

Dynamics

A-Level Physics

Mass, momentum and force

Mass

Mass 质量 tells you how hard it is to change an object's motion. The larger the mass, the larger the **force** 力 needed to give it a certain **acceleration** 加速度. Mass is measured in kilograms (kg) and is a **scalar** 标量.

Momentum

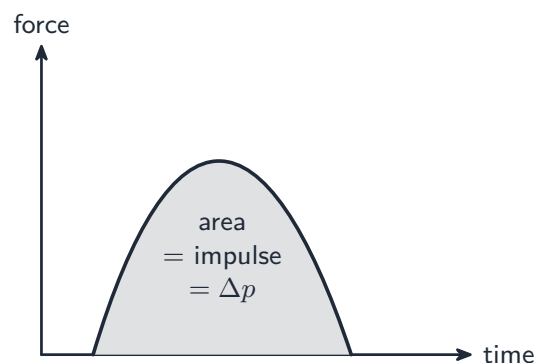
Linear momentum 动量 is the product of mass and velocity:

$$p = mv.$$

Momentum is a **vector** 矢量—it points the same way as the **velocity** 速度. Its unit is kg m s^{-1} , which is the same as N s .

Force as the rate of change of momentum

Newton's second law, in its general form: the **resultant force** 合力 on an object equals the rate of change of its momentum.



Impulse is the area under a force-time graph, equal to the change in momentum

$$F = \frac{\Delta p}{\Delta t}.$$

When the mass is constant this becomes $F = ma$, because $\Delta p = m \Delta v$ and $\Delta v / \Delta t = a$. Cambridge questions often want you to use $F = \Delta p / \Delta t$ directly for a **collision** 碰撞 or an **impulse** 冲量: the average force equals the change in momentum divided by the contact time.

A ball hits a wall with momentum p_1 and bounces back with momentum p_2 in the opposite direction. The change in momentum is $\Delta p = p_2 - p_1$ (give each direction the correct sign). The average force is $\Delta p / \Delta t$, where Δt is the contact time.

Worked example. A 0.20 kg ball hits a wall at 8.0 m s^{-1} and bounces straight back at 6.0 m s^{-1} . The contact lasts 0.050 s. Find the average force on the ball.

Take the rebound direction as positive, so $u = -8.0 \text{ m s}^{-1}$ and $v = +6.0 \text{ m s}^{-1}$:

$$\Delta p = m(v - u) = 0.20 \times (6.0 - (-8.0)) = 2.8 \text{ kg m s}^{-1},$$

$$F = \frac{\Delta p}{\Delta t} = \frac{2.8}{0.050} = 56 \text{ N}.$$

When you know the momentum but not the speed, the change in **kinetic energy** 动能 is

$$\Delta E_k = \frac{p_2^2 - p_1^2}{2m}.$$

This comes from $E_k = \frac{1}{2}mv^2 = p^2/(2m)$.

Newton's three laws of motion



A rocket pushes gas down; by Newton's third law the gas pushes the rocket up.

Image: NASA, Public domain (commons.wikimedia.org)

First law

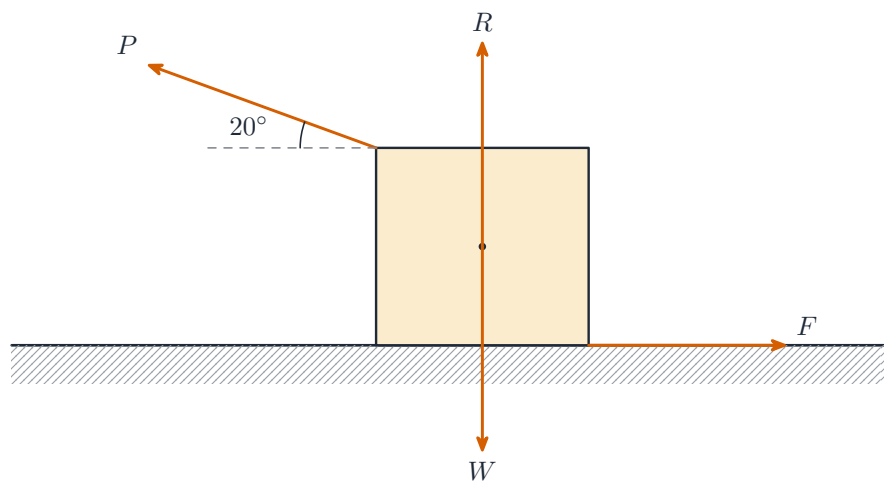
An object stays at rest, or keeps moving at constant velocity in a straight line, unless a resultant **external force** 外力 acts on it. In short: zero resultant force means zero acceleration.

Second law

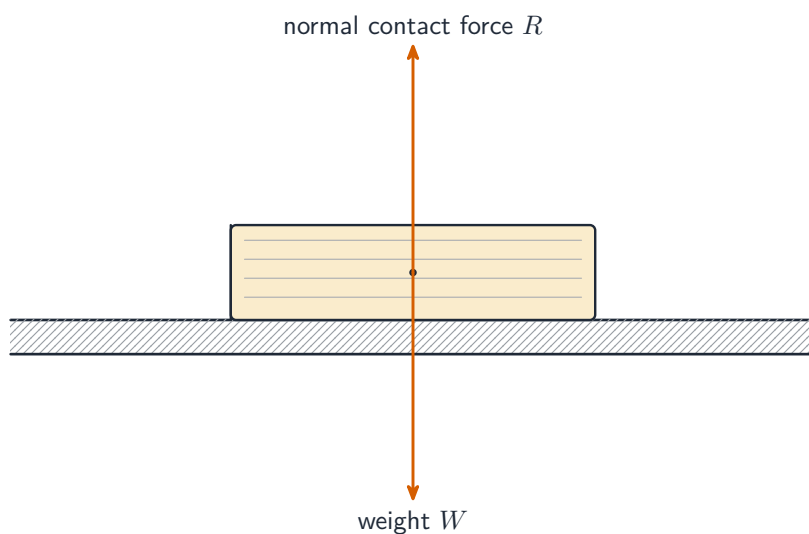
The resultant force on an object equals its rate of change of momentum, and acts in the same direction as that change. In SI units,

$$F = \frac{\Delta p}{\Delta t} = ma \quad (\text{for constant mass}).$$

Acceleration and resultant force always point the **same way**.



Free-body diagram showing all forces on a block being pulled at an angle



Weight and normal contact force on a book resting on a table

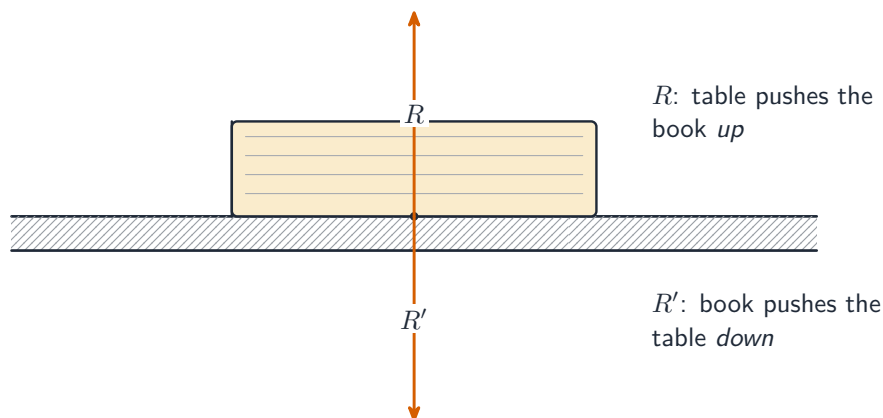
Third law

When body A pushes on body B, body B pushes back on body A with an **equal and opposite** force. The two forces:

- act on **different objects**,
- are of the **same type** (both gravitational, both contact, both **electrostatic** 静电, and so on),
- have the same size and opposite direction.

A common trap: the **weight** 重力 of a block on a table and the **normal contact force** 支持力 from the table are **not** a third-law pair (they act on the same object and are different types). The third-law partner of the block's weight is the pull the block makes on the Earth. The third-law partner of the table's contact force is the push the block makes on the table.

For a rocket: the **thrust** 推力 on the rocket and the force on the gases are a third-law pair (the engine pushes the gas down, the gas pushes the engine up). Weight and air resistance are not part of this pair.



Newton's third-law pair: R on the book (up) and R on the table (down)

Weight

Weight is the force on an object from a **gravitational field** 重力场. Near the Earth's surface,

$$W = mg,$$

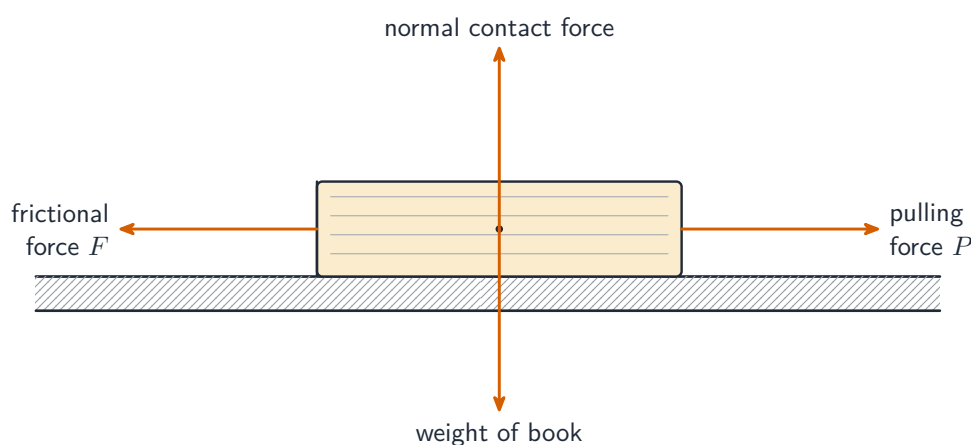
where $g \approx 9.81 \text{ m s}^{-2}$ is the acceleration of free fall. Weight is a vector that points towards the centre of the Earth. Do not mix it up with mass: mass is the same everywhere, but weight changes with place.

Non-uniform motion: friction, drag and terminal velocity

Friction and drag forces

A **friction** 摩擦力 force between two solid surfaces acts along the surface and opposes the sliding. A **viscous** 黏性 or **drag** 阻力 force is the resistive force from a **fluid** 流体 (a liquid or gas) on an object moving through it; air resistance is the case for air. You do not need to use any **coefficient** 系数 of friction or viscosity.

A simple model: the drag gets bigger as the speed gets bigger. At zero speed, the drag is zero.



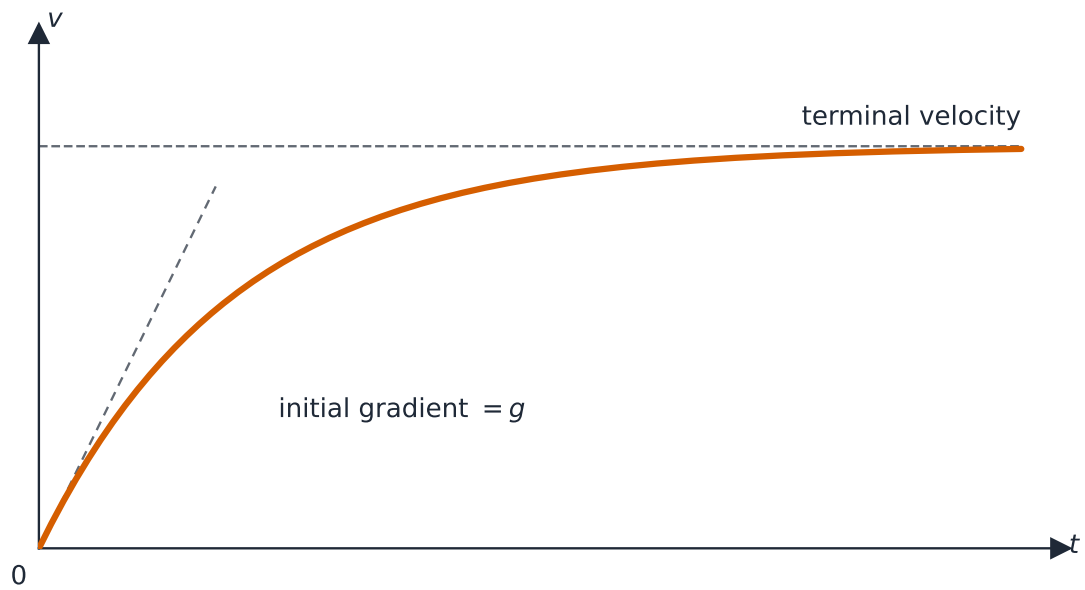
Free-body diagram of a book being pulled on a table

An object falling through air

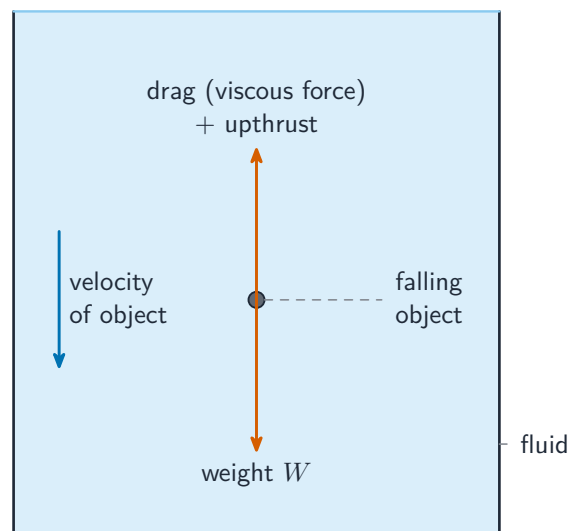
For an object dropped from rest and falling through air:

1. At first, only weight acts, so the object speeds up downwards at g .
2. As the speed grows, the upward drag grows. The resultant force gets smaller, so the acceleration gets smaller.
3. In the end, the drag equals the weight. The resultant force is zero, the acceleration is zero, and the speed stays constant —the **terminal velocity** 收尾速度.

On a velocity–time graph, the line starts straight with gradient g , then bends and flattens at the terminal velocity. This shape (fast start, then slowing acceleration, then constant speed) is how "falling with air resistance" differs from "free fall in a vacuum".



Velocity-time graph for an object falling through air



Forces on a falling object in a fluid



Why the spread-eagle pose? Spreading out gives the largest area, so the most drag. The bigger the drag, the sooner drag balances weight —and the lower the steady terminal velocity they fall at

Image: Rstpch, CC BY-SA 4.0 (commons.wikimedia.org)

Energy during a terminal-velocity fall

At terminal velocity, a **parachutist** 跳伞者 has constant kinetic energy. But the **gravitational potential energy** 重力势能 keeps falling as they go down. Where does it go? Almost all of it turns into **thermal energy** 热能 of the air around them. It does **not** become kinetic energy of the parachutist —that stays constant.

Cyclist or car at constant speed

A vehicle at constant speed on a flat road has zero resultant force. The forward driving force is equal and opposite to the total resistive force (friction, air resistance, and rolling resistance). At higher speed the drag is larger, so the driving force must be larger too — and so the **power** 功率 must be larger.

Conservation of linear momentum



In a crash, a large force acts over a very short time to change momentum.

Image: Kalispera Dell, CC BY 3.0 (commons.wikimedia.org)

The principle

For a system with no resultant external force, the total momentum stays constant. This is **conservation of momentum** 动量守恒.

It always holds when there is no outside resultant force—in collisions, **explosions** 爆炸, and **recoil** 反冲. In two dimensions, momentum is conserved along each direction on its own.

Worked example. A 2.0 kg trolley and a 3.0 kg trolley are held together against a compressed spring, then released from rest. The 2.0 kg trolley flies off at 6.0 m s^{-1} . Find the speed of the other trolley.

The total momentum stays zero (it started at rest), so

$$0 = (2.0)(6.0) + (3.0)(-v) \quad \Rightarrow \quad v = \frac{12}{3.0} = 4.0 \text{ m s}^{-1}$$

in the opposite direction.



equal size, opposite direction
(forces act on different objects)

Newton's third law in an isolated two-particle system: equal and opposite forces

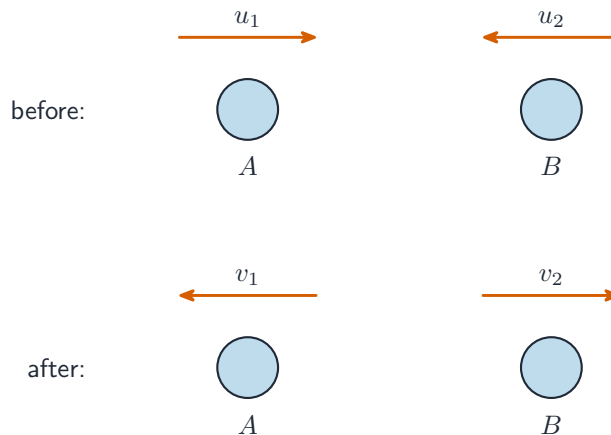
Elastic and inelastic collisions

In **every** collision, momentum is conserved (if there is no outside resultant force).

An **elastic collision** 弹性碰撞 is one where the total kinetic energy is **also** conserved. A quick test: in an elastic collision, the **relative speed** 相对速率 of approach equals the relative speed of separation.

In an **inelastic collision** 非弹性碰撞, momentum is conserved but kinetic energy goes down —some becomes thermal, sound, or **deformation** 形变 energy. If the two objects stick together, the collision is perfectly inelastic.

Solving collision problems (one dimension)



Head-on collision: velocities before and after

For two objects with masses m_1, m_2 and starting velocities u_1, u_2 that hit **head-on** 正面, write

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2.$$

Use **signed** velocities (positive in one chosen direction). If the collision is elastic, add the relative-speed equation

$$u_1 - u_2 = -(v_1 - v_2),$$

or, the same thing, $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$. That gives two equations for two unknowns.

Worked example. A 1500 kg car moving at 12 m s^{-1} runs into a stationary 1000 kg car and they lock together. Find their common velocity just after the collision.

Momentum is conserved (the cars stick, so $v_1 = v_2 = v$):

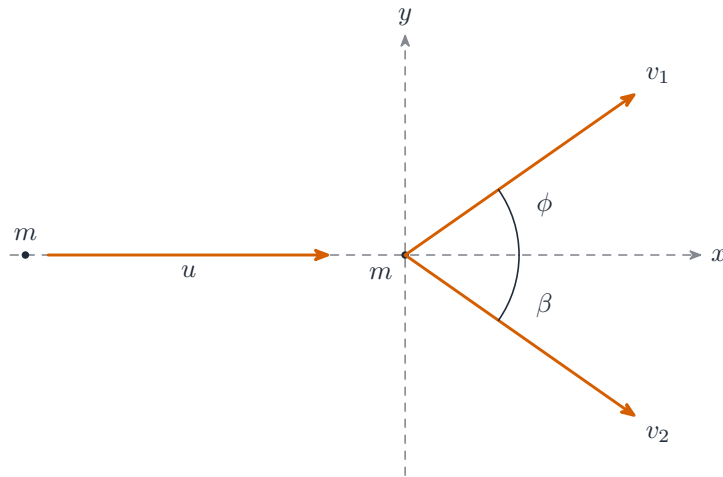
$$1500 \times 12 + 1000 \times 0 = (1500 + 1000) v \quad \Rightarrow \quad v = \frac{18\,000}{2500} = 7.2 \text{ m s}^{-1}.$$

A useful result for a head-on elastic collision of mass m with a **stationary** 静止 mass M :

$$v_m = \frac{m - M}{m + M}u, \quad v_M = \frac{2m}{m + M}u.$$

Collisions in two dimensions

If the objects move in two dimensions, split the velocities into **perpendicular** 垂直 **components** 分量 and apply conservation of momentum along each direction on its own. For a collision where the objects hit at an angle, choose one axis along the first object's motion and one across it. The total momentum is conserved along each axis.



A glancing collision resolved along two perpendicular axes

Rocket / pushing out mass

A rocket pushes out gas at velocity u (relative to itself) at a **mass-flow rate** 质量流率 $\dot{m}$ (kg per second). It feels a thrust

$$F = \dot{m} \cdot u,$$

which comes from $F = \Delta p / \Delta t$. The momentum given to the gas each second equals the thrust on the rocket (Newton's third law: the rocket pushes the gas one way, the gas pushes the rocket the other way).

Exam tips

- **Newton's second law** is $F = \Delta p / \Delta t$ (rate of change of momentum); $F = ma$ is the special case for constant mass.
- Identify **third-law pairs** correctly: the same type of force, acting on **two different bodies** —not the balanced forces on one body.
- **Momentum is conserved** in every collision; kinetic energy is conserved only in an **elastic** collision.
- For **terminal velocity**, explain that drag rises with speed until drag = weight, so the acceleration becomes zero.