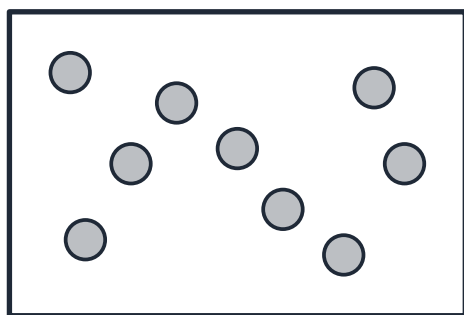


Ideal gases

A-Level Physics

The mole

Amount of substance 物质的量 is an SI base quantity. Its unit is the **mole** 摩尔 (mol) —one of the seven SI base units, with the kilogram, metre, second, ampere and **kelvin** 开尔文 from Topic 1.



1 mole

$$= 6.02 \times 10^{23}$$

particles (N_A)

One mole contains the Avogadro number of particles

One mole of any substance has a number of particles equal to the **Avogadro constant** 阿伏伽德罗常量:

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}.$$

A "particle" means whatever you are counting —**atoms** 原子 for a **monatomic** 单原子 element like helium, **molecules** 分子 for O_2 or H_2O . Always say what you are counting.

For n moles, the number of particles is $N = nN_A$.

The **molar mass** 摩尔质量 M_m is the mass of one mole (kg mol^{-1} or g mol^{-1}). Mass of n moles is $M = nM_m$. Mass of one particle is $m_0 = M_m/N_A$.

Equation of state of an ideal gas



Compressed gas cylinders store a fixed mass of gas at high pressure.

Image: Cjp24, CC BY-SA 4.0 (commons.wikimedia.org)

An **ideal gas** 理想气体 obeys $pV \propto T$ exactly, where T is the **thermodynamic temperature** 热力学温度.

The **equation of state** 状态方程 can be written two equal ways:

$$pV = nRT \quad \text{or} \quad pV = NkT.$$

Here:

- p — **pressure** 压强 (Pa).
- V — **volume** 体积 (m^3).
- T — thermodynamic temperature in kelvin (never $^{\circ}\text{C}$).
- n — number of moles; N — number of molecules.
- R — **molar gas constant** 摩尔气体常量, $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$.
- k — **Boltzmann constant** 玻尔兹曼常量, $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$.

Since $N = nN_A$, we get $k = R/N_A$: k is the gas constant **per molecule**, as R is per mole.

Using the equation of state

List the variables you have, find the unknown, and choose the form that matches your "amount" (moles $\rightarrow nRT$; molecules $\rightarrow NkT$). Always use SI units: Pa, m^3 , K.

Worked example. A cylinder of volume 0.020 m^3 holds gas at 27°C and a pressure of $2.0 \times 10^5 \text{ Pa}$. How many moles of gas are there? ($R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$.)

Convert to kelvin: $T = 27 + 273 = 300$ K. Then from $pV = nRT$,

$$n = \frac{pV}{RT} = \frac{(2.0 \times 10^5)(0.020)}{8.31 \times 300} \approx 1.6 \text{ mol.}$$

If a fixed amount of gas changes from state 1 to state 2:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}.$$

Worked example. A fixed mass of gas at 300 K occupies 0.50 m^3 . It is heated to 450 K at constant pressure. Find the new volume.

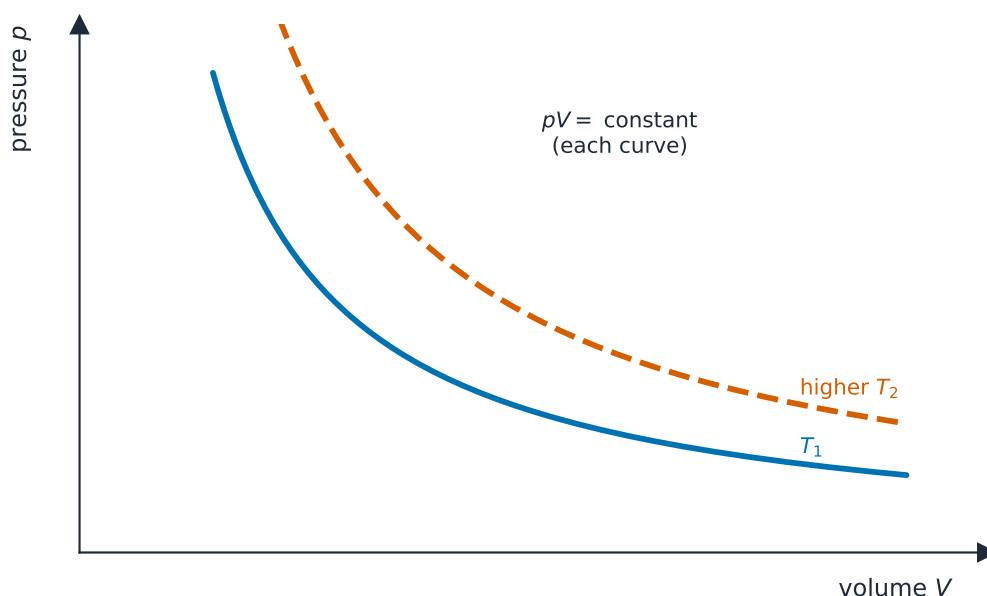
At constant pressure V/T is constant, so

$$V_2 = V_1 \frac{T_2}{T_1} = 0.50 \times \frac{450}{300} = 0.75 \text{ m}^3.$$

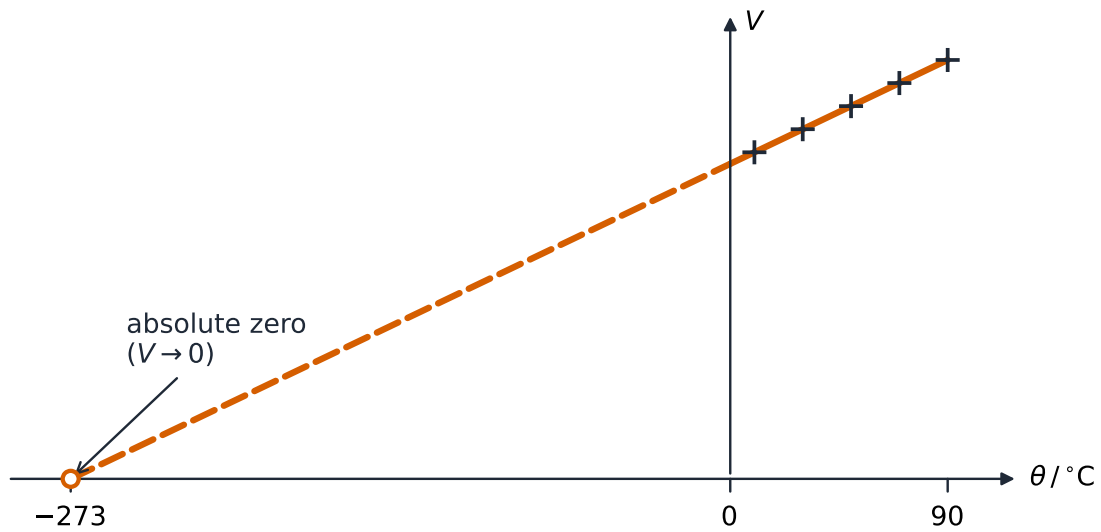
Special cases:

- **constant temperature (Boyle's law 玻意耳定律):** $p_1 V_1 = p_2 V_2$.
- **constant pressure (Charles's law 查理定律):** $V/T = \text{constant}$.
- **constant volume (pressure law 气体压强定律):** $p/T = \text{constant}$.

A common mistake is using $^{\circ}\text{C}$ instead of K — $pV \propto T$ only holds with T in kelvin.



Boyle's law: at constant temperature pV is constant, so a p - V graph is a hyperbola



At constant pressure the volume of a gas rises linearly with temperature, reaching zero at absolute zero (Charles's law)

Kinetic theory of gases



A scuba diver breathes compressed gas; its pressure, volume and temperature are all linked.

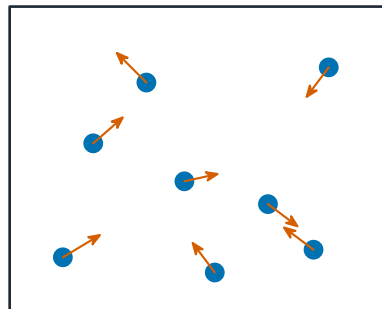
Image: Yu Diving, CC BY-SA 2.0 (commons.wikimedia.org)

The **kinetic theory** 分子动理论 explains a gas's large-scale behaviour from the **random motion** 无规则运动 of its molecules.

Assumptions

For an ideal gas:

1. a **large number** of identical molecules in continuous random motion.
2. the molecules' own volume is too small to matter compared with the container.
3. the time of each collision is too short to matter compared with the time between collisions.
4. **intermolecular** 分子间 forces are ignored except during collisions (molecules go in straight lines between them).
5. collisions (with the walls and with each other) are **elastic** —an **elastic collision** 弹性碰撞 loses no kinetic energy, so the gas does not cool down by itself.
6. Newton's laws apply.



1. many identical molecules, in continuous **random motion**
2. their own volume is **tiny** next to the container
3. straight lines between hits - intermolecular forces **ignored**
4. collisions are **elastic**: no kinetic energy is lost

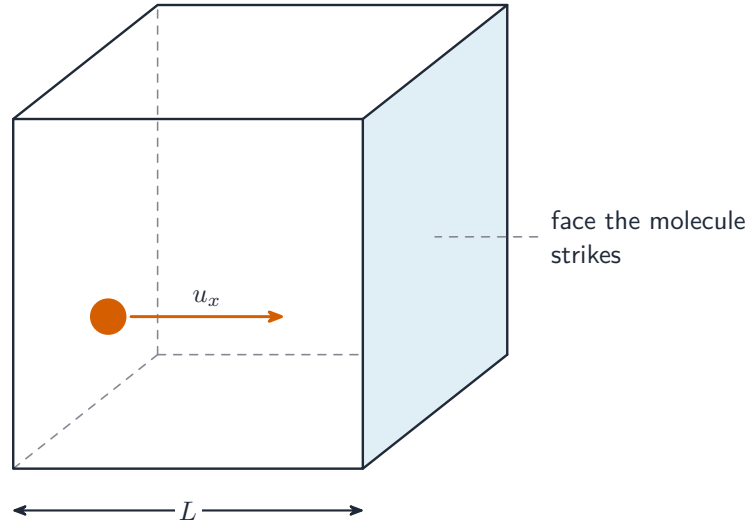
poor at very high pressure (volume matters) or very low temperature (forces matter)

The key assumptions of the kinetic theory of an ideal gas

These assumptions become poor at very high pressure (molecular volume matters) or very low temperature (intermolecular forces matter).

Pressure of a gas —outline of the derivation

Take a cubic box of side L with N molecules, each of mass m . Look at one molecule moving along the x -axis with velocity u_1 .



The pressure derivation considers one molecule's velocity component u_x normal to a face of a cube of gas

- **one collision with the right wall:** velocity reverses to $-u_1$, change in **momentum** 动量 $\Delta p_x = -2mu_1$. By Newton's third law the wall gets an **impulse** 冲量 of $+2mu_1$.
- **time between hits on that wall:** travel $2L$ there and back, so $\Delta t = 2L/u_1$.
- **average force from this molecule:** $F_1 = \Delta p / \Delta t = mu_1^2 / L$.
- **add over all molecules:** $F = (Nm/L)\langle u_x^2 \rangle$, where $\langle u_x^2 \rangle$ is the **mean square** 均方 of the x -velocity.
- **pressure:** $p = F/L^2 = Nm\langle u_x^2 \rangle / V$.

In 3-D, by symmetry $\langle u_x^2 \rangle = \frac{1}{3}\langle c^2 \rangle$, where $\langle c^2 \rangle$ is the **mean-square speed** 均方速率. So

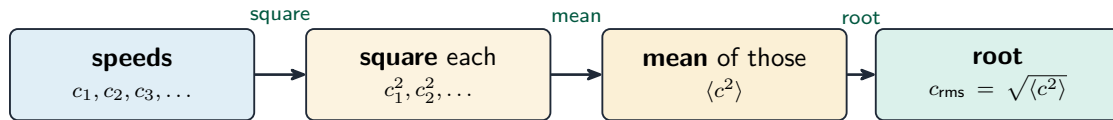
$$pV = \frac{1}{3}Nm\langle c^2 \rangle.$$

Root-mean-square speed

The square root of $\langle c^2 \rangle$ is the **root-mean-square** 均方根 (r.m.s.) speed:

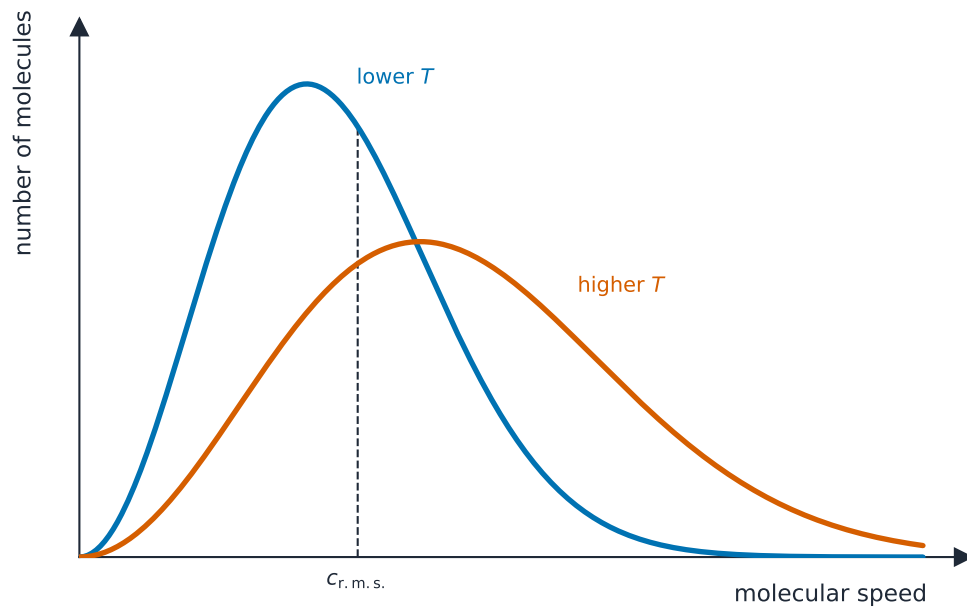
$$c_{\text{r.m.s.}} = \sqrt{\langle c^2 \rangle}.$$

It is a useful single measure of how fast the molecules move, slightly larger than the mean speed (squaring weights fast molecules more).



read the name **backwards**: root of the **mean** of the **squares** -
squaring weights fast molecules more, so c_{rms} is slightly above the mean speed

Root-mean-square speed: square each speed, take the mean, then the square root



The spread of molecular speeds: a higher temperature broadens the curve and shifts it to faster speeds

Average translational kinetic energy

Compare the two expressions for pV :

$$pV = NkT \quad \text{and} \quad pV = \frac{1}{3}Nm\langle c^2 \rangle.$$

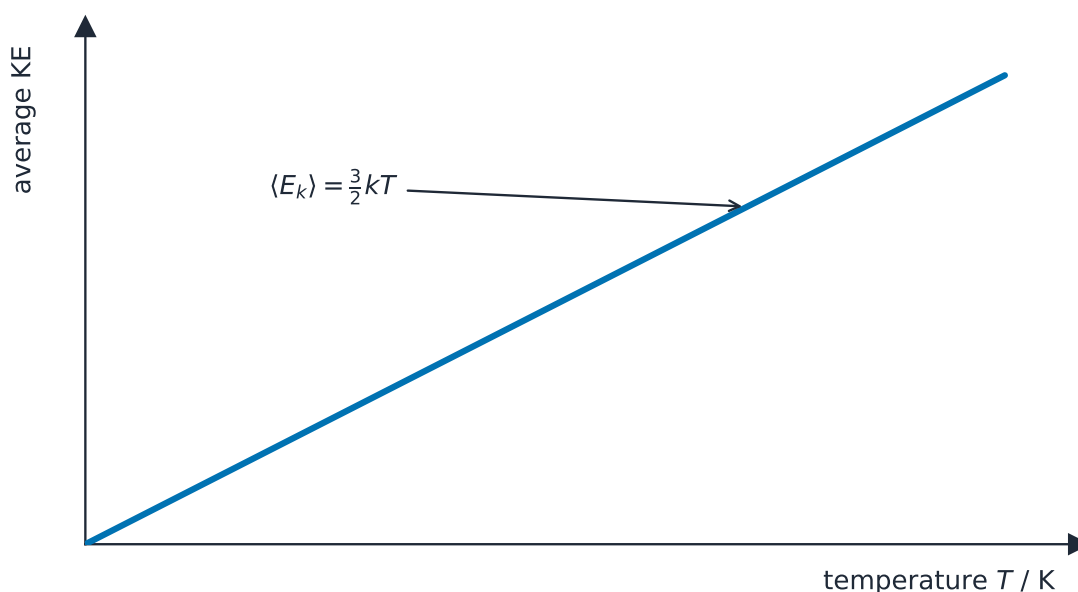
Set them equal, cancel N , and multiply by $\frac{3}{2}$:

$$\frac{3}{2}kT = \frac{1}{2}m\langle c^2 \rangle.$$

The right side is the **average translational kinetic energy** 平动动能 $\langle E_k \rangle$ of one molecule. So

$$\langle E_k \rangle = \frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT.$$

This is a key result: **the average translational kinetic energy of an ideal-gas molecule depends only on the thermodynamic temperature**, not on the type of gas or its pressure.



Average molecular KE is proportional to thermodynamic temperature: $\langle E_k \rangle = \frac{3}{2}kT$

Worked example. Find the root-mean-square speed of oxygen molecules at 300 K. (Mass of one O_2 molecule = 5.3×10^{-26} kg, $k = 1.38 \times 10^{-23}$ J K^{-1} .)

From $\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$, the mean-square speed is $\langle c^2 \rangle = 3kT/m$:

$$c_{\text{r.m.s.}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23})(300)}{5.3 \times 10^{-26}}} \approx 480 \text{ m s}^{-1}.$$

Consequences

- **doubling the absolute temperature doubles the average KE** of each molecule, so $\langle c^2 \rangle$ doubles and $c_{\text{r.m.s.}}$ grows by $\sqrt{2}$.
- for **two gases at the same temperature**, the lighter gas has a larger $\langle c^2 \rangle$. Hydrogen molecules move faster on average than oxygen molecules in the same room.
- **total translational KE** of N molecules: $\frac{3}{2}NkT = \frac{3}{2}nRT$.

Internal energy of an ideal gas

For an ideal gas the molecules are point particles with no intermolecular potential energy and (in this simple model) no rotation or vibration. So the **internal energy** 内能 is just the total **kinetic energy** 动能:

$$U = \frac{3}{2}NkT = \frac{3}{2}nRT.$$

So the internal energy of an ideal gas is proportional to the thermodynamic temperature—doubling T doubles U .

Changing pressure at fixed temperature

Doubling p at fixed T (by squeezing the gas to half its volume) does **not** change $\langle E_k \rangle$ —that depends only on T . There are more wall collisions per second, but each molecule has the same average kinetic energy.

Exam tips

- Use $pV = nRT$ (n in mol) or $pV = NkT$ (N molecules), with temperature in **kelvin**.
- Learn the **kinetic-theory assumptions** (random motion, negligible molecular volume, elastic collisions, no intermolecular forces).
- Mean translational KE = $\frac{3}{2}kT$ —it depends **only on temperature**.