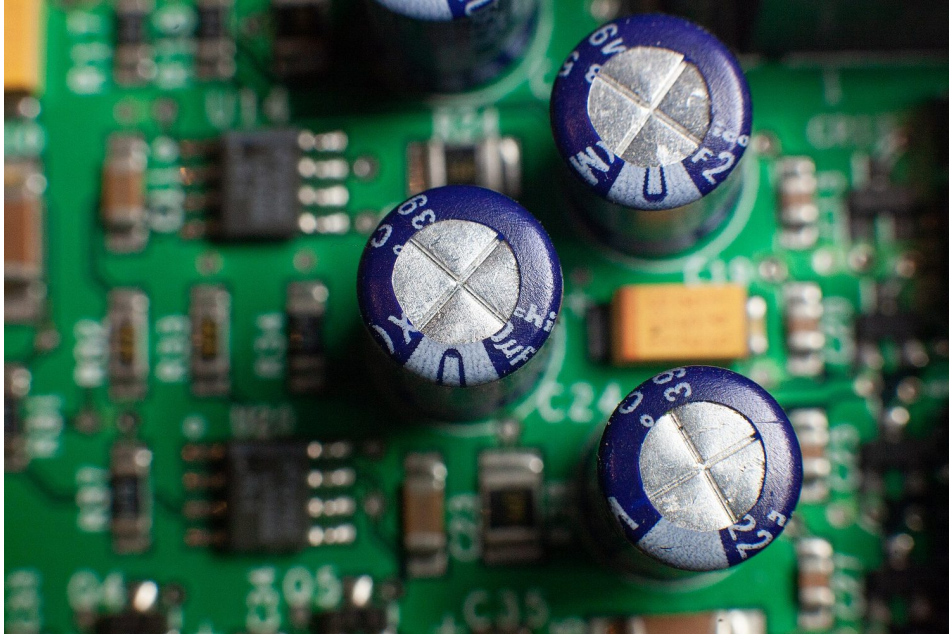


# Capacitance

## A-Level Physics

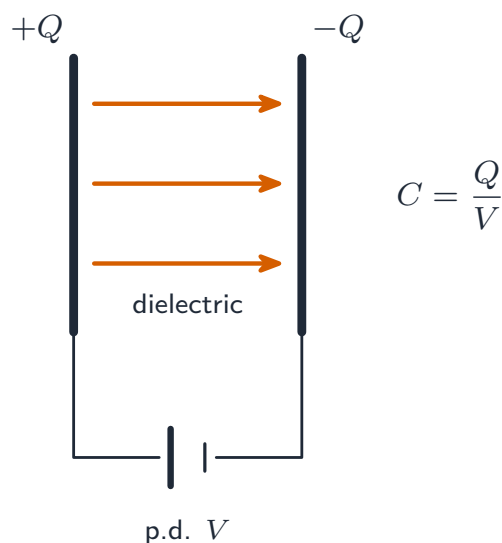
### Capacitance



*Capacitors store electric charge in a circuit.*

Image: Giovanna 27, CC BY 4.0 (commons.wikimedia.org)

A **capacitor** 电容器 stores charge. The simplest one is two parallel **conductor** 导体 plates with an **insulator** 绝缘体 (a **dielectric** 电介质, or just **vacuum** 真空 / air) between them. Connected to a battery, charge  $+Q$  builds up on one plate and  $-Q$  on the other, with a **potential difference** 电势差  $V$  across the gap.



*A parallel-plate capacitor: equal and opposite charges on two plates with a p.d.  $V$  across the gap*

The **capacitance** 电容  $C$  of any capacitor (or any isolated conductor) is

$$C = \frac{Q}{V}.$$

This applies to:

- an **isolated sphere** holding charge  $Q$  at potential  $V$  (zero at infinity). For radius  $r$ ,  $V = Q/(4\pi\epsilon_0 r)$ , so  $C = 4\pi\epsilon_0 r$ .
- a **parallel-plate capacitor**: charges  $\pm Q$  on the plates, p.d.  $V$  between them.

Unit: **farad** 法拉 (F) = C V<sup>-1</sup>. A farad is huge, so real capacitors run from pF to mF.

Capacitance is **constant** for a given capacitor (set by its size and dielectric). Doubling the charge doubles the voltage, so  $C = Q/V$  stays the same.

**Worked example.** A 100  $\mu\text{F}$  capacitor is charged to 12 V. Find the charge stored.

$$Q = CV = (100 \times 10^{-6})(12) = 1.2 \times 10^{-3} \text{ C } (= 1.2 \text{ mC}).$$



*Real capacitors range from large electrolytic cans (high capacitance) down to tiny film and ceramic types – from pF up to mF*

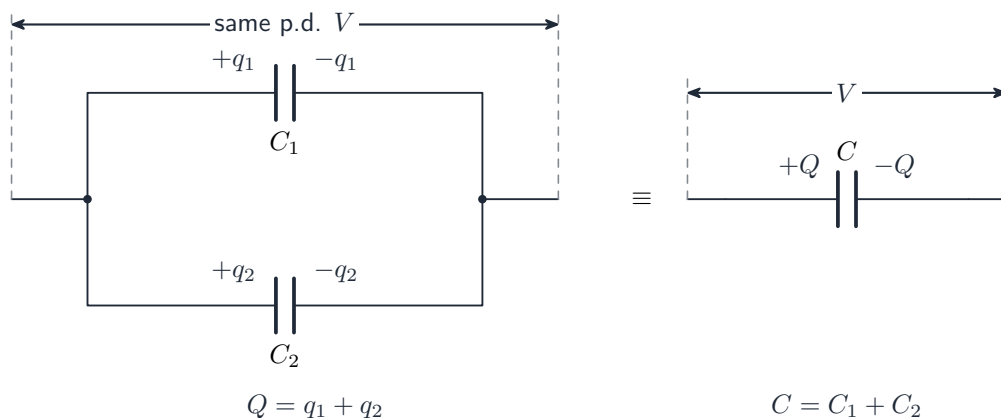
Image: Intiti, Public domain (commons.wikimedia.org)

## Combining capacitors

**Capacitors in parallel** share the same p.d.  $V$ . The total charge is the sum:

$$Q_{\text{total}} = C_1 V + C_2 V + \dots, \quad \text{so} \quad C_{\text{parallel}} = C_1 + C_2 + \dots$$

A parallel combination has **larger** capacitance than any one capacitor.

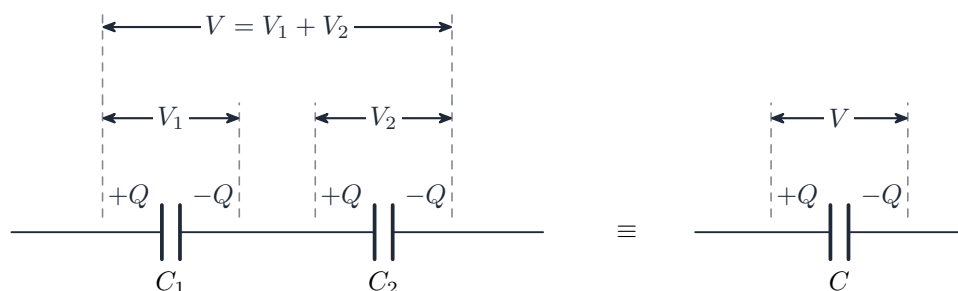


*Capacitors in parallel share the same p.d.; the charges add*

**Capacitors in series** carry the **same charge**  $Q$ . The total p.d. is the sum:

$$V_{\text{total}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \dots, \quad \text{so} \quad \frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

A series combination has **smaller** capacitance than any one capacitor.



*Capacitors in series carry the same charge; the p.d.s add*

Note: these rules are the **opposite** of those for resistors (resistors sum in series; capacitors sum in parallel), because  $C = Q/V$  has  $V$  on the bottom while  $R = V/I$  has  $I$  on the bottom.

## Energy stored in a capacitor

Charging a capacitor from 0 to  $Q$  needs work, because each extra bit of charge is pushed against the p.d. already there. When the charge is  $q$ , the p.d. is  $V(q) = q/C$ , so adding a small charge  $dq$  needs work  $V dq$ . The total work is

$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}.$$

Using  $V = Q/C$ , this is the **energy** 能量 stored:

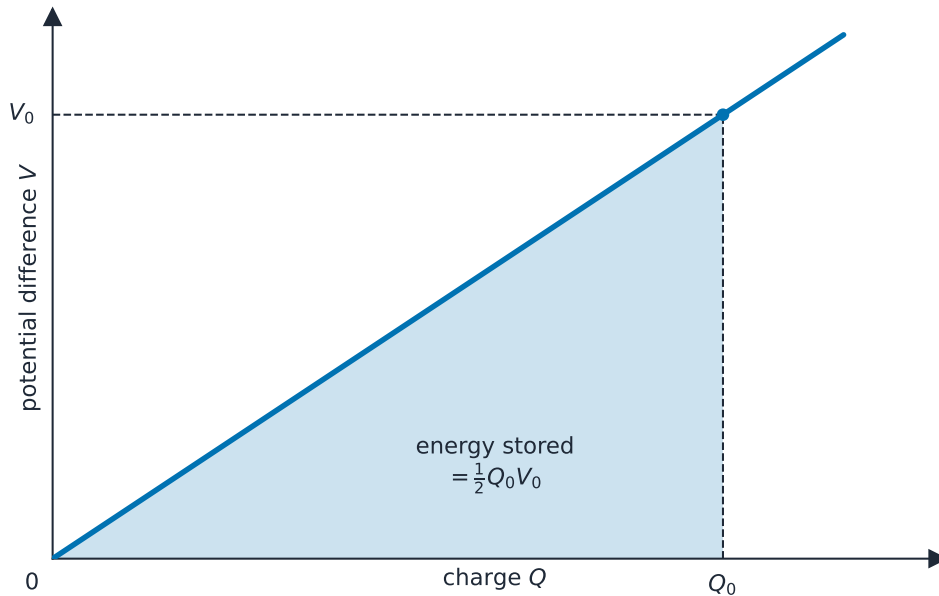
$$W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}.$$

**Worked example.** Find the energy stored in a  $100 \mu\text{F}$  capacitor charged to 12 V.

$$W = \frac{1}{2}CV^2 = \frac{1}{2}(100 \times 10^{-6})(12)^2 = 7.2 \times 10^{-3} \text{ J } (= 7.2 \text{ mJ}).$$

## Reading the $Q$ - $V$ graph

A plot of  $V$  against  $Q$  is a straight line through the origin with gradient  $1/C$ . The energy stored is the **area** under the line up to a given charge  $Q$ , which is the triangle  $\frac{1}{2}QV$ . The factor  $\frac{1}{2}$  is there because the average p.d. during charging is  $V/2$  (it grows from zero to  $V$ ), not  $V$ .



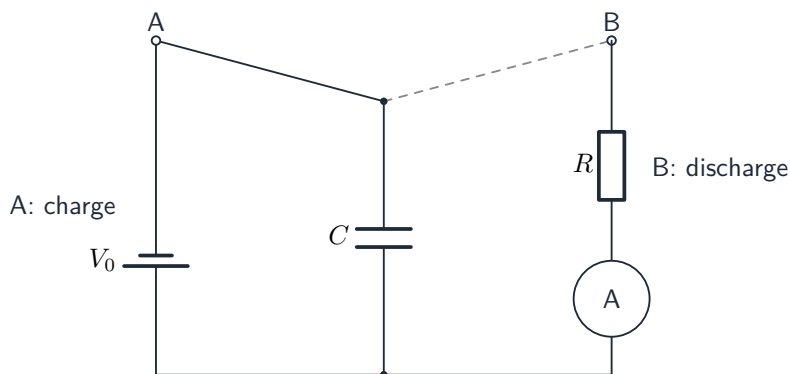
*The energy stored is the area under the potential-charge line (the triangle  $\frac{1}{2}QV$ )*

## Why charging is "half efficient"

Connect a capacitor  $C$  to an ideal battery of e.m.f.  $V$  through a wire. The capacitor stores  $\frac{1}{2}CV^2$ , but the battery supplies charge  $Q = CV$  at e.m.f.  $V$ , giving out  $QV = CV^2$ . The other half is lost as heat in the wire—whatever the wire's resistance.

## Capacitor discharging through a resistor

A capacitor  $C$  charged to  $V_0$  is connected through a switch to a **resistor** 电阻器 of **resistance** 电阻  $R$ . When the switch closes at  $t = 0$ , the capacitor discharges.



*A capacitor charges through switch A, then discharges through the resistor via switch B*

## Setting up the equation

By **Kirchhoff's second law** 基尔霍夫第二定律 around the loop,  $V_C = V_R$ . Using  $V_C = Q/C$ ,  $V_R = IR$  and  $I = -dQ/dt$ :

$$\frac{Q}{C} = -R \frac{dQ}{dt}.$$

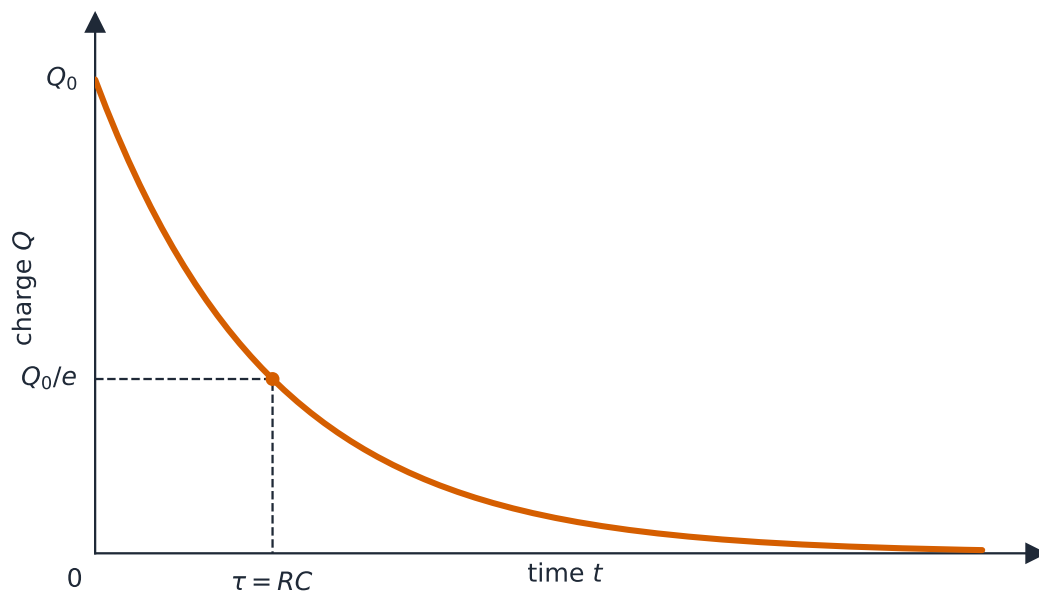
This is solved by an **exponential decay** 指数衰减 with time constant  $RC$ .

## Discharge equations

Charge  $Q$ , p.d.  $V$  and current  $I$  all decay exponentially with the same time constant:

$$Q = Q_0 e^{-t/(RC)}, \quad V = V_0 e^{-t/(RC)}, \quad I = I_0 e^{-t/(RC)},$$

with  $I_0 = V_0/R$ .



*Charge decays exponentially during discharge, falling to  $Q_0/e$  after one time constant  $RC$*

## Time constant

$$\tau = RC$$

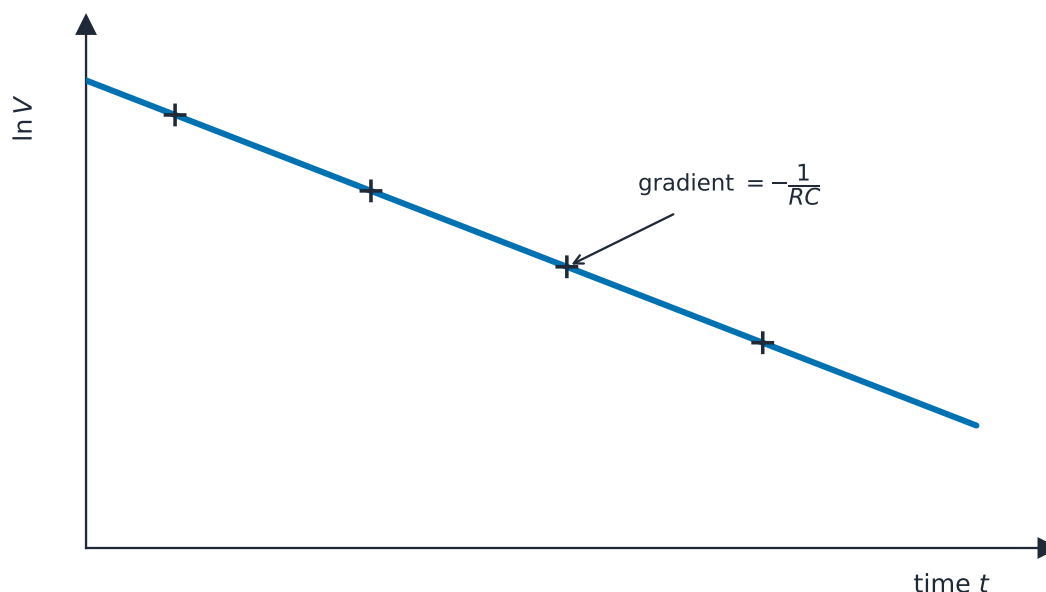
is the **time constant** 时间常数 (in seconds:  $\Omega \cdot \text{F} = \text{s}$ ). It is the time for a decaying quantity to fall to  $1/e \approx 0.37$  (about 37%) of its starting value. After  $2\tau$  it is at about 13.5%; after  $5\tau$ , below 1%.

**Worked example.** A  $100 \mu\text{F}$  capacitor charged to 12 V is discharged through a  $47 \text{ k}\Omega$  resistor. Find the time constant and the voltage after one time constant.

$$\tau = RC = (47 \times 10^3)(100 \times 10^{-6}) = 4.7 \text{ s}.$$

After one time constant the voltage falls to  $1/e$  of its start:  $V = 12 \times 0.37 \approx 4.4 \text{ V}$ .

To find  $\tau$  from a curve: read the time to fall to  $1/e$  of the start. Or take logs:  $\ln(V/V_0) = -t/(RC)$ , so a plot of  $\ln V$  against  $t$  is a straight line with gradient  $-1/(RC)$ .



*A graph of  $\ln V$  against time is a straight line of gradient  $-1/(RC)$*

## Reading graphs during discharge

- $Q$  against  $V_C$ : since  $Q = CV$  always, this is a straight line through the origin with gradient  $C$ . Discharge moves the point from  $(V_0, Q_0)$  down to  $(0, 0)$ .
- $I$  against  $V_C$ : since  $I = V_C/R$ , this is a straight line through the origin with gradient  $1/R$ , so you can find  $R$ .

## Common exam questions

Given a discharge curve  $V(t)$  or  $Q(t)$ :

- read the start value  $V_0$  or  $Q_0$  at  $t = 0$ .
- read the time to fall to  $V_0/e \rightarrow$  time constant  $\tau = RC$ .
- given  $R$ , find  $C = \tau/R$  (or the other way round).
- predict a later value with the exponential formula.

## Exam tips

- $C = Q/V$ ; energy stored  $W = \frac{1}{2}QV = \frac{1}{2}CV^2$  (the  $\frac{1}{2}$  is the area under the  $Q$ - $V$  graph).
- Combine capacitors the **opposite way to resistors** (parallel add, series reciprocal).
- Discharge is exponential:  $Q = Q_0 e^{-t/RC}$ ; the time constant  $\tau = RC$  is when the charge falls to 37%.