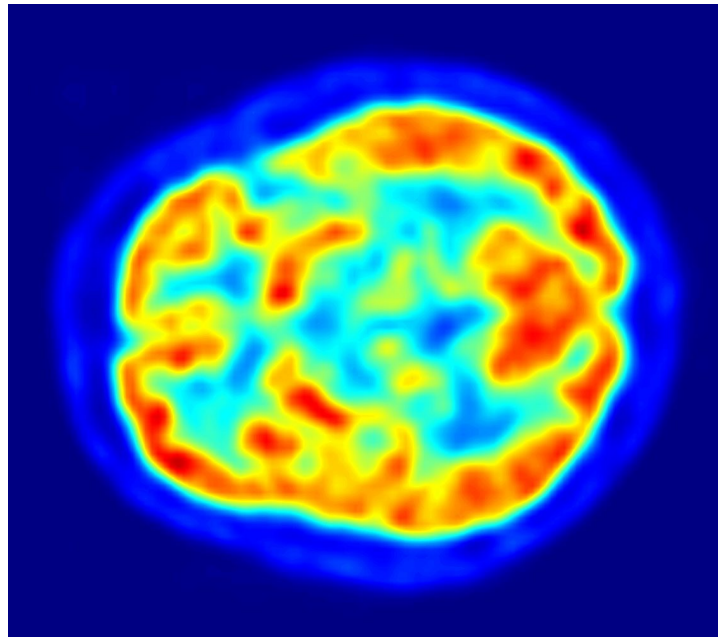
$$2hf = 2m_e c^2, \quad hf = m_e c^2.$$

Each photon has  $hf = m_e c^2 \approx 8.2 \times 10^{-14}$  J  $\approx 0.51$  MeV, with  $\lambda \approx 2.4 \times 10^{-12}$  m.

## Reconstructing the image

The two photons leave the body in opposite directions and hit **detector rings** around the patient. Recording the **two simultaneous arrivals** (a "coincidence") fixes the line the annihilation happened on. Many coincidences from many angles let the computer

build a 3-D map of the tracer —showing tissues with high metabolic activity. Comparing the two **arrival times** can refine the position along that line (time-of-flight PET).



*A finished PET image of the brain: warm colours (red, yellow) mark where the tracer collected – the most active tissue*

Image: Jens Maus (<http://jens-maus.de/>), Public domain ([commons.wikimedia.org](https://commons.wikimedia.org/))

## Exam tips

- **Ultrasound** reflects at boundaries (acoustic impedance); a coupling gel reduces reflection at the skin.
- **X-rays** are attenuated as  $I = I_0 e^{-\mu x}$ ; contrast media (barium, iodine) absorb strongly.
- **PET** uses a positron emitter; annihilation gives two  $\gamma$ -rays in opposite directions, located by the detector ring.