Work, energy and power

A-Level Physics

Work, energy and power

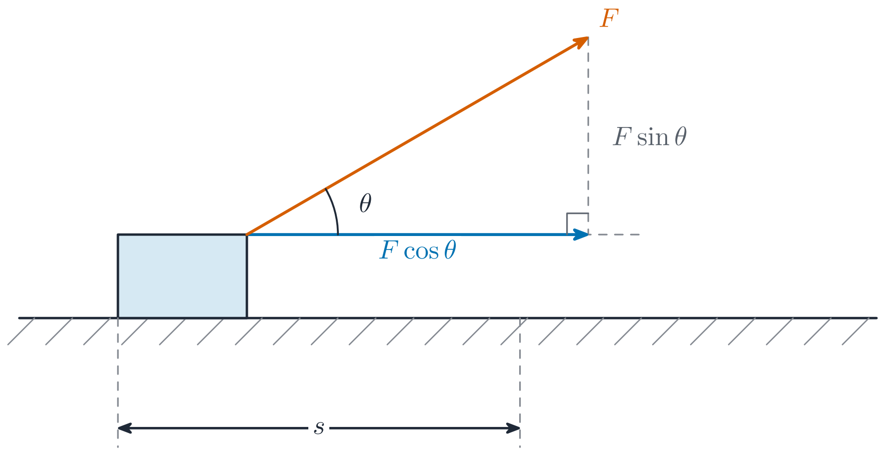
Wind turbines transfer the kinetic energy of the wind into electrical energy.

Work done by a force

Work 功 is done when a force moves its point of contact along the line of the force. The work done by a constant force F that causes a **displacement** 位移 s is

$$W = F \cdot s \cdot \cos\theta,$$

where θ is the angle between the force and the displacement. Only the **component** 分量 of the force along the displacement does work.



Only the component of the force along the displacement ($F\cos\theta$) does work

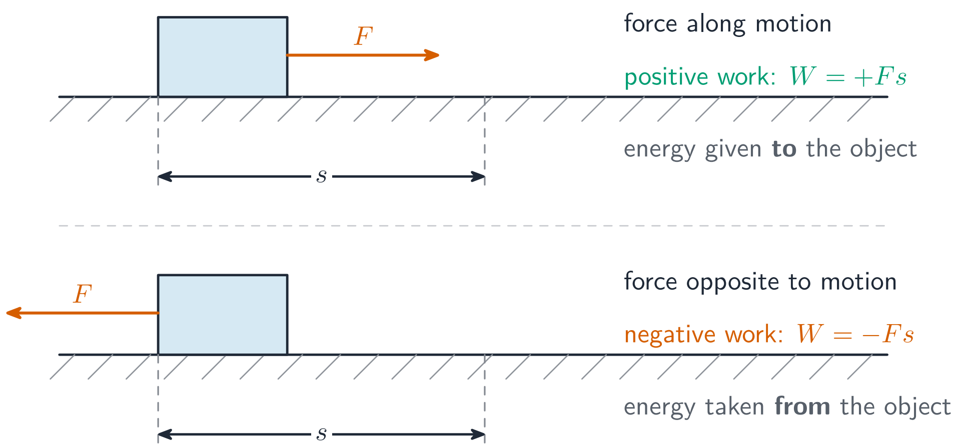
Worked example. A child pulls a sledge 5.0 m across the snow with a rope, using a force of 20 N at 60° to the ground. Find the work done by the rope.

$$W = Fs\cos\theta = 20 \times 5.0 \times \cos 60^\circ = 20 \times 5.0 \times 0.50 = 50 \text{ J}.$$

Unit: J = N m. Work is a **scalar** 标量.

Special cases:

- force in the same direction as the motion ($\theta = 0$): $W = Fs$, positive work, **energy** 能量 given **to** the object.
- force at right angles to the motion ($\theta = 90^\circ$): $W = 0$. The **normal contact force** 支持力 on a car on a flat road does no work.
- force opposite to the motion ($\theta = 180^\circ$): $W = -Fs$, negative work, energy taken **from** the object (for example **friction** 摩擦力).



Positive work ($W = +Fs$): force along the motion (top). Negative work ($W = -Fs$): force opposite to the motion, e.g. friction (bottom)

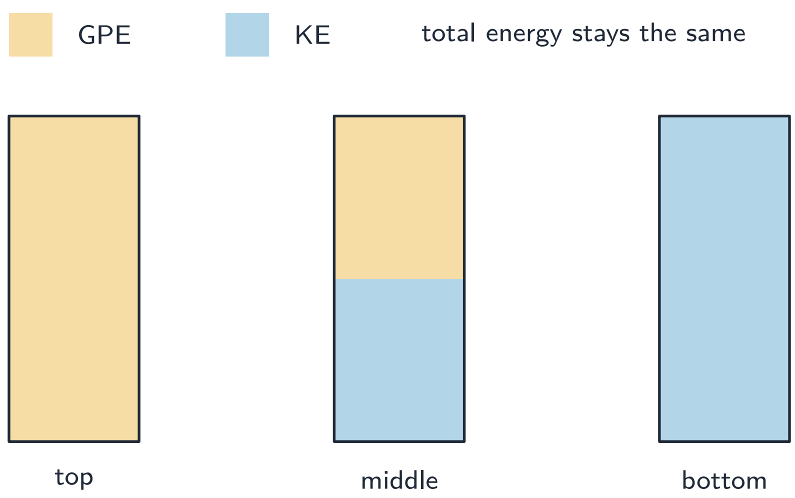
For an object moving up a slope at angle α to the **horizontal** 水平, the work done against gravity in rising a height h is mgh, while the work done by a horizontal push over the slope length L uses $\cos\alpha$.

Conservation of energy

Energy is **never made or destroyed** —it only changes from one form to another, or moves from one object to another. In a **closed system** 封闭系统, the total energy stays constant. This is **conservation of energy** 能量守恒.

When you write an energy equation, list every form the energy starts as and ends as. Common forms in this syllabus: kinetic, gravitational potential, **elastic potential energy** 弹性势能, **electrical** 电能, thermal, sound, **chemical energy** 化学能.

A ball rolling down a **frictionless** 无摩擦 **ramp** 斜坡 turns **gravitational potential energy** 重力势能 into **kinetic energy** 动能: $mgh = \frac{1}{2}mv^2$, so $v = \sqrt{2gh}$. With friction, some of this energy becomes **thermal energy** 热能 of the ramp and the air.



On a frictionless ramp, GPE turns into KE while the total energy stays constant

Worked example. A ball is released from rest at the top of a smooth ramp 1.2 m high. Find its speed at the bottom (take $g = 9.81$ m s^{-2}).

All the gravitational potential energy becomes kinetic energy, so $v = \sqrt{2gh}$ (the mass cancels):

$$v = \sqrt{2 \times 9.81 \times 1.2} \approx 4.9 \text{ m s}^{-1}.$$

Efficiency

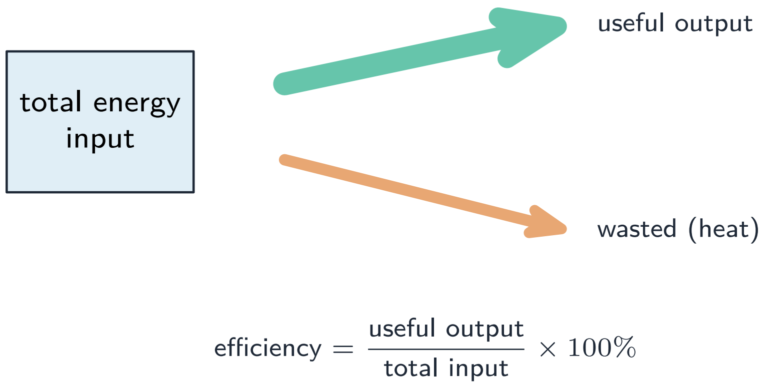
The **efficiency** 效率 of a system is

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%.$$

The same idea with power:

$$\text{efficiency} = \frac{\text{useful power output}}{\text{total power input}} \times 100\%.$$

Efficiency is always less than 100% in a real system, because some input energy becomes ”useless” forms —usually thermal energy.



Efficiency: only part of the energy input leaves as useful output; the rest is wasted, usually as heat

For an electric motor lifting a load with efficiency η at **voltage** 电压 V and **current** 电流 I, the useful output power is ηVI. From this you can find a force, a lifting speed, or a **tension** 张力.

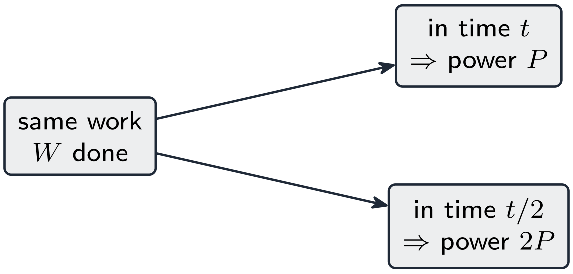
Worked example. An electric motor lifts a 50 kg load at a steady 0.40 m s^{-1} while drawing 250 W of electrical power. Find its efficiency (take $g = 9.81$ m s^{-2}).

The useful output power is $P = mgv = 50 \times 9.81 \times 0.40 = 196$ W, so

$$\text{efficiency} = \frac{196}{250} \times 100\% \approx 78\%.$$

Power

Power 功率 is the rate of doing work, or the rate of transferring energy:



$$P = \frac{W}{t} \text{ — less time, more power}$$

The same work done in less time means more power

$$P = \frac{W}{t} = \frac{\Delta E}{\Delta t}.$$

Unit: W = J s^{-1}. Power is a scalar.

Power, force and velocity

For an object moving at **velocity** 速度 v with a force F along the direction of motion, in a short time Δt the displacement is $v\,\Delta t$ and the work done is $Fv\,\Delta t$. Dividing by Δt:

$$P = Fv.$$

This is one of the most useful results in mechanics.

Worked example. A car travels at a steady 25 m s^{-1} against a total resistive force of 600 N. Find the output power of its engine.

At constant speed the driving force equals the resistive force, so

$$P = Fv = 600 \times 25 = 15\,000 \text{ W} = 15 \text{ kW}.$$

- For a car at constant velocity v on a flat road, the engine power must balance the total resistive force: $P = F_{\text{resist}} \cdot v$. If the **drag** 阻力 grows with v^2, doubling the speed roughly **quadruples** the power needed.
- For lifting a **weight** 重力 mg straight up at constant speed v, the useful output power is $P = mg \cdot v$.
- For an aircraft hovering at a fixed height, the lift force equals the weight, and a large power is needed because air must be pushed downwards all the time.

Gravitational potential energy

In a **uniform** gravitational field (close to a planet's surface), the change in gravitational potential energy of **mass** 质量 m rising or falling through a height Δh is

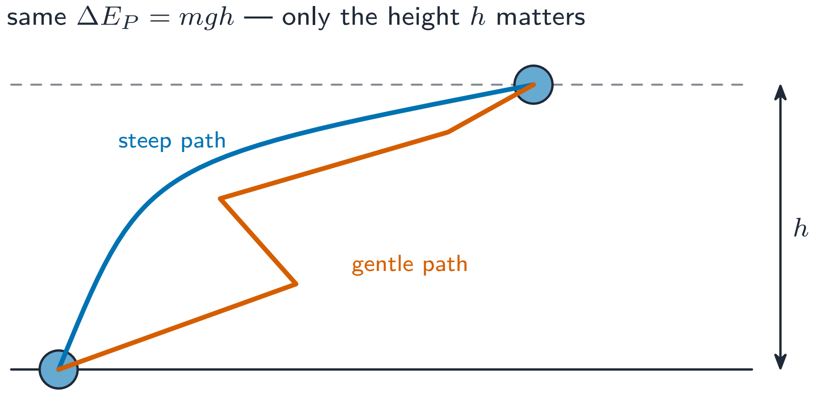
$$\Delta E_{\mathrm{P}} = mg\Delta h.$$

Where it comes from

The work done **against** gravity to raise a mass m slowly (no change in kinetic energy) through height Δh equals the gravitational potential energy gained:

- the gravitational force on the mass is mg downwards,
- the force needed to lift it slowly is mg upwards,
- the work done by this force is $W = F \cdot s = mg \cdot \Delta h$,
- this work becomes ΔE_{P}.

So $\Delta E_{\mathrm{P}} = mg\Delta h$. To use it you need m and Δh (and g). You do not need speed or time.



Gravitational PE depends only on the height risen, not the path taken

Kinetic energy

The kinetic energy of an object of mass m moving at speed v is

$$E_{\mathrm{K}} = \tfrac{1}{2}mv^2.$$

Where it comes from

Apply a resultant force F to a mass m that starts at rest. It speeds up evenly from 0 to v over a displacement s. From $v^2 = u^2 + 2as$ with $u = 0$,

$$s = \frac{v^2}{2a}.$$

The work done on the mass is

$$W = F \cdot s = ma \cdot \frac{v^2}{2a} = \tfrac{1}{2}mv^2.$$

All this work becomes kinetic energy, so $E_K = \frac{1}{2}mv^2$.

Kinetic energy and momentum

Combining $p = mv$ and $E_K = \frac{1}{2}mv^2$:

$$E_K = \frac{p^2}{2m}.$$

This is handy when the **momentum** 动量 is given but not the velocity. For a momentum change from p_1 to p_2 at constant mass, the change in kinetic energy is $(p_2^2 - p_1^2)/(2m)$.

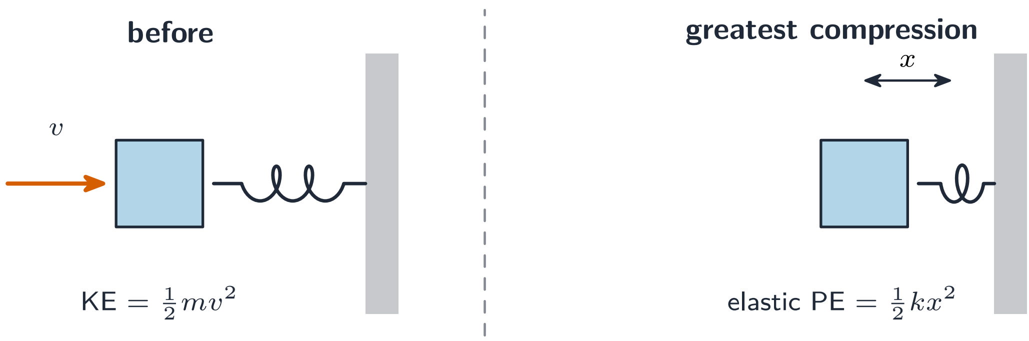
Using energy methods

A useful plan for problems that mix forces and energy:

1. **Find the start and end states.** Write the kinetic and potential energies in each.
2. **List any work done by outside forces** (friction, a push). Friction usually takes energy out; a push can add it.
3. **Conservation of energy**: $E_{\text{start}} + W_{\text{in}} = E_{\text{end}} + W_{\text{lost as heat etc.}}$.

Examples:

- A box pushed at **constant velocity** up a ramp of length L rising by h: E_K does not change, so the work done by the push goes into ΔE_P plus the work done against friction.
- A block sliding into a **spring** 弹簧 with kinetic energy E_K on a frictionless surface: at greatest **compression** 压缩 x, all the kinetic energy has become elastic potential energy $\frac{1}{2}kx^2$ (where k is the **spring constant** 劲度系数).
- A ball dropped from height h_1 that bounces to height h_2: the ratio h_2/h_1 is the fraction of mechanical energy kept, $h_2/h_1 = (v_{\text{up}}/v_{\text{down}})^2$.
- A **projectile** 抛体 thrown to the same height at different angles: the final speed is the same (only the height matters); use components to get its direction.



A block's kinetic energy becomes elastic potential energy as it compresses the spring

Exam tips

- **Work = force × distance moved in the direction of the force** —use $Fs\cos\theta$ when they are at an angle.
- Use **conservation of energy**: loss in GPE = gain in KE (+ work done against resistance).
- **Power = work / time** = Fv; efficiency = useful output ÷ total input.
- GPE uses the **vertical** height gained, not the distance along a slope.