Kinematics

A-Level Physics

Key definitions

A speedometer shows speed: the distance travelled per unit time.

These five quantities come up in almost every **kinematics** 运动学 question. Learn the exact words —the examiner gives marks for precise wording.

- **distance** 距离—the total length of the path travelled. A **scalar** 标量.
- **displacement** 位移—the straight-line distance from the start to the end, with a direction. A **vector** 矢量.
- **speed** 速率—the rate of change of distance with time. A scalar.
- **velocity** 速度—the rate of change of displacement with time. A vector.
- **acceleration** 加速度—the rate of change of velocity with time. A vector.

The unit of speed and velocity is m s^{-1}; the unit of acceleration is m s^{-2}.

A common mistake: **deceleration** 减速度 just means acceleration in the opposite direction to the velocity. It is not a separate quantity.

Motion graphs

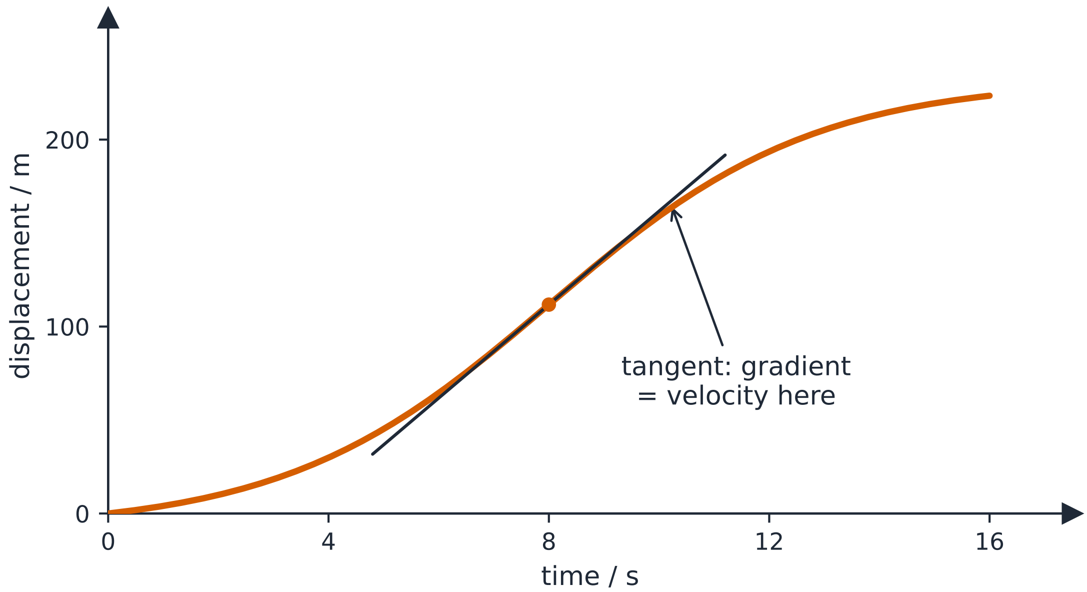
A high-speed train: its motion can be shown on a distance-time graph.

Many marks come from reading or drawing motion graphs. Two graphs matter.

Displacement–time graph

The **gradient** 斜率 (steepness) of a displacement–time graph at a point gives the velocity at that moment.

- flat line → the object is at rest.
- straight sloping line → constant velocity (gradient = velocity).
- curved line → changing velocity. Draw a **tangent** 切线 at the point and find its gradient.



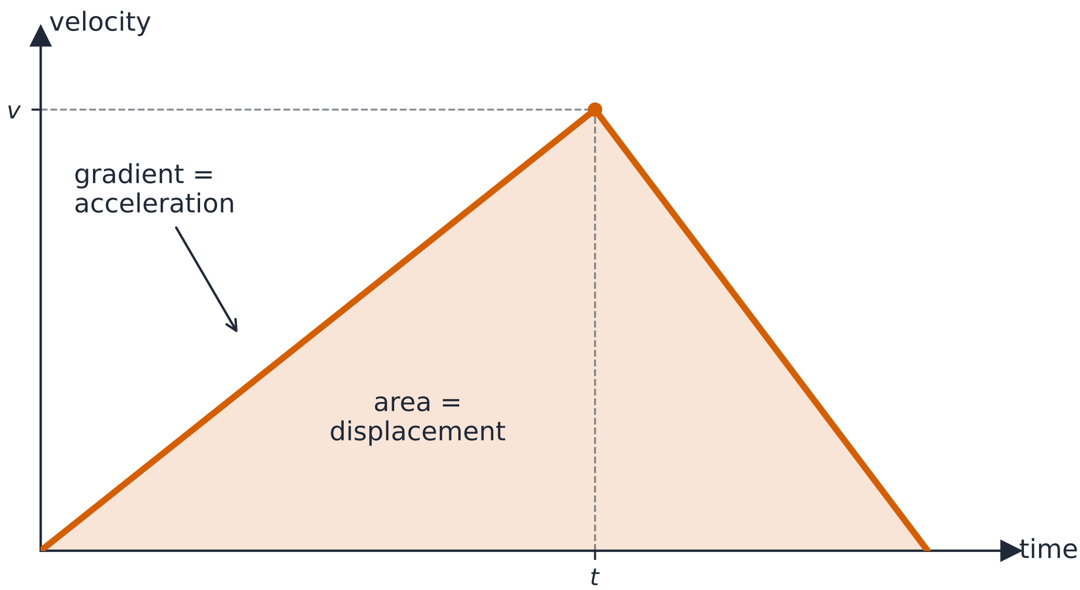
Displacement–time graph of a car on a test track

Velocity–time graph

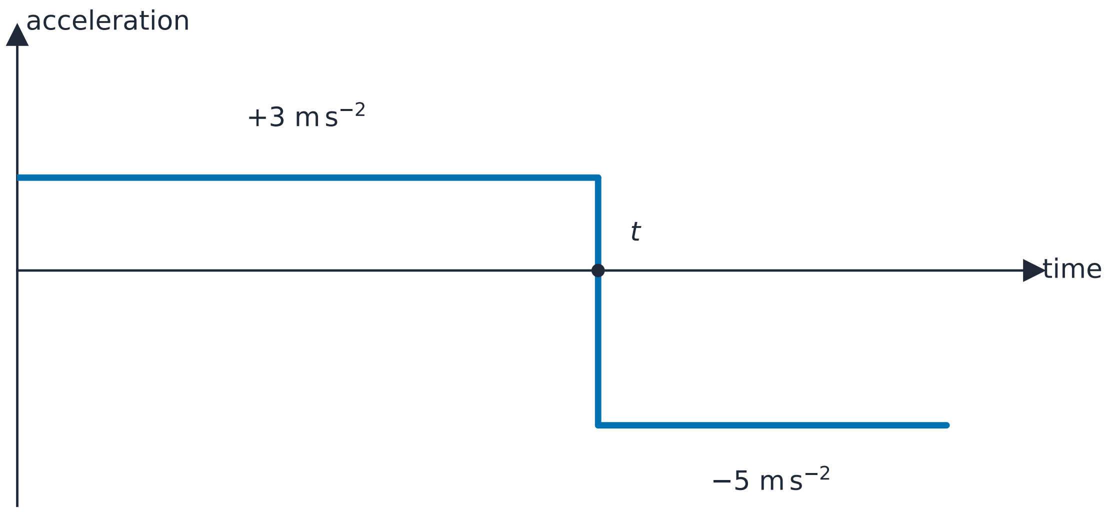
The gradient of a velocity–time graph gives the acceleration at that moment.

The **area** between the line and the time axis gives the displacement in that time.

- flat line → constant velocity (zero acceleration).
- straight sloping line → **uniform acceleration** 匀加速 (constant acceleration).
- curved line → changing acceleration.
- area above the time axis is positive displacement; area below is negative (the object moved backwards).



Velocity–time graph —gradient gives acceleration, area gives displacement



Acceleration–time graph derived from the same motion

To find the displacement, split the area into triangles and rectangles, or count grid squares. Area of a triangle is $\frac{1}{2}$ × base × height; area of a rectangle is base × height.

The four SUVAT equations

For motion in a straight line with uniform acceleration, we use five symbols: starting velocity u, final velocity v, acceleration a, displacement s, and time t. Four equations link them:

$$v = u + at$$

$$s = ut + \tfrac{1}{2}at^2$$

$$s = \tfrac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

Each equation uses four of the five symbols. To pick the right one: write down what you know and what you want, then choose the equation with exactly those four.

Worked example. A car accelerates uniformly from 8 m s^{-1} to 20 m s^{-1} over a distance of 56 m. Find its acceleration.

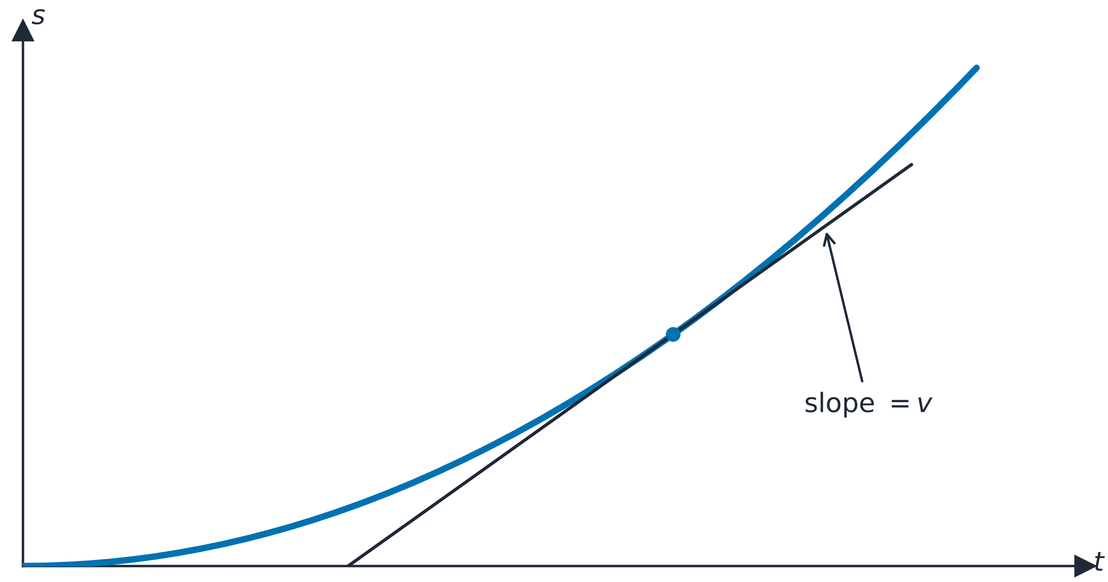
We know u, v and s and want a, so use $v^2 = u^2 + 2as$:

$$20^2 = 8^2 + 2a(56) \quad \Rightarrow \quad 336 = 112\,a \quad \Rightarrow \quad a = 3.0 \text{ m s}^{-2}.$$

Where the SUVAT equations come from

You should be able to get these from the definitions of velocity and acceleration:

- $v = u + at$ comes from $a = (v - u)/t$, the gradient of the line.
- $s = \frac{1}{2}(u + v)t$ is the area under the line —a **trapezium** 梯形 with parallel sides u and v and width t.
- $s = ut + \frac{1}{2}at^2$ comes from putting $v = u + at$ into the area.
- $v^2 = u^2 + 2as$ comes from removing t from the first two.



Displacement–time graph for uniform acceleration —the slope at any point equals the instantaneous velocity

If a question asks ”which equation can be found using only the gradient of a velocity–time graph?”, the answer is $v = u + at$ (the gradient is the acceleration).

Choosing a positive direction

Pick a positive direction at the start and keep it. Anything pointing the other way gets a minus sign. For a ball thrown straight up, if ”up” is positive: u is positive, $a = -g$ (**gravity** 重力 pulls down), and at the highest point the displacement is positive but the velocity is zero.

Free fall under gravity

When **air resistance** 空气阻力 can be ignored, an object in **free fall** 自由落体 has a constant acceleration $g \approx 9.81$ m s^{-2} downwards. This is the same for every mass.

For a ball dropped from rest and falling a distance h:

$$h = \tfrac{1}{2}gt^2, \qquad v = gt, \qquad v^2 = 2gh.$$

For a ball thrown straight up with speed u:

- greatest height: put $v = 0$ in $v^2 = u^2 - 2gh$, giving $h = u^2/(2g)$.
- time to reach the top: put $v = 0$ in $v = u - gt$, giving $t = u/g$.
- total time to fall back to the start height: $2u/g$ (the motion is **symmetric** 对称).

Worked example. A ball is thrown straight up at 20 m s^{-1}. Find the greatest height it reaches (take $g = 9.81$ m s^{-2}).

At the highest point $v = 0$, so from $h = u^2/(2g)$:

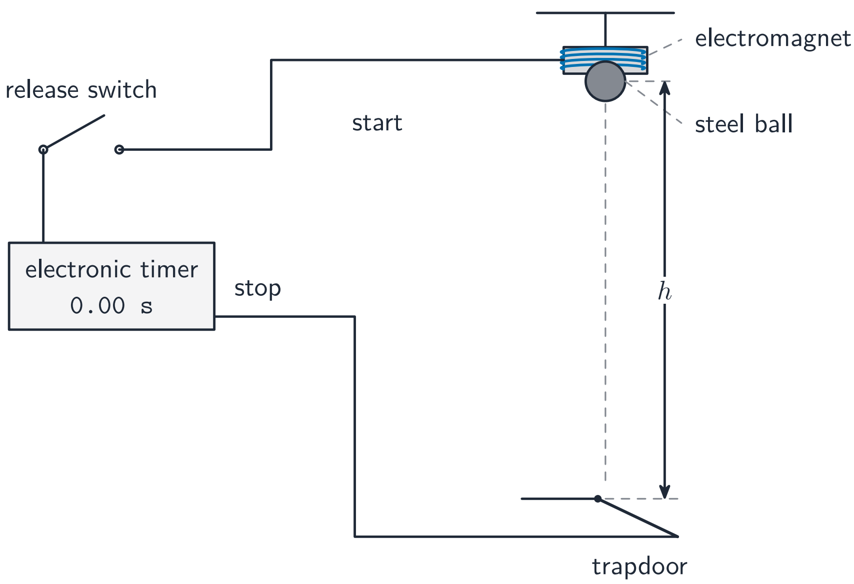
$$h = \frac{20^2}{2 \times 9.81} = \frac{400}{19.62} \approx 20.4 \text{ m}.$$

Experiment to find g

A common method: drop an object from rest, then measure the distance h it falls and the time t it takes. Then

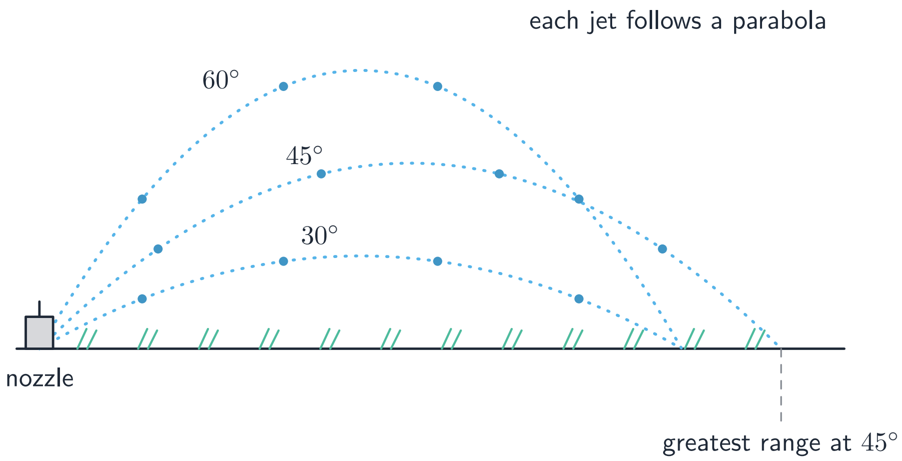
$$g = \frac{2h}{t^2}.$$

Repeat for several heights and plot h against t^2. The gradient of the best straight line is $g/2$, so g is twice the gradient. Repeating reduces **random error** 随机误差. An electronic timer —using **light gates** 光电门, or a switch the ball hits —removes **reaction-time** 反应时间 error.



Experimental set-up for measuring the acceleration due to free fall

Motion in two directions



Water jets from a sprinkler trace parabola paths —a real example of projectile motion

When an object moves at constant velocity in one direction (say **horizontal** 水平) and speeds up in a direction at right angles to it (say **vertical** 竖直, under gravity), the two motions do not affect each other. Treat each direction on its own, with its own SUVAT equation.

Horizontal throw

An object thrown horizontally with speed u_{H} from height h, with air resistance ignored:

- **horizontal**: constant velocity u_{H}. After time t, the horizontal distance is $x = u_{\mathrm{H}}t$.
- **vertical**: starts from rest and speeds up downwards at g. After time t, it has fallen $y = \frac{1}{2}gt^2$ and has vertical velocity $v_{\mathrm{V}} = gt$.

The time to reach the ground depends **only** on the height h, not on u_{H}. Solve $h = \frac{1}{2}gt^2$ for t; then the horizontal **range** 射程 is $u_{\mathrm{H}}t$.

Worked example. A ball is thrown horizontally at 15 m s^{-1} from the top of a cliff 20 m high. Find the time it takes to land and how far from the base it lands (take $g = 9.81$ m s^{-2}).

Vertical motion gives the time: from $h = \frac{1}{2}gt^2$,

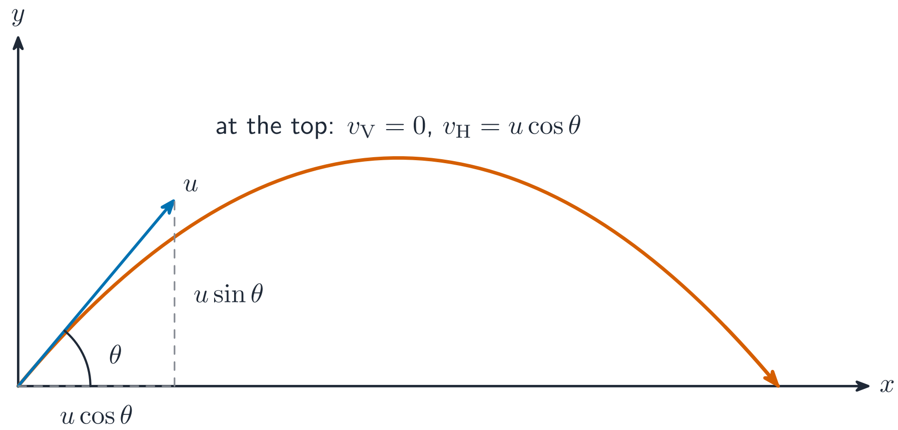
$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 20}{9.81}} \approx 2.0 \text{ s}.$$

Horizontal motion then gives the range: $x = u_{\mathrm{H}}t = 15 \times 2.0 \approx 30$ m.

The horizontal-velocity graph is a flat line at u_{H}. The vertical-velocity graph is a straight line from the origin with gradient g.

Projectile at an angle

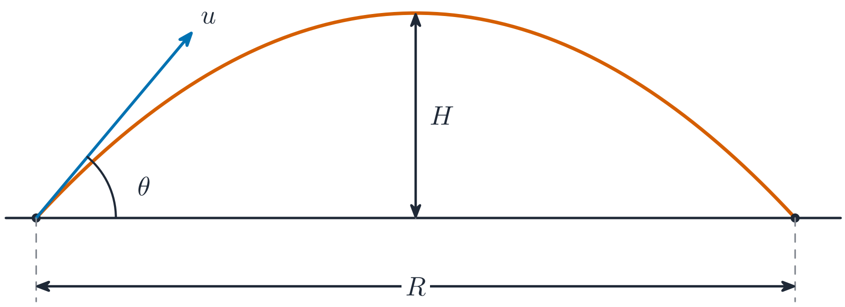
A **projectile** 抛体 thrown at speed u at angle θ above the horizontal:



Projectile launched at angle θ —horizontal and vertical motions are independent

- horizontal **component** 分量 of the starting velocity: $u_H = u\cos\theta$ (stays constant during the flight).
- vertical component of the starting velocity: $u_V = u\sin\theta$ (gets smaller, becomes zero at the top, then grows downwards).

At the highest point, $v_V = 0$, but v_H is still $u\cos\theta$. The time to the top is $t_{up} = u\sin\theta/g$; the total flight time (back to the start height) is $2t_{up}$.



Range R of a projectile launched from and landing on level ground

Bouncing ball

When a ball bounces, its velocity–time graph is a set of straight sloping lines (constant g) with a sudden jump at each bounce (the velocity flips direction, and gets smaller if some **energy** 能量 is lost). Add up the times and the distances across the bounces.

Two objects meeting

When two objects move along the same line in different ways, write a displacement equation for each. Use the same start time and the same positive direction. Then set the two displacements equal (or set their difference to a given gap).

For a goods train at constant velocity u_G and an express train starting from rest with acceleration a, both passing the same point at $t = 0$:

$$s_G = u_G t, \qquad s_E = \tfrac{1}{2}at^2.$$

They are level again when $s_G = s_E$, giving $t = 2u_G/a$.

Tips for solving problems

1. **Draw a diagram** and mark the positive direction.
2. **List the SUVAT symbols** with their known and unknown values, including signs.
3. **Choose the SUVAT equation** with exactly the four symbols you have, plus the one you want.
4. For projectile motion, **split into horizontal and vertical** SUVAT problems, linked only by the time t.
5. Always **check the units** of your answer, and that its size is sensible.

Exam tips

- Use the **SUVAT** equations only for **constant acceleration**; list s, u, v, a, t and pick the equation missing your unknown.
- Choose one direction as **positive** and keep signs consistent (usually $g = -9.81$ m s^{-2}).
- On a **velocity-time graph**, gradient = acceleration and area = displacement.
- For projectiles, treat **horizontal (constant velocity) and vertical ($a = g$) motion separately**, linked by the same time.