MATHEMATICS

Paper 9709/11
Pure Mathematics 1

Key messages

The question paper contains a statement in the rubric on the front cover that 'no marks will be given for unsupported answers from a calculator.' This means that clear working must be shown to justify solutions, particularly in syllabus items such as quadratic equations and trigonometric equations. In the case of quadratic equations, for example, it is necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient for certain marks to be awarded. It is also insufficient to quote only the formula; candidates need to show values substituted into it. When an exact answer is required, a rounded decimal is not sufficient.

General comments

Some very good responses were seen, but the paper proved challenging for many candidates. In AS and A Level Mathematics papers, the knowledge and use of fundamental algebraic and trigonometric methods is expected, as stated in the syllabus.

Comments on specific questions

Question 1

Strong responses used $b^2 - 4ac$ to obtain $(k + 8)^2 - 4 \times 3k \times 3$ and hence $k^2 - 20k + 64 = 0$ or $k^2 - 20k + 64 > 0$, which they solved using a suitable method to find $k < 4$, $k > 16$. Some responses did not include the method for solving the quadratic equation, and some gave the solution as $k = 4, 16$ rather than $k < 4$, $k > 16$.

Question 2

(a) The strongest responses stated $a\cos\theta = 8$ and $a\cos^4\theta = \frac{1}{8}$, divided these two equations to find $\cos^3\theta = \frac{1}{64}$ and solved this to obtain $\theta = 1.32^c$. Other successful candidates used $ar = 8$ and $ar^4 = \frac{1}{8}$ to find $r^3 = \frac{1}{64}$. Some candidates gave the final solution in degrees rather than in radians as asked for in the question.

(b) Successful responses found a using $a = \dfrac{8}{r}$ and then substituted into $S_\infty = \dfrac{a}{1-r}$. Again, some responses did not give an exact answer as asked for in the question.

Question 3

Stronger responses found the relevant terms of $5p^4x^4 \times 3$ and $-6p^2x^4$ using the binomial formula. Responses which included the full expansions were usually less successful than those who only found the relevant terms. Stronger responses went on to find that $15p^4 - 6p^2 - 216 = 0$ and solved this as a quadratic in p^2 using a suitable method to conclude $p = 2$. Some responses did not include a full and detailed method for solving the quadratic equation.

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Question 4

(a) This was an accessible question, and many responses achieved full credit. Stronger responses complete the square directly but the expansion of $a - (x + b)^2$ and the equating of coefficients was also seen.

(b) The most successful responses stated the correct answer directly from their result in **4(a)**. Weaker responses restarted the question rather than using their prior work.

Question 5

(a) Most responses began by stating $\tan^4\theta = \dfrac{\sin^4\theta}{\cos^4\theta}$. Stronger candidates then worked on the left-hand side of the equation to give $\dfrac{\sin^4\theta - \cos^4\theta}{\cos^4\theta} \rightarrow \dfrac{(\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)}{\cos^4\theta}$ $\rightarrow \dfrac{(1 - \cos^2\theta - \cos^2\theta)}{\cos^4\theta} \rightarrow \dfrac{1 - 2\cos^2\theta}{\cos^4\theta}$, or $\dfrac{\sin^4\theta - \cos^4\theta}{\cos^4\theta} \rightarrow \dfrac{\left(1 - \cos^2\theta\right)^2 - \cos^4\theta}{\cos^4\theta}$ $\rightarrow \dfrac{1 - 2\cos^2\theta + \cos^4\theta - \cos^4\theta}{\cos^4\theta} \rightarrow \dfrac{1 - 2\cos^2\theta}{\cos^4\theta}$.

(b) Stronger responses were structured as $\dfrac{1 - 2\cos^2\theta}{\cos^2\theta} = 7 \rightarrow 9\cos^2\theta = 1 \rightarrow \cos\theta = \pm\,\frac{1}{3} \rightarrow$ $\theta = 70.5°$ and $109.5°$. Weaker responses did use the result of **5(a)**, which made solving the equation more difficult with several more steps involved.

Question 6

(a) Stronger responses stated $\mathrm{f}(x) \geqslant -3$ or $y \geqslant -3$. A common error was to state that $y \geqslant -12$.

(b) This was an accessible question and there were many fully correct answers. Some responses stated that $\mathrm{f}^{-1}(x) = -3 \pm \sqrt{x+12}$ rather than $\mathrm{f}^{-1}(x) = -3 + \sqrt{x+12}$.

(c) Successful responses stated $2((x + 3)^2 - 12) - 5 = 69 \rightarrow 2\left(x+3\right)^2 = 98$ or $2x^2 + 12x - 80 \rightarrow x = 4$. A common error was to include -10 in the final solution.

Question 7

(a) Successful responses usually subtracted the area of the triangle from the area of the sector or used the formula for the area of a segment. Weaker responses only found the area of the triangle or the sector and did not proceed to the stated expression.

(b)(i) This question proved to be challenging for many. Stronger responses stated $\dfrac{\mathrm{d}A}{\mathrm{d}r} = 1.228r$ and then used $\dfrac{\mathrm{d}A}{\mathrm{d}t} = \dfrac{\mathrm{d}A}{\mathrm{d}r} \times \dfrac{\mathrm{d}r}{\mathrm{d}t}$, or equivalent, with $r = 20$ to obtain 9.8. Weaker responses attempted to answer the question without using the chain rule.

(b)(ii) As with **7(b)(i)**, this question proved to be challenging for a majority of candidates. Stronger responses stated or implied $\frac{2}{3}\pi r$ for length of arc A, differentiated and applied the correct use of chain rule and obtained 0.84.

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Question 8

(a) There were many correct solutions to this question, with stronger responses integrating to obtain $\frac{1}{3}x^{\frac{3}{2}}$, evaluating the integral between the limits of 0 and 9 and subtracting the result from $\frac{27}{2}$ to give a final answer of $\frac{9}{2}$. Some responses incorrectly used the limits of 0 and $\frac{3}{2}$ in this approach.

A less common approach was to state $x = 4y^2$ and evaluate the integral between limits of 0 and $\frac{3}{2}$ to obtain the area.

(b) Successful responses stated $x^2 = y^4$, stated that the volume = $\pi\int 16y^4\ \mathrm{d}y$ and integrated between limits 0 and $\frac{3}{2}$ to obtain $\frac{243}{10}\pi$. Some responses incorrectly calculated the volume of revolution about the x-axis rather than about the y-axis.

Question 9

(a) Successful responses obtained $S_2 - 1 = 3 + d$, $S_4 = 8 + 6d$, $S_9 = 18 + 36d$, stated $(3 + d) + (18 + 36d) = 2(8 + 6d)$ and hence $d = -\frac{1}{5}$. A common error was to give the first term of the sequence as S_2 rather than $S_2 - 1$. Another common error was to use d as the difference between the terms $3 + d$, $8 + 6d$ and $18 + 36d$.

(b) Most responses began successfully by finding the 15th term of the first progression as $2 + 14 \times$ *'their' d.* Stronger responses then obtained the first term = $\frac{14}{5}$ and the common difference = 4 for the second progression, found the 15th term of the second progression as $\frac{294}{5}$ and hence the difference = $\frac{298}{5}$ or 59.6.

Question 10

(a) This was an accessible question for most, and there were many fully correct solutions seen. Successful responses substituted for x or y to obtain $5x^2 - 40x + 75 = 0$ or $5y^2 - 20 = 0$ and solved to obtain coordinates $(3, 2)$ and $(5, -2)$.

(b) Many responses began by correctly finding the coordinates of the centre of the circle as $(0, -2)$. They then attempted to find the area of the triangle in a variety of ways, the simplest of which was to use the base of the triangle as 5 and the vertical height as 4, giving $\frac{1}{2} \times 5 \times 4 = 10$.

Question 11

(a) This was an accessible question and there were many correct solutions. Successful candidates substituted $x = 4$ to obtain the gradient of curve as $\frac{1}{10}$, then found the equation of the normal using (4, 3) and -10 for the gradient to give $y = -10x + 43$. Some candidates integrated the given expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ rather than differentiating.

(b) Unlike **11(a)**, this proved to be a challenging question, and few fully correct solutions were seen. Stronger candidates differentiated $\frac{\mathrm{d}y}{\mathrm{d}x}$ to give $-16x^{-3} + 40(2x - 3)^{-3}$ and substituted $x = 4$ to obtain $\frac{7}{100}$. A common mistake was to substitute $x = 4$ into the expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

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(c) Successful responses integrated to obtain $y = -8x^{-1} + 5(2x-3)^{-1} + c$, substituted $x = 4$ and $y = 3$ to find c and hence $y = -8x^{-1} + 5(2x-3)^{-1} + 4$ before then substituting $x = -1$ to find q. A common mistake was to substitute $x = -1$ into $y = -8x^{-1} + 5(2x-3)^{-1} + c$.

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MATHEMATICS

Paper 9709/12
Pure Mathematics 1

Key messages

Candidates would benefit from having more confidence in applying fundamental algebraic techniques. On this paper, common misapplications included:

- cancelling the b in $\frac{b}{a} = \frac{c}{b}$
- expanding $(a-3c)^2$ to give $a^2 + 9c^2$
- squaring both sides of the equation $1 = 2x^{\frac{1}{2}}$ to obtain $1 = 2x$
- expressing $5 - \cos^2 x$ as $5\sin^2 x$, or $5\cos^2 x - 1$ as $5\sin^2 x$
- concluding that $\sin x(3 + 5\sin x) = 2$ leads to $\sin x = 2$ or $3 + 5\sin x = 2$.

Candidates are also reminded that alternative possible angles may exist as solutions to trigonometric equations other than those given by their calculators.

General comments

The paper was generally found to be accessible for most candidates. Many good responses were seen, and candidates generally seemed to have sufficient time to finish the paper.

Presentation of work was mostly good, although some answers were written in pencil and then overwritten with ink. This practice can produce an answer which is unclear and difficult to read during marking; candidates are therefore strongly advised not to do this.

Comments on specific questions

Question 1

(a) Responses usually stated the correct values of p and q, but showed had problems finding r.

(b) Very few responses showed they understood the link with **1(a)**, or followed the 'Hence' instruction. Candidates should be aware of the significance of this type of linking statement. Most instead used the discriminant, but often did not then simplify $(-36)^2 - 4 \times 9 \times (8-k) < 0$ correctly.

(c) Again, very few candidates continued from their work in **1(a)**, but instead rearranged the equation and used the quadratic formula. Weaker responses sometimes gave decimal approximations rather than the exact roots that were requested in the question.

Question 2

This question proved to be accessible to candidates of all levels of ability, with many responses achieving full credit. Many responses included the full expansion, but weaker ones sometimes stopped after the first three or four terms before reaching the term independent of x. $(2x^2)^2$ quite often incorrectly became $2x^4$, and the minus sign caused issues for some candidates.

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Question 3

(a) This part was generally well answered. Some responses used the term “transformation” instead of “translation”, some thought that the stretch was parallel to the *y*-axis rather than the *x*-axis, and some stated that the scale factor was 3 rather than $\frac{1}{3}$. Responses which stated the translation as a vector were generally more successful, and this approach should be encouraged.

(b) Again, this was generally well answered. Common errors included placing the 3 inside the function, and the minus sign was sometimes outside.

Question 4

(a) Many strong responses for this question part were seen, with candidates generally understanding the key equation to form from the information stated.

(b) Most candidates realised that integration was required, but many did not include the constant of integration and substituted the point *S* rather than the point *T*.

Question 5

(a) Many strong responses to this question were seen. Weaker responses often confused the methods required, and often set $y = 0$ rather than $\frac{dy}{dx} = 0$ to find the value of *a*.

(b) Most realised that integration was required but the lower limit was often set as 0, which is inconsistent with the given diagram. Candidates are encouraged to check that their answers are consistent with any diagrams given, as these are often provided to support understanding of the situation.

Question 6

(a) Many graphs incorrectly started at the origin or finished at $(0,\ 2\pi)$. Responses which used a transformation approach were often more successful, although it is important to indicate which is the final answer if candidates use a staged approach.

(b) Many candidates were unclear how to proceed with this type of question, with many not attempting to solve **6(b)(ii)** and a significant number trying to solve the equations rather than considering the number of points of intersection with their drawn sketch.

(c) This part was more familiar, and many responses correctly used the identity $\cos^2 x = 1 - \sin^2 x$ and then found the correct values of $\sin x$. Weaker responses did not appreciate that the value of $-\frac{1}{2}\pi$ given by calculators for $\sin^{-1}(-1)$ was outside the given range for *x*, did not find the required value of $\frac{3}{2}\pi$ or assumed that no solutions were possible.

Question 7

(a) Most responses found the centre of the circle correctly and were familiar with the formula for the equation. Some responses incorrectly used the calculated diameter as the radius in the circle equation.

(b) Most were able to correctly find the gradient of *PQ,* but a significant number used this in the equation of the tangent rather than the negative reciprocal of it.

(c) Weaker responses did not include a diagram, and were therefore often unaware of the symmetry of the situation. Drawing a diagram for this type of question should always be encouraged. Some substituted $x = 7$ into their circle equation and eventually found that the y-coordinate was 1, but this could have been identified immediately from a diagram. Some weaker candidates had the same gradient for both of their tangents but still attempted to find their intersection. The few stronger responses which did include a diagram were often also able to see that the required *y*-

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coordinate would be 6, which led to an efficient solution, but many still tried to find the equation at *Q* and solve the two lines simultaneously.

Question 8

(a) Many weaker responses began by assuming the equation given in **8(a)** and then trying to solve it, rather than showing it from the information given. Those who did try to show it often appeared unclear as to which series they were considering and whether it was arithmetic or geometric. Stronger responses were able to form two correct equations given the information provided and to correctly eliminate *b* to find the given result.

(b) Most responses showed understanding that the given result from **8(a)** could be used. Those which used the given information and the equation were usually able to find the required values of *r* and *d,* and find the two sums correctly.

Question 9

(a) The vast majority of responses understood that differentiation was required, although weaker responses tried to find the inverse function instead. Most showed correct differentiation, although some omitted multiplication by 3 and others incorrectly handled the negative indices by increasing them rather than decreasing them. Many candidates did not state the connection between the differential obtained and what type of function that it was.

(b) Only the strongest responses realised the connection with **9(a),** and the fact that the inverse function existed because it was a decreasing function.

(c) The majority of responses were not worthy of credit for this question part.

(d) The majority of responses were not worthy of credit for this question part, and no response was offered by almost a third of candidates.

Question 10

(a) Weaker responses often did not realise that the triangle *BCD* was equilateral and were therefore unable to make any progress with the question. Those who did realise used a variety of techniques to show the given result. The cosine rule, sine rule, Pythagoras theorem and splitting one of the isosceles triangles in half were often all used correctly, but candidates need to be aware that all of the necessary working needs to be shown in this type of ‘show that’ question and that care needs to be taken when presenting their work. For example, some responses stated $s = r^2 + r^2 - 2r^2\cos\left(\frac{2}{3}\pi\right)$ when using the cosine rule, rather than the correct $s^2 = ...$

(b) Stronger responses understood the correct radii and angles to use with the different triangles and sectors involved. Weaker responses were often confused about which to use and sometimes ignored the triangles completely.

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MATHEMATICS

Paper 9709/13
Pure Mathematics 1

Key messages

When asked to show or verify that two expressions are equivalent, candidates are discouraged from initially putting the expressions equal and then manipulating both sides until they are the same. Instead, candidates are encouraged to be detailed and organised in their algebraic working, showing clearly the steps taken to manipulate the equation from one side of the equivalence to the other.

General comments

The paper was generally found to be accessible for most candidates. Many strong responses were seen for questions involving trigonometry and differentiation. Many good scripts were seen, and candidates generally seemed to have sufficient time to finish the paper.

Comments on specific questions

Question 1

This question proved to be accessible to almost all candidates, and many strong responses were seen for both parts **(a)** and **(b)**.

(a) Common errors included omitting the minus signs in the expansion, as well as omitting the $\frac{1}{2}$ when raising the x-terms to the appropriate powers.

(b) After an incorrect response to **1(a)**, it was possible for candidates to gain credit for choosing the 'correct' products in **1(b)**. Many candidates were able to gain credit here even if they did not do so in **1(a)**.

Question 2

(a) This part was again generally well answered by most candidates. The minus sign from $\tan\left(-\frac{1}{4}\pi\right)$ was the most common omission in the responses seen.

(b) Almost all responses correctly used $\sin^2\theta + \cos^2\theta = 1$ to obtain a quadratic equation in either $\sin\theta$ or $\cos\theta$. The correct quadratic was then, generally, correctly solved. Some responses did not appreciate the given range for the equations in radians, and therefore did not arrive at the correct angles from the principal values given by their calculators.

Question 3

(a) Almost all responses achieved full credit for this question part.

(b) Almost all responses achieved full credit for this question part.

(c) This question part tested candidates' understanding of the relationship between the chord gradients and the gradient of the tangent at A, here referred to as $f'(8)$. Many responses did not give a clear statement of the relationship between these, and hence did not receive credit for their response.

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Question 4

(a) Nearly all responses achieved full credit for this question part.

(b) This was also generally well answered by the majority of candidates. Some, however, transferred values from **4(a)**, despite this part starting with the words "Given instead". Candidates are advised to read the question carefully to look for key information such as this.

Question 5

(a) Responses which used the cosine rule usually proceeded to the correct value for θ. Responses considering a right-angled triangle with $\frac{1}{2}\theta$ did not always make their method clear, and sometimes did not double their answer at the end to find the value of θ.

(b) Many responses did not fully calculate the area required in the question, evaluating only the area of the sector *PXQ* but not including the area of the triangle.

Question 6

(a) Many responses achieved full credit for this part of the question. Of those which did not, many achieved almost full credit by stating the correct transformations but in the wrong order.

(b) Responses which had correct numerical values in **6(a)** generally went on to answer this question correctly as well.

Question 7

This question was algebraically demanding, and some candidates found this challenging. Some responses replaced *a* with 6 from the start, and could therefore not arrive at the correct inverse.

Verifying that, when $a = 6$, $g^{-1}(x) \equiv g(x)$, also proved challenging too. Again, many responses replaced *x* with a numerical value, and consequently only verified the equivalence for that particular value.

Question 8

(a) Many strong responses for this question part were seen. The most common error was, having found the length of the diameter, not dividing it by 2 in order to find the radius to use in the circle equation.

(b) Candidates who realised that solving the equations of the appropriate diameter and circle would produce the required values, were generally successful in achieving full credit. Other methods of solution achieved varying degrees of success; using the gradient of the diameter and taking appropriate *x*- and *y*-'steps' from the centre of the circle was the most successful of these.

Question 9

Most responses used the equations of the line and curve to form a quadratic equation and solved it to find the required limits. It was clear that, generally, candidates were aware of the need to integrate y^2, and most responses appropriately included multiplication by π in order to find the volume of revolution. Stronger responses dealt with the curve and line separately. Weaker responses omitted terms when squaring the equations, or focussed only on the curve and omitted the volume produce by rotating the line.

Question 10

(a) (i) Completing the square was achieved successfully by the vast majority of candidates.

(ii) Many responses did not follow through on their working from **10(a)(i)**, not appreciating the significance of their value of *c* in this question part.

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(b) Candidates found this the most challenging question on the paper. At least four different approaches were seen across all responses. Many responses focussed on using the discriminant of $f(x) = 0$, and were not able to proceed to an answer worthy of credit.

Question 11

(a) Stronger responses differentiated, replaced x with 3 in $\frac{dy}{dx}$, used this gradient to get the equation of the tangent and then equated that with the given line equation. Many responses which incorrectly used the chain rule still gained significant credit for their later algebraic work. Weaker responses attempted to solve the curve and the given line from the start, which was rarely worthy of credit.

(b) (i) Candidates who had not differentiated in **11(a)** often realised that they needed an expression for $\frac{dy}{dx}$ here. Many completed this correctly, before equating it to 0 and solving for x.

(ii) Responses which accurately found the second derivative generally achieved full credit here. Weaker responses did not apply the chain rule correctly, as was the case in **11(a)**.

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MATHEMATICS

Paper 9709/15
Pure Mathematics 1

Key messages

Most candidates displayed strong algebraic manipulation and problem-solving skills, although these were not always clearly demonstrated. For example, many responses omitted key algebraic working and showed the use of calculators to solve quadratic equations. When sufficient detail in a response is absent, full credit cannot be awarded. Candidates are therefore encouraged to achieve proficiency in factorising quadratic equations.

When a given result has to be reached, a strong response shows all the steps leading to it logically so there is no doubt over the solution's validity.

Candidates are reminded that questions involving definite integration, or the use of a differential, must show the algebraic result of integration or differentiation to gain full credit.

General comments

The paper provided many question parts which were accessible to candidates of all abilities. It appeared that there was sufficient time for those candidates who were able to complete the paper to do so. Most scripts were clear and legible.

Comments on specific questions

Question 1

(a) Those candidates who used the formula $(x-a)^2+(y-b)^2=r^2$ with the given centre and radius were able to reach the correct equation in one step. Most answers used this route, with a few instances of alternative circle equations being used.

(b) Nearly all candidates were able to substitute $(8, k)$ into their equation from **1(a)** and solve the resulting quadratic equation, either by making k the subject of the equation or by forming a three-term quadratic equation in k and using factorisation to solve for the two values. A small number of candidates used a diagram with the given centre, radius and point to find the two possible y-coordinates.

Question 2

(a) The solution to the calculation of the common ratio using division of the second term by the first term was seen in nearly all answers, although some gave an incomplete response by omitting the calculation step.

(b) The formula for the sum to infinity was applied efficiently with many correct answers seen, either as an irrational number or a three significant figure decimal.

Question 3

(a) A correct process for completing the square to the given form was often seen. Some sign errors were reported, and some responses gave an answer in the incorrect form $(ax+b)^2+c$.

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(b) Although the value of *k* could be found directly as the value of '*c*' in the required completed square form found in **3(a)**, many candidates chose to use another method to find *k*. Nevertheless, many correct answers or follow through answers were seen.

Question 4

(a) The required arc length was usually found correctly either by using the angle at the centre of the major sector or by subtracting the arc length of the minor sector from the circumference. Very occasionally degrees and radians were confused with each other.

(b) Many completely correct methods were seen with candidates realising their expression from **4(a)** had to be used to find the radius. When this was found to at least two decimal places, a correct value of the required area usually followed. Responses which found accurate answers to the areas they used generally fared better than those who left the overall answer in algebraic form and then used a calculator to evaluate this, as the more complex expression provided more opportunities for calculator input errors.

A common error, easily avoided by checking working, was the omission of $\frac{1}{2}$ from the area formula for a triangle.

Question 5

This was a question answered well by many. A form of quadratic equation was obtained by most candidates, and these were correctly solved to find the required values. When the solution of the quadratic was shown clearly, an answer worthy of full credit usually followed.

Question 6

(a) The definition of tangent in terms of sine and cosine and the trigonometric identity connecting sine and cosine were well known and used effectively to reach the required result. Most responses showed all the necessary steps, but a small minority made unacceptable leaps in their working without justification.

(b) The given result in **6(a)** was nearly always the starting point for solutions of this part. As in other questions, candidates were expected to clearly show their solution of the quadratic equation in cosine. The correct angles were often found, and the accuracy of the non-exact answers was usually sufficient. Some candidates did not realise the negative solution of the quadratic led to two solutions and some used an incorrect relationship to find the reflex angles, but many completely correct responses were seen.

Question 7

(a) For many candidates, this was the most challenging question on the paper. Although the correct surface area formula for the open cylinder was often seen, so was the formula for a closed cylinder and instances of incorrect calculation of the curved surface area. Where a correct surface area formula was found, a correct expression for the height of the cylinder usually followed. Full credit was achieved when this height expression was used in the volume formula with clear working and an indication of cancelled and factorised terms.

(b) Use of differentiation to find the radius which gave the maximum volume did not appear to be obvious in many solutions. Those who did appreciate this were not always able to differentiate the expression given in **7(a)**.

(c) Many candidates were able to substitute their result from **7(b)** into the volume equation given in **7(a)** and gain the available marks for this question.

Question 8

(a) Where the correct integration of πy^2 was used, the correct given inequality was usually obtained. Most candidates realised that a clear substitution of the integration limits must be required when

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reaching a given result. Very few candidates omitted π, although some appeared to include it late in their working when they noted the given volume inequality.

(b) Evaluating the given inequality was generally handled well, although the correct critical values often appeared without sufficient supporting work. Many correct inequalities involving both critical values were seen, with some candidates correctly discarding the negative region. Strong responses often included diagrams to eliminate the negative range of answers.

Question 9

(a) Correct methods for finding the required coefficients were a feature of most responses. The algebra required to reach the ratio was well understood but the rearranging of this to a correct ratio in the required form proved to be challenging for a significant number of candidates.

(b) The result from **9(a)** was usually effectively used with an expression for the coefficient of x^3 to reach an equation in *p* or *q*. These equations were often then used to correctly find *p* and *q*. Responses which included negative values were not worthy of full credit.

Question 10

(a) This part proved to be the most accessible question part on the paper. Candidates are however reminded that when a question asks for the value of a particular variable, *a* in this case, they should quote the answer as '$a =$' rather than '$x =$'.

(b) Many good responses were seen to obtain the correct inverse function. As $\mathrm{f}^{-1}(x)$ was required, it was essential this notation featured in the expression for the inverse. Similarly, the domain of $\mathrm{f}^{-1}(x)$ was only worthy of credit when presented correctly, either as $x > 3$ or $x \in (3, \infty)$.

(c) Most responses chose to find the composite function and simplify this. The composite function was usually found correctly, and candidates with sufficient algebraic skills were able to simplify this to the required result. Responses which used $\mathrm{g}(x) = \mathrm{f}^{-1}(kx)$ were mostly able to solve the resulting equation to find *k*.

Question 11

(a) The procedure for finding the equation of the tangent to a curve at a given point was well understood and the solution to this part was often presented completely correctly. Some responses confused the tangent with the normal, but few other errors were seen.

(b) When the point of intersection of the curve and the *x*-axis was deduced correctly and used as a limit of integration, a complete solution usually followed. There were a significant number of responses where no integration was seen and correct numerical results appeared; without showing working to justify answers, full credit cannot be awarded.

(c) (i) The effect of the one-way stretch on the equation of the curve was not widely understood, but stronger responses usually featured a correct simplified answer.

(ii) Reducing the *x*-coordinate and the area found in **11(b)** by a factor of $\frac{1}{3}$ to obtain the coordinates of *Q* and the transformed area was seen in very few responses.

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MATHEMATICS

Paper 9709/21
Pure Mathematics 2

Key messages

Candidates need to read questions carefully to ensure that their demands are met in full and that the final answer is given in the required form. For example, candidates often gave decimal answers instead of exact answers. More care is sometimes needed with general algebraic and arithmetic processes. Diagrams are given to help candidates visualise a problem and should be referred to in the context of the question. Candidates should also ensure that when they are required to show a given answer, each step in the process is shown in full.

General comments

Most candidates were able to show understanding of the syllabus requirements, with a wide variation of performance seen across the range of available marks. Timing did not appear to be an issue, and most candidates had sufficient room in which to provide their solutions.

Comments on specific questions

Question 1

Where responses made use of a double angle formula to obtain an integrand in the correct form, a correct solution was usually obtained. There were occasional arithmetic slips and sign errors seen throughout responses.

Question 2

It was essential that an expansion of the given equation be made initially to show that this equation was a quadratic equation in terms of e^{2x}. Equating to zero and subsequent factorisation led to only one possible solution of $e^{2x} = 12$. The final form of the answer was not specified in the question, so either an exact form or a decimal form to the correct level of accuracy was acceptable.

Question 3

(a) Most candidates were able to obtain the correct answers by either using a squaring method or by considering two appropriate linear equations.

(b) The word 'Hence' should have alerted candidates to the fact they needed to make use of their response to **3(a)**. Of those responses which did this, most stated that $\cos\theta = -\frac{3}{5}$ and then gave their resulting answer in radians as required,

Question 4

Provided the correct expansion and simplification of $\tan(\theta + 45°)$ was given, responses usually progressed to obtain the correct quadratic equation $7\tan^2\theta - 6\tan\theta + 1 = 0$. Provided they were working in degrees, most candidates were able to obtain at least one correct solution.

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Question 5

(a) In order to obtain full credit, complete working needed to be seen. A statement of $8e^{-\frac{1}{2}x} - 1 = 0$ or equivalent was an essential starting point. Correct use of logarithms leading to $-\frac{1}{2}x = \ln\frac{1}{8}$ or similar needed to be seen next. The statement $x = -2\ln\frac{1}{8}$ was often obtained, but many responses did not show the next essential step of stating this as $x = -2\ln(2)^{-3}$ and then $x = 6\ln 2$, or stating $x = 2\ln 8$ and then $x = 6\ln 2$.

(b) Candidates should refer to the given diagrams to assist with solutions. In this question part, to find the shaded area, the simplest way is to find the area of the triangle *OAB* and then subtract the area under the curve. Some candidates chose to find the equation of the line *AB* and use integration and subtraction. Whilst this is acceptable, it is a lengthier method with more opportunities to make errors. The most successful responses found the area of the triangle. Most were able to make a reasonable attempt at integration, but errors often occurred when the limits were substituted in and during the subsequent simplification of exponential terms involving logarithms.

Question 6

(a) It was essential that candidates attempt use of the chain rule to find an expression for $\frac{dy}{d\theta}$. Most candidates were able to obtain $\frac{dx}{d\theta} = \sec^2\theta$ and then attempt a correct method to find $\frac{dy}{dx}$. Again, it was essential that sufficient steps be shown as the answer was given. There were few correct solutions due to frequent incorrect application of the chain rule.

(b) To make progress, candidates needed to find the value of θ at the point where $y = 0$. It was given that $0 < \theta < \frac{\pi}{2}$, so the only solution for θ was obtained from $1 - \sin^2\theta = 0$. Once a value for θ was obtained, the gradient of the curve at this point could be found using the given result from **6(a)**. Finding the gradient of the normal and then the equation of the normal followed.

Question 7

(a) The majority of responses made correct use of the factor theorem and obtained the correct value of 4.

(b) Some responses did not include the value of α as required, but most attempted algebraic long division to obtain a correct quotient of $2x^3 + 4x + 9$. A rearrangement of this quotient equated to 0 led to the correct result, which many candidates were able to obtain.

(c) Most responses used a change of sign method by considering the value of $\beta - \sqrt[3]{-2\beta - 4.5}$, or similar, when $\beta = -1.4$ and $\beta = -1.0$. Alternative approaches were acceptable provided sufficient detail was seen and an explanation was given.

(d) Many strong responses to this question were seen. Responses usually included an appropriate number of iterations to the correct level of accuracy, with a final answer of -1.26.

Question 8

Most candidates recognised that they needed to differentiate the given equation and equate the result to 0. Errors sometimes occurred when dealing with indices and subsequent simplification. Very few responses obtained a correct simplified or unsimplified expression to conclude that a stationary point occurred when $3 - 4(3x - 1) = 0$, or equivalent. As a result, very few fully correct solutions were seen.

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MATHEMATICS

Paper 9709/22
Pure Mathematics 2

Key messages

Candidates need to ensure that they read questions carefully and that the demands are met in full, with the final answer given in the required form. For example, responses often showed decimal answers instead of exact answers, or did not consider all the possible angles for a given range in the solution of a trigonometric equation. More care is sometimes needed with general algebraic and arithmetic processes. Diagrams are usually given to help candidates visualise a problem, and should be referred to in the context of the question.

General comments

Most candidates were able to show that they had some understanding of the syllabus requirements, with a wide variation of performance across the range of available marks. Timing did not appear to be an issue, and most candidates has sufficient room in which to provide their solutions.

Comments on specific questions

Question 1

Most candidates were able to simplify the logarithmic terms correctly to $\ln\left(\frac{3x+5}{x-2}\right)$. However, few responses showed subsequent progress to an equation without logarithms. Although many candidates obtained $\left(\frac{3x+2}{x-2}\right) = e^4$, there were often algebraic slips when it came to solving for x. Some candidates chose to give their answer in decimal form, even though an exact form of the answer was specified in the question demand.

Question 2

Most candidates made use of the correct identity and obtained the correct equation $2\sec^2\theta + 3\sec\theta - 20 = 0$ or the equivalent form in $\cos\theta$. Many candidates obtained two correct positive angles as the solutions to a correct quadratic equation, but did not consider the possibility of negative values.

Question 3

(a) Most candidates were successful in obtaining the critical values 9 and $-\frac{1}{5}$, either by using a squaring method or by finding two linear equations or inequalities. Each method appeared to be equally successful. While there were many correct answers, some candidates mistakenly chose to reject the critical value of $-\frac{1}{5}$.

(b) Although the word 'Hence' indicated the result from **3(a)** needed to be used, some responses showed attempts to solve the given equation using other, often erroneous, methods. It was essential to relate the term $7^{0.01N}$ to the values of x obtained in **3(a)**, and also realise that the only possible solution would be obtained by solving the equation $7^{0.01N} = 9$. Of the responses which included the correct approach, most obtained the value of 112.915. Sometimes this was given as a final answer, without using the fact that the final value of N had to be an integer as specified in the question demand. Of the responses that did give integer answers, an answer of 113 was frequently

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given rather than the correct 112, having 'rounded up' rather than appreciating the nature of the inequality.

Question 4

(a) Although many correct examples of algebraic long division were seen, some candidates found it challenging to work with a divisor not containing a term in *x*. Candidates should set their work out clearly and align their terms in powers of *x* so that they are directly underneath each other in the division process. This should help avoid algebraic errors in questions such as these. Algebraic working was often unjustifiably engineered to give the remainder –11, which was stated in the question.

(b) As in the previous question, candidates were expected to make use of the result from **4(a)** and show clearly that they were using the quotient correctly. Errors occurred when the term of 11 was misused, with some candidates solving the equation $x^2 - 10x + 17 = 0$ and adding 11 to their solution, or by solving $x^2 - 10x + 26 = 0$.

Question 5

(a) It was essential that candidates take careful notice of the given diagram, which shows symmetry and implies that points *A* and *B* have the same *y*-coordinates. Most candidates differentiated the given equation and equated this to zero, then attempted to find the values of *x* at which stationary points occurred. Some errors in coefficients in the differentiation were seen, but many candidates obtained $\sin x = \frac{1}{2}$ and $\cos x = 0$. Subsequently, values of $\frac{1}{6}\pi$ and $\frac{1}{2}\pi$ were obtained, however many responses stated that these two values were the *x*-coordinates of the maximum points rather than the *x*-coordinate of the maximum point at *A* and the minimum point between the two maximum points. This conclusion resulted in two different *y*-values, which should have alerted candidates as this contradicts what the diagram was showing.

(b) Integration was usually well attempted to find the area of the shaded region, with the occasional algebraic error seen in the coefficients of the resulting trigonometric terms. Responses generally used their limits and the *x*-values obtained in **5(a)** in a correct form of integral. An exact solution was expected here, and this was not always seen.

Question 6

(a) It was essential that each step of the integration and subsequent substitution of limits be shown clearly. Many correct integrals were seen, but errors usually occurred when the lower limit of –2a was substituted and sign errors were made. The resulting equation, $\frac{1}{2}e^{2a} - \frac{1}{4}e^{-a} - \frac{1}{4}e^{-4a} = 5$, was usually an indication that correct integration, substitution of limits and simplification had taken place. Several further steps were then needed to show the given answer.

(b) Most candidates chose to consider the sign of $a - \frac{1}{2}\ln\left(10 + \frac{1}{2}e^{-a} + \frac{1}{2}e^{-4a}\right)$ or equivalent for values of $a = 1.0$ and $a = 1.4$. Alternative approaches were acceptable, but a full and clear explanation was always needed.

(c) Most candidates performed the iterative processes well. Solutions were usually given using an appropriate number of iterations to the correct level of accuracy, including the final answer of 1.159.

Question 7

(a) Most candidates were able to make a reasonable attempt at differentiating the given equation. Errors occurred most commonly when implicit differentiation was not used, and also with the rearranging of terms to give $\frac{dy}{dx}$. Few candidates realised that there was sufficient information to find the value of *x* when *y* = 0, and did not substitute this to find a numerical value of $\frac{1}{4}$ rather than $\frac{7}{5x^2 + 8}$.

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(b) Candidates found this question part challenging, and very few realised that the denominator of the gradient function needed to be considered. A full explanation was needed as to why this denominator could never be 0 in order to gain full credit.

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MATHEMATICS

Paper 9709/23
Pure Mathematics 2

Key messages

Candidates need to ensure that they read questions carefully and that the demands are met in full, with the final answer given in the required form. For example, responses often showed decimal answers instead of exact answers, or did not consider all the possible angles for a given range in the solution of a trigonometric equation. More care is sometimes needed with general algebraic and arithmetic processes. Diagrams are usually given to help candidates visualise a problem, and should be referred to in the context of the question.

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Most candidates were able to show that they had some understanding of the syllabus requirements, with a wide variation of performance across the range of available marks. Timing did not appear to be an issue, and most candidates has sufficient room in which to provide their solutions.

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(b) Although the word ‘Hence’ indicated the result from **3(a)** needed to be used, some responses showed attempts to solve the given equation using other, often erroneous, methods. It was essential to relate the term $7^{0.01N}$ to the values of x obtained in **3(a)**, and also realise that the only possible solution would be obtained by solving the equation $7^{0.01N} = 9$. Of the responses which included the correct approach, most obtained the value of 112.915. Sometimes this was given as a final answer, without using the fact that the final value of N had to be an integer as specified in the question demand. Of the responses that did give integer answers, an answer of 113 was frequently

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(b) Integration was usually well attempted to find the area of the shaded region, with the occasional algebraic error seen in the coefficients of the resulting trigonometric terms. Responses generally used their limits and the x-values obtained in **5(a)** in a correct form of integral. An exact solution was expected here, and this was not always seen.

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(b) Candidates found this question part challenging, and very few realised that the denominator of the gradient function needed to be considered. A full explanation was needed as to why this denominator could never be 0 in order to gain full credit.

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Paper 9709/25
Pure Mathematics 2

There were too few candidates for a meaningful report to be produced.

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MATHEMATICS

Paper 9709/31
Pure Mathematics 3

Key messages

Candidates are advised to:

- Organise their working clearly, showing sufficient detail for the examiner to be confident that the method is fully correct.
- Check carefully for slips in the arithmetic and algebra.
- Read reports on past papers and be aware of common errors, such as not spotting improper fractions.

General comments

There was only a small number of candidates for this paper. They generally fell into two categories: a group of candidates who gave fully correct answers to the majority of questions, and a group who found this paper highly challenging.

Of the candidates who performed well, some of these provided overly brief evidence of working and reasoning for their solutions to be sufficiently convincing. When making extensive use of calculators, it is important to ensure that the key stages in the working are all documented. The rubric on this paper states that "no marks will be given for unsupported answers from a calculator", and some candidates did not obtain full marks as they showed insufficient working to support their answers.

Comments on specific questions

Question 1

Those candidates who recognised the need to use integration by parts usually made good progress with this question. When applying the correct method, the most common error seen was $\frac{\mathrm{d}}{\mathrm{d}x}(\ln 3x) = \frac{1}{3x}$. There were some errors after the use of limits when simplifying to obtain an answer of the required form.

Question 2

(a) Most candidates made correct use of a law of logarithms to rearrange the given equation. Those who remembered that $2 = \log_4 4^2$ were often able to get as far as an equation without logarithms, which was usually quadratic in structure.

(b) This was answered well by those candidates who had a quadratic equation to work with. Candidates who obtained two solutions usually remembered to check that both were valid solutions to the original equation.

Question 3

(a) Those candidates who started with a correct expansion of $\sin(x + 45°)$ usually obtained the correct expression $3\sin x + 4\cos x$ and went on to complete this part correctly. Candidates who did not show full working sometimes made the error $\tan\alpha = \frac{4}{3}$.

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(b) Candidates with an answer to **3(a)** usually obtained at least one correct solution here. The second solution was often missing, as $180° + \cos^{-1}\left(\frac{4}{5}\right)$ had not been considered.

Question 4

Most candidates attempted to differentiate the function using the product rule. Common errors included sign errors and some mistakes in coefficients. Some candidates were not sure how to proceed once they had set their derivative equal to zero, but most were able to take out the exponential factor and obtain an equation in one trigonometric function.

Question 5

(a) Most candidates were able to identify at least one of the inequalities that define the shaded region. The simplest of these is $\operatorname{Re} z \geqslant 2$, although some candidates preferred to describe this as $\left|z-(1-2\mathrm{i})\right| \geqslant \left|z-(3-2\mathrm{i})\right|$.

(b) This part of the question proved to be more challenging. The first step was to identify the correct point. Having done this, the next step is to substitute $x = 2$ into the equation of the circle to find the *y*-coordinate of the point. Several candidates offered no response to this part of the question.

Question 6

The most common approaches were to use the coefficients in the quadratic formula to obtain expressions for the values of *w*, or to use the substitution $w = x + \mathrm{i}y$ and consider the real and imaginary parts of the resulting equation. Arithmetic errors aside, both methods were frequently successful. Candidates making extensive use of a calculator were at risk of not showing a sufficiently detailed method for simplifying $w = \dfrac{-4+2i}{4+2i}$, as discussed in the General comments.

Question 7

Most candidates gave a correct expression for $\dfrac{\mathrm{d}x}{\mathrm{d}t}$. In order to find $\dfrac{\mathrm{d}y}{\mathrm{d}t}$, it was necessary to use either the quotient rule or the product rule; candidates who used the quotient rule were more likely to state the correct simplified form of the derivative. Candidates generally understood where there was a need to use the chain rule, and most used their derivatives of *x* and *y* correctly.

Candidates were expected to deal with any nested fractions within their expression and to cancel any common factors between the numerator and the denominator to obtain a simplified expression; there were several algebraic errors seen at this stage.

Question 8

Candidates who understood the process of separating the variables were usually able to complete this correctly and make progress with the integration. Most integrals were of the correct form, but errors in the coefficients were often seen. In order to complete the solution, candidates needed to use the two pieces of information given to find the value of the constant *k* and the constant of integration. The constant of integration was often correct, but there were some errors in the value of *k*. Some of the weaker responses tried to use the given information without making any attempt to integrate.

Question 9

(a) There were some good sketches, but candidates need to ensure that they show the coordinates of the points of intersection with the axes, and indicate any asymptotes. Whether they were working with the given functions or with their reciprocals, +1 and –1 should have been clear on the *y*-axis, and $\frac{1}{4}\pi$ and $\frac{1}{2}\pi$ on the *x*-axis. Several candidates could not obtain the second mark because they did not demonstrate how their sketch shows the required result. The point of intersection needs to be emphasised, or an appropriate comment alongside the sketch needs to be seen.

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(b) Candidates were familiar with the work required here, and several gave correct responses. A few candidates were seen to be incorrectly working in degrees.

(c) Most candidates showed some understanding of what was required here. Some candidates started with the given equation, got as far as $\cos 2x = -e^{-x}$ and went no further. Candidates starting from $\sec 2x = -e^{x}$ often stopped without stating the iterative formula.

(d) Most candidates started from an appropriate point, worked to the required accuracy and completed sufficient iterations to obtain the correct value for the root.

Question 10

(a) Many candidates did not recognise this as an improper fraction. Strong responses started with the form $A + \frac{B}{3+x} + \frac{Cx+D}{2+x^2}$, or divided the numerator by the denominator to obtain $1 + \frac{-3x^2-17}{(3+x)(2+x^2)}$ and worked with that as a starting point. Fully correct partial fractions were unusual.

(b) Most candidates were familiar with the binomial expansion and were able to obtain an expansion based on their fractions. The expansions were often correct, but there were frequently errors seen in the coefficients.

Question 11

(a) Most candidates started with a correct statement of the direction vector of the line. The most common error was that the candidates did not use the correct form for the equation of the line.

(b) Several candidates understood the correct process for finding the point of intersection of two lines. Most errors were due to slips in the arithmetic or in transcription of vectors.

(c) Candidates found this part more difficult. Several did not understand that they needed to start by finding a vector from *C* to a general point on *m*. If this step was correct, then the candidate usually found the correct value for the parameter at the foot of the perpendicular and the correct position vector.

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MATHEMATICS

Paper 9709/32
Pure Mathematics 3

Key messages

- Take care in the presentation of solutions and use mathematical terminology accurately. This applies to all aspects of the work, but particularly to the use of brackets. Essential brackets are often left out, leading to serious errors early in a solution. When the brackets are there, care needs to be taken over the signs when the brackets are removed.
- If an error is made when working, cross it out and rewrite it. Writing over an existing solution usually leads to illegible working which is difficult to mark accurately. This also applies to drafting a solution in pencil and then writing over it in ink, as it is often not possible to work out what was intended.
- Do not expect every question to be solvable in the same way as a similar question on a previous paper. Keep an open mind, and think about the relevant methods.
- Be mindful of the messages about common errors in past papers: do not make up new rules of algebra, logarithms and trigonometry, and look out for basic forms such as improper fractions.
- Show sufficient working to make the method clear and detailed. If an answer is worth more than one mark, then simply writing down an answer from the calculator will not be sufficient.

General comments

Candidates found this a challenging paper, with many gaining fewer than 10 marks and very few scoring more than 60. There were frequent unforced arithmetic errors, and many responses showed errors due to incorrect presentation of working; the omission of brackets was common, and very few candidates recovered from this.

Question 1 and **Question 5** were more accessible to candidates who were able to use the first part as an introduction to the second part. In both cases, the second part could be completed independently, but this often resulted in longer methods.

In **Question 1** and **Question 9**, some candidates were confused by the constant *a*. Some responses immediately assigned a value to *a*, some thought they needed to solve for *a* and some thought it was another variable. The specification mentions integration as ‘reverse differentiation’, but very few candidates recognised the form $\sin x\cos^n x$ in **Question 8(b)** as an exact derivative, resorting instead to a variety of long and fruitless approaches. The majority of candidates did not recognise the fraction in **Question 9** as improper.

Comments on specific questions

Question 1

(a) Most responses showed a ‘V’-shaped sketch, but several incorrectly positioned the graph relative to the axes. Some sketches were clearly not intended to be symmetrical. Some candidates assigned a value to *a*, or ignored it completely.

(b) Most responses showed the correct value of $-\frac{2}{3}a$, although it was unusual for this to follow reference to the diagram in **1(a)**. Adding the line $y = a - 2x$ to the sketch in **1(a)** would have shown that there was only one critical value to be found. Most responses chose to consider the two linear equations separately or to square the given equation. Errors introduced when squaring frequently led to incorrect critical values. The incorrect conclusion of $-\frac{2}{3}a < x < 4a$ was common.

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Question 2

There were several fully correct responses seen to this question. Those who chose to take logs immediately were usually successful, but those who chose to separate the indices often made errors; some were sign errors, but the most common was stating $3^3 = 9$. Some responses showed little understanding of the laws of indices, with the incorrect start $6^{x+1} = 12^{2x-3}$ being common.

Question 3

(a) The crucial starting point was to recognise that the real part of the number represented by *P* is equal to 2. After this, the most successful method was to use the equation of the circle and substitute $x = 2$. This led to a quadratic in *y*, where one of the solutions led to the correct point *P*. The alternative method seen in responses was using Pythagoras' Theorem to obtain the *y*-coordinate of *P*. Some candidates overlooked the request for the answer to be in the form $x + \mathrm{i}y$.

(b) The majority of responses identified the correct point to work on. The simplest method was to split the required angle into two parts. Many candidates found the angle $\tan^{-1}\left(\frac{1}{3}\right)$, but finding the other part proved to be more difficult. A few responses were successful in finding the equation of the tangent to the circle passing through the origin. Some responses incorrectly thought that the greatest value lay at the intersection of the line and the circle near the top of the circle. A small number of candidates thought that the diameter from the required point met the circle on the real axis, which is close but not correct.

Question 4

Most candidates realised that integration by parts was required, although there was some confusion about the choices of *u* and *v*. Many responses started well, but subsequent success was limited as many found the second stage of the integration challenging. The use of natural logarithms was often seen and there were some algebraic errors noted as responses tried to manipulate the rational expression, including some which tried to cancel the x^2. Successful responses often included a long division process to change the form of the second integrand and were then able to complete the solution correctly. Most of the candidates who completed the integration accurately showed that they had considered both limits when finding the answer.

Some candidates did not understand the difference between $\tan^{-1} x$ and $(\tan x)^{-1}$.

Question 5

(a) This part of the question proved particularly challenging for many candidates**.** Many responses did not recognise that $\mathrm{f}(x)$ was expressed as a product and that they therefore needed to start by applying the product rule. Having differentiated correctly, the simplest way to complete the solution was to factorise to demonstrate $(x - a)$ is a factor. Most candidates preferred to substitute $x = a$ to demonstrate that $\mathrm{f}'(a) = 0$. Several responses achieved this, but they did not go on to draw the appropriate conclusion. The question asks candidates to demonstrate a result, so a conclusion is necessary.

(b) Although the question does not require the candidates to use the result from **5(a)**, this was the simplest method to use, and it avoids all the errors introduced by attempts at long division or false assumptions. Only a minority of responses used $\mathrm{f}'(3) = 0$. The majority did use $\mathrm{f}(3) = 0$, but they went on to divide the cubic expression by $(x-3)^2$. This process often resulted in the correct quotient, but there were several slips in the remainder. Many solutions stopped at that point. The simplest next step would have been to equate the components of the remainder to zero. Use of division by $(x-3)^2$ and equating the remainder to zero is a complete method in itself. Having completed the division by $(x-3)^2$, some responses multiplied their quotient by $(x-3)^2$ and compared coefficients, which was usually successful. The most common incorrect method was to

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use $f(-3) = 0$. Most responses were worthy of some credit, but the main difficulty was seeing how to draw the information together to complete the solution.

Question 6

(a) It was rare to see a completely correct answer to this question. Although many responses included graphs of the correct shape, they frequently omitted the key values on the axes, or did not indicate clearly the asymptotes on the cotangent graph. Some incorrect curvature was seen on the sine graph; for example, implying stationary points at the ends. A few responses incorrectly showed the cot graph had a discontinuity at $x = \frac{1}{4}\pi$. Those candidates who did manage to draw two correct graphs, often did not obtain the second mark as they omitted how their diagram showed that the equation had only one root.

(b) Most responses rearranged the given equation into the form f(x) = 0 and proceeded to check for a change of sign. Occasionally, different rearrangements of the equation were used, but this was often less successful, with candidates not able to produce the correct comparison of values at the end. A few responses made the error of working in degrees, which gained no marks.

(c) Most candidates understood the process of using an iterative formula. Those who chose to start with 0.5 found that the iterations converged quickly, whereas a large number of values had to be found if starting with either 0.4 or 0.6. In the latter cases, it was unfortunate that several candidates ceased iterations just before the point where the root would have been justified to 2 decimal places. Few candidates were efficient in deciding when they had shown sufficient iterations. While it is a good idea to continue for one or two iterates after the point where the root seems to have been confirmed, some responses continued well beyond what was expected. Occasionally, candidates chose to check for a change of sign to verify that they had found the root, which is a valid alternative to continuing iterations. Again, a few responses made the error of working in degrees, which gained no marks.

Question 7

(a) The technique of differentiating implicitly was well understood, although sign errors in the derivative of the second term were common. Several candidates had to go back and amend their solutions in the light of not achieving the given result; when doing this, it is important to trace the error right back to its source. As the question involved proving a given answer, any inaccuracy meant the final mark could not be awarded. Candidates should be reminded to pay attention to the accuracy of their notation on each line of their proof. In particular, incorrect statements such as declaring they were dividing the fraction by 3 (as opposed to cancelling the common factor of 3) often caused withholding of the final accuracy mark.

There was, occasionally, evidence of conceptual misunderstanding of $\frac{\mathrm{d}y}{\mathrm{d}x}$ where, for example, $3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$ was written as $\frac{3x^2\mathrm{d}y}{\mathrm{d}x}$ (i.e. thinking of $\frac{\mathrm{d}y}{\mathrm{d}x}$ as meaning d*y* divided by d*x*, rather than recognising it as a single expression for the derivative of *y* with respect to *x*). A very small number of candidates did not differentiate the constant 16 on the right-hand side.

(b) Fully correct answers to this question part were uncommon. Many responses did not appear to know where to start, not realising that the normal being parallel to the *y*-axis meant that the derivative obtained in **7(a)** had to be set equal to zero. Those who did realise this usually went on to make some progress, although the solution from $x = 0$ was often neglected. Having obtained $x = 0$ or $x = -2y$, some candidates neglected to use the fact that the points were on the curve, and made no further progress. Candidates who did attempt to form a cubic equation in *x* or *y* only often made errors in the substitution or in simplifying their equation. There was some confusion between quadratics and cubics, with the error $x^3 = 64 \Rightarrow x = \pm 8$ occurring frequently. The most common incorrect approach was to set the denominator of the derivative equal to zero.

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Question 8

(a) Many candidates were successful in proving the identity by working from left to right using the double angle formulae. Working from right to left was more challenging, but there were several correct solutions. The common incorrect statements were $\sin 4x \equiv 4\sin x \cos x$, $\sin 4x \equiv 4\sin 2x \cos 2x$, $\sin 4x \equiv \sin 2x + \sin 2x$ and $\sin 4x \equiv \sin^2 2x$.

(b) The two simple methods to complete this integral were to recognise the form $\sin x \cos^n x$ as an exact derivative or to use the substitution $u = \cos x$. Very few candidates used the former, but several used the substitution method correctly. The question does say 'hence', but few candidates could see how to use the result from **8(a)** to get the function into a form that they could integrate. Those candidates who completed the integral correctly were not always able to simplify the answer using $\left(\sqrt{2}\right)^7 = 8\sqrt{2}$ and $\left(\sqrt{2}\right)^5 = 2\sqrt{2}$.

Question 9

(a) Most responses showed some understanding of partial fractions. Many though did not realise that the fraction was improper and chose an incorrect form for splitting it. Those who did recognise that they were dealing with an improper fraction used long division or chose a correct form to split the fraction. There were many algebraic errors when candidates chose to equate coefficients. Some responses appeared unsure about the presence of the constant a in this question and decided to assign it a value, which made these responses not worthy of full credit.

(b) Many candidates integrated their partial fractions correctly here. The majority recognised that the answer involved logarithms, but there were errors in the coefficients. There was again some confusion between the constant a and the variable x. Candidates dealt well with the substitution of the limits. There was mixed success with the simplification of the logs in the answer to get to the required form.

Question 10

(a) There was a minority of candidates who clearly understood the meaning of each of the differential coefficients and the need for the use of the chain rule. Many, however, struggled to translate the information given in the question into correct mathematical expressions. There was confusion about the symbols for the rate of change of depth and the rate of change of volume, and many candidates could not process the information about the rate being proportional to h^2. Some candidates compounded their difficulties by claiming that the volume of water in the tank was $250h\text{ cm}^3$. Many responses consisted of disconnected ideas on the page, some correct and some false, but with no final thread linking these together as a coherent argument; it is essential to lay out work in a logical way to provide convincing evidence of a complete process.

(b) Many candidates did not make any meaningful progress with this part of the question, often as they did not seem to realise that they needed to solve the differential equation stated in **10(a)**. Candidates who completed the integration correctly usually went on to find the constant of integration and the required time.

Question 11

(a) Very few responses stated correct answers for $\overrightarrow{MB}$ and $\overrightarrow{MC}$. Many started with the vector $-\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$, but they equated this to $\overrightarrow{OM}$ and went on to obtain an answer using $\overrightarrow{MO} + \overrightarrow{OB}$.

(b) The question did not require the use of a scalar product, but that was the method chosen by most candidates. The majority of candidates used their answers to **11(a)** correctly. Candidates should be aware that in order to find the cosine of the angle at M, they need to be using $\overrightarrow{MB}$ and $\overrightarrow{MC}$, or $\overrightarrow{BM}$ and $\overrightarrow{CM}$. The question asks for the cosine of the angle, so it was not necessary to go on to find the value of the angle. However, it was necessary to state the value of the cosine. A few responses were successful in finding the lengths of the sides of the triangle and using the cosine rule.

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(c) Candidates were not directed to use any particular method here, so there were several options. Some started work on triangle *ABC*, obtained a correct value for the cosine of one of the angles but then could not obtain an exact value for the sine of the angle. The method requiring the least work was to use the answer from **11(b)** to obtain an exact value for the sine of the angle at *M*, and then to understand that the area of triangle *CMB* is half the area of triangle *ABC*. Many solutions were not worthy of credit because they used the angle at *M* as an angle in *ABC*, or they used values from *ABC* incorrectly in their working for *CMB*.

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MATHEMATICS

Paper 9709/33
Pure Mathematics 3

Key messages

Candidates need to:

- Know how to sketch simple transformations of standard functions, such as ln 5x and $\frac{1}{5x}$, see **Question 8(b)**.
- Be able to apply the chain rule accurately, see **Question 11(a)**.
- Choose a sensible starting point when using an iterative method, see **Question 8(d)**.
- Know what is meant by a line being perpendicular to a given vector, namely that the vector is perpendicular to the direction vector of the line, see **Question 9(a)**.

General comments

Candidates found this to be an accessible paper, with strong responses generally seen. In particular, candidates were able to answer **Question 1**, **Question 3** and **Question 8** very well, albeit with some difficulties seen on **Question 8(b)** (as mentioned in the Key messages). In contrast, **Question 9** and **Question 10** proved the most challenging.

Candidates generally presented their solutions clearly and there did not appear to be any significant timing issues, with most candidates having attempted all questions.

Comments on specific questions

Question 1

The most popular method by far was to solve a non-modular quadratic inequality or equation, which was performed successfully in the vast majority of responses which contained this approach. The most common mistake seen in this approach was neglecting to square the factor 3 on the right-hand side. A few candidates solved four inequalities or equations, usually successfully choosing the correct two. A small minority gave the final answer as a single 'sandwiched' inequality, and a similar number spoilt their final answer by using the word 'and' in place of 'or'.

Question 2

This question was slightly less well answered than the previous, but many strong responses were seen. Almost every response utilised long division, with the remainder theorem seldom seen. A significant minority made errors, usually with one or more signs, or failed to complete the method which led to a remainder containing a term in x.

Question 3

Many responses took logarithms of both sides as a first step, which often led to bracket omissions on the left-hand side. However, these candidates often recovered this slip later in their answer. A minority of responses used the alternative method successfully, or started on this path before switching to the main method.

Question 4

Several candidates did not attempt this question. However, those who did generally took great care with their drawing, producing detailed sketches. Common errors included the incorrect position of the centre of the

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circle or point (2, –4), or shading the wrong side of their perpendicular bisector or unequal scales on the axes.

Question 5

(a) The majority of candidates performed very well on this question, which is uncommon for a proof question involving trigonometric identities. However, a minority of candidates found this question extremely challenging, not understanding that the angle in this identity should not be replaced by a specific value, and that the symbol being used to denote the angle should be retained and not replaced by an alternative unless rectified at the end. Many candidates either gave up or switched to the right-hand side of the identity, but fared no better. Many of the successful candidates started with $1 - 2\sin^2 2x$, whereas others who started with $\cos^2 2x - \sin^2 2x$ faltered as they omitted the second term from this expression.

(b) The main error seen in responses to this question was when finding the possible values of $\sin x = 0$, often by stating only $x = 0$ or by getting this and then rejecting it, or using it to get only 0° and 180°, omitting the value −180°. The other common error was the failure to obtain all four angles from $\sin x = \frac{\pm\sqrt{3}}{2}$, often omitting either −60° or −120°.

Question 6

Most responses recognised the need for integration by parts, chose u and v correctly and used the correct formula. Many completed the first step well, but far too many overcomplicated further steps by not consolidating the negative signs. Some candidates did not recognise the need for a second application of integration by parts. Slips were often seen with incorrect signs, coefficients being not quite right or the occasional miscopy from $2x$ to x. Substitution of limits was usually accurately done, although again candidates sometimes overcomplicated by trying to evaluate during the process. Some candidates omitted substituting 0 in the final term as the other 2 terms produced a zero value.

Question 7

Many responses understood the principles of what they needed to do, i.e. replace z, rationalising the fraction and then equate real and imaginary parts. Most knew the meaning of z^*, and that $i^2 = -1$. However, errors with signs commonly led to a loss of accuracy. Those who rationalised the fraction before multiplying ended up with easier equations to solve than those who multiplied through by (2 – i). Candidates who achieved correct equations by the first method normally completed the question accurately. Those who just multiplied by (2 – i) did not always finish even if their equations were correct.

Question 8

(a) Several responses correctly identified the function as a product of two terms and applied the product rule appropriately. Many produced fully correct solutions, and those who made early slips were often able to detect and correct their errors, especially since this was a 'show that' question. However, a common mistake was in differentiating $\ln 5x$ as $\frac{1}{5x}$ instead of $\frac{1}{x}$. Many candidates did not simplify their expressions early enough in their working, e.g. factors of 5 not removed, or incorrectly multiplying negative terms leading to far more complicated expressions than necessary.

(b) While a substantial number of candidates were able to sketch the graph correctly, many incorrectly labelled the intercept as 1. The sketch of the graph was frequently inaccurate, indicating limited understanding of asymptotic behaviour. A sizeable number of responses showed an exponential curve instead of a reciprocal function. It was also common for responses to omit a clear conclusion, such as marking the point of intersection with a dot or cross or adding an arrow, any of which would have been acceptable. Candidates are reminded that they must provide justification for their answers, especially in 'show that' questions.

(c) Most responses carried out the required calculations correctly, either by forming a combined expression and showing that one value was positive and the other negative, or by evaluating two separate expressions and demonstrating that the inequality changed direction. However, some candidates did not draw a clear conclusion from their working, while others introduced arithmetic or rearrangement errors.

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(d) Many candidates sensibly chose a point within the given domain as their starting value for the iteration. Only a few did not obtain full credit from not showing a sufficient number of iterations. A sizeable number of candidates began their iteration from 0.2 or 0.6, or from even further outside the stated interval. Whilst, if done correctly, this would lead to the required root, candidates should have utilised the knowledge gained in **8(c)** about the value of the root to conduct this process more efficiently.

Question 9

(a) Most candidates found this question challenging. The requirement was to form a vector equation, so the final answer needed to be expressed in the standard form using **r**. Although many candidates correctly identified a suitable position vector and found an appropriate direction vector, they did not present their final answer as a vector equation. Instead, they wrote an equation with l_1 on the left-hand side or some other incomplete form, which did not meet the demand of the question. Most candidates were also unable to find the equation of the second line. Many realised that they needed to take a scalar product with a vector that had a no x-component, but could not determine how to proceed from there.

(b) The most straightforward method for this part was to use the scalar product. However, many candidates were unable to find a correct direction vector form in **9(a)**, and therefore did not attempt this part of the question. Even among those who managed to obtain an equation for this in **9(a),** several left this part unanswered. A small number of responses also confused direction vectors with position vectors of points on the lines, leading to incorrect scalar products.

(c) This part of the question could be answered independently of the previous parts, provided that the x-component of the direction vector was set to 0. Candidates would still have been able to earn full credit, as only the x-component was required to determine the value of the parameter, enabling them to find the point of intersection. However, this part was also poorly answered by many candidates, largely because they were unable to determine the equation of the second line and mistakenly thought they could not proceed.

Question 10

(a) Many responses were determined to force the question into a pre-learned order of terms in the denominator. This led to many sign inaccuracies. Candidates needed to realise that the question was arranged to give a direct and suitable answer to be carried forward to the next part, and that formulaic processes only work when an expression is written in the required form.

(b) The separation of variables was often not performed well, with the '2' from the partial fraction unnecessarily placed with the $\sec^2(3x)$ term, or worse the $\sec^2 3x$ term written as $\cos^2 3x$. Many responses were not careful with the use of brackets or modulus signs, with logarithmic terms leading to $\ln(-1)$ in the evaluation of the arbitrary constant. This should have indicated that the response contained an error, but this fact was often missed.

Question 11

(a) Candidates who dealt with the two parts of the product separately and differentiated them before combining the results in the product rule were usually more successful than those trying to do the whole differentiation in a single attempt. It was also simpler to keep to the $\sec^2 x$ term given in the question rather than replacing it by $(\cos x)^{-2}$, or by $\dfrac{2}{\cos 2x+1}$. Candidates found it challenging to deal with the factors of $(3 + 2\tan x)^{-1/2}$ and $(3 + 2\tan x)^{1/2}$. Solving the equation resulting from equating the derivative to 0 often successfully led to $2(3 + 2\tan x)(\tan x) = -\sec^2 x$, but many did not then continue to a quadratic equation in $\tan x$ and therefore the completion of the question.

(b) This question was answered well by the majority of candidates. Stronger responses used notation well and kept track of their dx and their du; whereas, the weaker responses did not. A small number did not manage to make the substitution correctly, although most did reach a multiple of $u^{1/2}$. Some made errors with the limits, or did not change them at all. A small number clearly did not understand the process correctly and tried to use integration by parts on a 'mixed' integral containing both u's and x's. Candidates are encouraged to think about the form of their final

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answer, considering if it is fully simplified, if fractions have been calculated and/or cancelled down, if powers have been evaluated and whether it is in the ‘exact form’ which the question requires.

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MATHEMATICS

Paper 9709/35
Mathematics 3

Key messages

Candidates should appreciate that the first part of a question may be used to help in the solution of subsequent parts, even if the word 'hence' has not been used. Questions should be read carefully to ensure that all the demands have been met and that any answers are in the required form. 'Show that' questions should include sufficient steps to justify the given answer. The rubric states "You must show all necessary work clearly, no marks will be given for unsupported answers from a calculator"; some candidates were not able to gain marks because this instruction was not followed.

General comments

Many candidates performed well, showing a good understanding of syllabus requirements and applying these correctly. There did not appear to be any timing issues and candidates had sufficient room to show their solutions clearly.

Comments on specific questions

Question 1

(a) The sketch needed to be a "V-shape" formed by symmetrical straight lines, with a vertex on the x-axis and lying in both the first and second quadrants. Most candidates produced such a sketch, but it was also essential that the intercepts of the graph with the coordinate axes be stated or marked in, so that the unique position of the graph could be shown.

(b) Most candidates used a correct approach to the solution of the given inequality. The sketch from **1(a)** was intended to guide candidates and inform them that there was only one solution. Some candidates sketched the line $y = 5x - 3$ on their sketch in **1(a)** to determine this, then obtained the critical value $\frac{9}{8}$ and therefore usually the correct answer of $x < \frac{9}{8}$. Many candidates used either a squaring method or considered two linear equations to obtain the values $-\frac{1}{2}$ and $\frac{9}{8}$. Often candidates included the value $-\frac{1}{2}$ in their final answer, not appreciating the significance of the first part of the question.

Question 2

It was important that there was some indication of scale seen or implied on both axes of the Argand diagram. Most candidates realised that $|z - 1 - 3i| \leqslant 2$ could be represented as a circle with centre $(1,\ 3)$ on the Argand diagram. However, it was sometimes not explicitly shown that the radius of the circle was 2 units. Dashes or numbers on the axes were usually sufficient to show or imply a radius of 2. Most candidates also sketched a correct circle, centre the origin and radius 4, and subsequently shaded the appropriate region as required. Some candidates mistakenly thought that the inequality $|z| \geqslant 4$ represented a straight line parallel to one of the axes.

Question 3

(a) Most candidates re-wrote the given equation as $\ln y + \ln A = x \ln b$ or correct equivalent. Candidates were asked to 'Show that the graph of ln*y* against ln*x* is a straight line', and it was expected that

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reference and direct comparison be made to an equation of a straight line such as $Y = mX + c$ or correct equivalent in order to obtain full credit. Few candidates did this explicitly.

(b) Many strong responses were seen here, following on from correct equations in **3(a)**. There were occasional sign errors from **3(a)** and some candidates did not give their final answers for *A* and *b* correct to 2 significant figures as required.

Question 4

Most candidates attempted to differentiate the left-hand side of the given equation, using the product rule and implicit differentiation. Errors usually occurred when dealing with the terms involving implicit differentiation, subsequent incorrect simplification or incorrect differentiation of $\ln 6$. Many correct unsimplified fractions for $\frac{dy}{dx}$ were obtained, usually in the form $\frac{2+4\ln 6}{\frac{4}{3}-\ln 6}$ or similar. As the question demand was for a simplified form, no nested fractions were permitted for full credit to be awarded.

Question 5

Candidates are reminded that they must read the question carefully and ensure that any given equations are written down or read correctly. In this particular question, there were candidates who omitted the "+2", which clearly affected the rest of the solution.

Most candidates made a correct substitution for *z* and *z**, with a valid attempt to simplify. Two equations were usually then obtained by equating real parts and imaginary parts, and many candidates were able to give two correct complex numbers. Some candidates did not give their final answers as complex numbers, just writing the values obtained for *x* and *y*; a check that an answer obtained is in the required form should always be made.

Question 6

(a) Many responses did not give their final answer as a simplified fraction as required. Most differentiated the equation of the curve as it was given in the question. These candidates tended to have more subsequent success than the candidates who chose to write $\tan x$ as $\frac{\sin x}{\cos x}$, thus introducing an extra term in the denominator which then sometimes involved differentiation of a product or simplification to a double angle form.

Errors in responses usually occurred when dealing with fractions within fractions, which was a necessary simplification when trying to obtain all terms in terms of $\sin x$. Some candidates chose to give their final answers as two separate fractions instead of the required single fraction. Several candidates unnecessarily expanded out the denominator of their already correct solution; a denominator in terms of factors is what was expected as a simplified fraction.

(b) Candidates were expected to equate the numerator of their answer to **6(a)** to 0. Most candidates did just this, although the simplification process was often completed in this part of the question rather than **6(a)**. Many candidates were able to set their numerator to zero and attempt some form of solution. As this was a 'Show that' question, it was essential that some reference be made to the fact that $|\sin x| \le 1$ or equivalent to obtain full credit.

Question 7

(a) Most candidates showed sufficient detail in their working to obtain full credit. It was essential each step and substitution be shown in full given that this was a "show that" question.

(b) Most candidates used the given result from **7(a)** and attempted to find the value of the given integral. There were many instances of the answer $\frac{112}{3}$ appearing with little or no working shown, which goes against the rubric on the front of the paper (as discussed in the Key messages).

In some cases, the answer $\frac{112}{3}$ appeared after incorrect work, and it was evident that a calculator's definite integral function had been used. Whilst it is fine to check that a correct answer has been

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obtained from full working, this question is testing the syllabus content for integration, not the use of a calculator; integrating terms in *u* and subsequent substitution of limits needs to be seen for full credit to be awarded.

Question 8

(a) Many candidates completed a fully correct solution and obtained full credit. Most realised that the given expression had to be expanded out and then simplified to $\frac{5}{2}\sin\theta + \frac{3}{2}\sqrt{3}\cos\theta$ before attempting to find R and α. Common errors included incorrect signs, giving R in decimal form rather than exact form and not giving α to the required degree of accuracy in terms of radians.

(b) Most candidates used their result from **8(a)** to attempt to solve the given equation. Whilst many correct solutions were seen, some candidates only found one solution rather than two. The methods of solution were sometimes vague, and it was not always sufficiently clear that a 'correct answer' had been obtained from full and correct working.

Question 9

(a) Very few incorrect responses were seen for this question. A few candidates mistakenly thought that the scalar product of the direction vectors was equal to -1 rather than 0.

(b) Again, very few incorrect responses were seen, with candidates using correct methods to find the point of intersection of the two lines. Any errors were usually arithmetic.

(c) Most responses attempted to use the scalar product to find the values of *a*. Most were able to obtain $\frac{8-3a}{\sqrt{14}\sqrt{13+a^2}} = \frac{5}{14}$. Subsequent squaring to obtain a three-term quadratic equation in *a* proved problematic for some, with some arithmetic errors and some instances of $\sqrt{13a^2}$ being used rather than $\sqrt{13+a^2}$. Some responses did not show the simplification of working to obtain a three-term quadratic equation in *a*, and simply wrote down the solutions. These were regarded as unsupported answers from a calculator. Use of a calculator to solve quadratic equations is acceptable on this paper if the equations have been simplified to a three-term form equated to 0. Some candidates erroneously discarded one of the values of *a*. Reading the demands of the question carefully would have shown that more than one value of *a* was required.

Question 10

(a) Many correct solutions were seen, with candidates making use of integration by parts to obtain the correct equation. Most candidates substituted the given limits correctly and showed sufficient detail to reach the given answer.

(b) The most common method used was to consider the function $\mathrm{f}(a) = a - 2 - \mathrm{e}^{-\frac{1}{2}a}$ or equivalent, and show a sign change when the values of 2.2 and 2.4 were substituted for *a*. It was essential that numerical values for $\mathrm{f}(2.2)$ and $\mathrm{f}(2.4)$ be seen for justification of a sign change. Other methods were equally acceptable, as shown in the mark scheme, but less common. It was equally important to show sufficient working and reasoning to justify the given answer.

(c) Most candidates obtained the correct answer from a series of iterations to the correct level of accuracy required.

Question 11

(a) Candidates needed to state that $(1-x)$ represented the fraction of unaffected trees. Too many candidates just rewrote the given information.

(b) Most candidates separated the variables correctly and used partial fractions to obtain $\int \frac{1}{x} + \frac{1}{1-x}\ \mathrm{d}x = \int k\ \mathrm{d}t$, or correct equivalent. There were some errors involving the partial fractions

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and candidates who did not involve k with their partial fractions were usually more successful subsequently. Many candidates were able to obtain a correct general solution.

It was important that the given information concerning the conditions be read carefully and interpreted correctly. It was intended that when the disease is first detected that $t = 0$ and then after 2 years, $t = 2$. Many candidates used values of $t = 1$ and $t = 2$, and were unable to progress correctly. Some candidates used values of $t = 1$ and $t = 3$ and were able to obtain full credit if a correct adjustment in their final answer was made. Other candidates used values of t and $t + 2$. These candidates were unable to gain full credit as they were unable to find the particular solution as required, unless a value of $t = 0$ was subsequently used. Again, some candidates did not read the demands of the question carefully and omitted the particular solution.

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MATHEMATICS

Paper 9709/41
Mechanics

Key messages

- When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming either an equilibrium situation or a Newton's Law equation. Such a diagram would have been particularly useful here in **Questions 4**, **5** and **8**.
- Non-exact numerical answers are required correct to 3 significant figures (or correct to 1 decimal places for angles in degrees) as stated on the question paper. Candidates would be advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.
- In questions such as **Question 7** in this paper, where velocity is given as a function of time, then calculus must be used, and it is not possible to apply the equations of constant acceleration.

General comments

The requests were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 1**, **2**, **3**, **7(a)** and **8(a)** were found to be the easiest questions whilst **Questions 4**, **5**, **6**, **7(c)** and **8(b)** proved to be the most challenging.

In **Questions 4** and **7(b)**, the angles were given exactly as $\sin^{-1}(0.6)$ and $\sin^{-1}(0.2)$ respectively. There is no need to evaluate the angle in these cases and in doing so can often lead to inexact answers (and so any approximation of the angle can lead to a loss of accuracy).

One of the rubric points on the front cover of the question paper was to take $g = 10$ and it was noted that almost all candidates followed this instruction.

Comments on specific questions

Question 1

(a) This part was answered extremely well with almost all candidates correctly applying Newton's second law to find the required driving force.

(b) This part too was answered extremely well with almost all candidates realising that to find the power of the car's engine they needed to multiply their answer to **1(b)** by the given speed of 15.

Question 2

(a) Almost all candidates correctly applied the formula for conservation of linear momentum to find at least one of the two possible speeds of *Q* after collision. The most common error was to only consider the velocity of *P* after collision as 1 and not ± 1 and so therefore several candidates only found one of the two possible speeds.

(b) This part was answered well with most candidates correctly finding the kinetic energy before and after collision and then subtracting these two values the correct way round to find the required loss. A few candidates though incorrectly used the formula $\frac{1}{2}mv$ for kinetic energy and some attempted

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to combine the kinetic energies of *P* before and after collision as a single expression of the form $\frac{1}{2} \times 3 \times (4-1)^2$.

Question 3

This was the best answered question on the entire paper with almost all candidates correctly resolving horizontally and vertically to find the magnitude of the resultant force and a relevant angle for the direction. However, many candidates did not score the final mark due to not being able to give an accurate description of the direction of the resultant force. It is advised in situations like this that candidates give a description based on the angle either to the positive/negative *x*-direction/horizontal (e.g. 25.3 degrees below the positive x-direction), positive/negative *y*-direction/vertical (e.g. 64.7 degrees to the downward vertical) or relative to one of the given forces (e.g. 24.7 degrees anticlockwise from the 72 N force).

Question 4

Although the question explicitly stated that an energy method was to be used, many candidates used a combination of Newton's second law together with the formulae for constant acceleration, therefore scoring a maximum of 3 (of the 6 possible) marks. While many candidates correctly considered the work-done by the resistive force and attempted both the gravitational potential energy lost by *A* and the gravitational potential energy gained by *B*, several candidates only considered the kinetic energy of one of the two particles. Another common error was to calculate the gravitational potential energy gained by *B* as $3g \times 0.75$ rather than the correct $3g \times 0.75 \times 0.6$ (so therefore using the distance travelled by *B* rather than the correct change in vertical height). Very few candidates could set up a work-energy equation with the correct number of dimensionally correct terms, and even those that did many of these equations contained sign errors.

Question 5

This was a question in which many candidates struggled, and it was clear that many were unsure where to begin with this type of extended response. Examiners commented that it was very unclear at times what candidates were doing and what mechanical principles were being applied. It would be extremely beneficial if candidates stated which principle (for example, resolving parallel to the plane) they were applying at each stage of their working.

One common error was to assume that the normal contact force of the plane on the particle was a component of the weight only when in fact it was a combination of this component and a component of the 8 *mg* N force. A second common error was to have the frictional force acting in the wrong direction (as *P* was on the point of slipping down the plane the friction should be acting up the plane).

Finally, while many candidates did form an equation in both sine and cosine, several did not realise that they had to use the fact that $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to then find the required angle.

Question 6

The responses to this question were mixed; examiners reported seeing several perfect responses (scoring all 6 marks) to some that were left blank or little in the way of progress was made. While many correctly calculated the speed of *A* before the collision many struggled to set up the correct equation for the speed of *A* after collision (with the most common errors being either sign errors when applying the principle of conservation of linear momentum, or surprisingly, thinking that the speed of *A* at collision was zero). The most common error when attempting to find the distance that *A* moved up the plane after collision was to assume that the acceleration was *g* rather than the correct –2 from $-g \times 0.2$.

Question 7

(a) This part was done extremely well with many candidates correctly differentiating the given expression for *v* and then substituting $t = 2$ to find the required acceleration. A small minority of candidates changed the correct answer of –1 to 1 even though the question explicitly asked for the acceleration and not the magnitude of the acceleration.

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(b) While most candidates correctly realised that they needed to integrate the given expression for v to find the total distance travelled by P in the first three seconds of motion, many did not realise that P changed direction at $t = 1.5$ (as indicated by the factorised form given in the opening stem) and therefore they only evaluated their expression for the displacement of P at $t = 0$ and $t = 3$ (instead of the correct consideration of $t = 0$ to $t = 1.5$ **and** then separately $t = 1.5$ to $t = 3$). Because of this, very few candidates obtained the correct distance in this part.

(c) Many candidates left this part blank or made little in the way of progress. Most candidates who employed a correct method in this part tended to set their expression for s equal to zero and then showed that the resulting cubic in t could be reduced (by ignoring the solution for $t = 0$) to a quadratic which had no real roots. The more efficient method though of integrating v between the limits of $t = 0$ and $t = 4$ and showing that this value was greater than zero (and so proving that P does not return to O) was rarely seen by examiners.

Question 8

(a) This first part of the final question was answered extremely well with many candidates correctly applying Newton's second law for B to find the required tension in the string. The most common error was to use $T\sin 15$ rather than $T\cos 15$ as the resultant force.

(b) As expected, the responses to this final part were mixed with very few candidates successfully finding the coefficient of friction between A and the surface. While many had some success in applying Newton's second law for the motion of A (although there were the usual sign errors and incorrect trigonometric ratios in the components of the forces), a similar error to the one made in **Question 5** was then made in this part too. This time the error was assuming that the normal contact force of the surface on A was the weight only, when in fact it was a combination of not only the weight but also a component of the 6 N force and a component of the tension in the string.

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MATHEMATICS

Paper 9709/42
Mechanics

Key messages

- In previous reports, it has been mentioned that when answering questions involving any system of forces, a well annotated force diagram could help candidates to include all relevant terms when forming either an equilibrium situation or a Newton's Law equation. Force diagrams are now being seen more often than in previous occasions when this examination has been sat.
- Non-exact numerical answers are required correct to 3 significant figures or angles correct to 1 decimal place as stated on the front of the question paper. Candidates are strongly advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.

General comments

The requests were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 1**, **3(a)** and **7(a)** were found to be the easiest questions whilst **Questions 4**, **6** and **7(b)** proved to be the most challenging.

In **Question 2**, the angle θ was given exactly as $\sin^{-1}0.04$. It was pleasing to see that many candidates used this exact value and fewer than in recent exam series evaluated the angle which would have led to a loss of accuracy. If the angles are found, then they should be used to at least 4sf to maintain accuracy to 3sf for a final answer.

One of the rubric points on the front cover of the question paper was to take $g = 10$ and it was noted that almost all candidates followed this instruction.

Comments on specific questions

Question 1

This question was answered well with the majority of candidates getting $x = 80$ and $a = 10$. A common error seen by examiners was to equate $\frac{x-0}{4-6}$ to a speed of 40 rather than a velocity of –40. Another common error was to have a correct distance of 80 m calculated, but then not giving this as x as requested.

Question 2

Candidates who drew a force diagram for this question were more successful than those who did not. The most successful approach was by candidates who had two separate equations of motion for the locomotive and the coach and then solving simultaneously. It was common to see candidates having equations that implied that the motion was up the hill, when the question clearly stated that the locomotive was towing the coach down the hill. Other common errors seen were regarding the weight components which were either omitted completely or using $240\,000g\sin 0.04$ or $240\,000g\sin^{-1}0.04$ for the locomotive and similar errors for the weight component of the coach.

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Question 3

(a) This was a well answered question by the majority of candidates.

(b) (i) There was a mixed response to this question. Most candidates found the change in kinetic energy correctly. The work done by the car to overcome the resistance was omitted by a significant number of candidates and those who did find this, combined it incorrectly with the kinetic energy. Most who got this far completed the calculation correctly using **Energy** $=$ **Power** $\times$ **time**. It was notable that a minority of candidates did not follow the instruction to use an energy method, but used a method based on Newton's second law.

(ii) This request was only successful for a minority of candidates. Many assumed that the car was still accelerating when the question indicated steady speed, so the force needed was 800 N, when many responses incorrectly used 1500 N.

Question 4

(a) This type of question with 2 particles in motion vertically is nearly always found difficult and this proved to be the case. Candidates should be clear on which direction they are taking as positive and consistently use that for both particles. Positive being upwards is recommended. Then the vertical distance moved in time t seconds for P and Q are $30t-\frac{1}{2}gt^2$ and $10t-\frac{1}{2}gt^2$. Then, as Q is initially 15 m above P, this gives the equation $10t-\frac{1}{2}gt^2+15=30t-\frac{1}{2}gt^2$ leading to $t=\frac{3}{4}$ for the time taken for the particles meet.

(b) This proved to be the most challenging question on the paper. Conservation of linear momentum is clearly well understood, but unfortunately a significant number of candidates used speed 30 ms^{-1} for Q and 10 ms^{-1} for P, giving no consideration to the fact the particles are 15 m apart, moving vertically and take $\frac{3}{4}$ seconds before they meet and hence their speeds will have changed. Even some candidates who had a speed of 6.5 ms^{-1} for the combined particle after collision assumed this was the required speed and did no further work.

Question 5

This proved to be a challenging question for a significant number of candidates. The most common error is for the normal reaction to be seen as $5g\cos 10$ and so had an incorrect friction. When another force acts on the particle in a direction not parallel to the surface, then that force will contribute to the normal reaction. So, the normal reaction in this situation is $5g\cos 10-20\sin 35$. The majority of candidates did get a correct Newton's second law equation, but with their incorrect friction. A minority of candidates used an energy method, but not successfully due to dimensionally incorrect terms, missing terms for gravitational potential energy appearing twice in an energy equation.

Question 6

Only a minority of candidates achieved full marks on this question. The majority of candidates made a good attempt at solving $a=0$, eventually getting to $t+1=4$ and solving to get $t=3$, but a significant number of candidates solved this to get $t=5$. The integration of a to get $4(t+1)^{\frac{1}{2}}-t+c$ was done well. The initial conditions of $v=0$ when $t=0$ where then used, but many candidates thought this gave $c=0$ and assumed that $4(t+1)^{\frac{1}{2}}$ is not 0 when $t=0$. Candidates who did get $c=-4$ usually proceeded well by a further integration then either using limits correctly or finding a further constant of integration and substituting $t=3$.

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Question 7

(a) This part was well answered and is a usual good source of marks. Most candidates resolved correctly in two directions and solved the resulting equations correctly for S and α.

(b) The common error in this question was regarding the normal reaction. Some candidates thought this was $0.6g$, others resolved vertically on the system of forces but omitted the $0.6g$. An acceleration had to be calculated to use Newton's second law by resolving horizontally. Many calculated the acceleration correctly using constant acceleration, but others mistakenly thought that the ring was in equilibrium even though the question stated it starts from rest and moved 2 m in 3 s. The candidates who had a correct acceleration and normal reaction invariably went on to complete the request successfully.

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MATHEMATICS

Paper 9709/43
Mechanics

Key messages

- When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming either an equilibrium situation or a Newton's Law equation. Such a diagram would have been particularly useful here in **Questions 3** and **7**.
- Non-exact numerical answers are required correct to 3 significant figures (or correct to 1 decimal places for angles in degrees) as stated on the question paper. Candidates would be advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.
- In questions such as **Question 6** in this paper, where acceleration is given as a function of time, then calculus must be used, and it is not possible to apply the equations of constant acceleration.

General comments

The requests were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 1**, **2**, **4(a)**, and **5(a)** were found to be the easiest questions whilst **Questions 3**, **5(b)**, **5(c)**, **6(b)** and **7(b)** proved to be the most challenging.

In **Question 2(b)** the angle was given exactly as $\sin^{-1}\left(\frac{3}{20}\right)$. There is therefore no need to evaluate the angle in this case and in doing so can often lead to inexact answers (and therefore a possible loss of accuracy).

One of the rubric points on the front cover of the question paper was to take g = 10 and it was noted that almost all candidates followed this instruction.

Comments on specific questions

Question 1

(a) This part was answered extremely well with almost all candidates giving the correct deceleration of the athlete 7 seconds after leaving A. Strictly speaking an answer of –1.6 for the deceleration was incorrect but was condoned on this occasion.

(b) This part was also answered extremely well with many candidates giving the correct total distance travelled by the athlete. Apart from calculation errors when working out the area of the required triangles, the most common conceptual error seen by examiners was to find the displacement of the athlete from time t = 0 by subtracting the area below the t-axis from the areas above (rather than working out the area of all three triangles and summing these three values together).

Question 2

(a) Most candidates correctly showed the required result that c = 80 by using a correct combination of $P = D \times v$ and Newton's second law.

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(b) The responses to this part were more mixed. While most candidates scored at least the first mark for correctly stating the weight component of the cyclist up the hill, some candidates either forgot to include the resistance force in their equation of motion or did not realise that 'steady' speed implied that the acceleration was zero. While most candidates who set up a correct three term quadratic in v went on to solve it correctly, some candidates obtained a correct speed (possibly from a calculator) but from clearly incorrect working.

Question 3

(a) There were two common errors in this part: the first was to assume that the normal contact force of the road on the block was given by the weight only, when in fact it was a combination of the weight and a component of the pulling force too. The second error was in not reading the question carefully and assuming that the block was being pulled up a road inclined at an angle of 36 degrees to the horizontal by a force acting parallel to the road. Of those candidates that did not make either of these two fundamental errors most went on to correctly find the work done against friction.

(b) The errors made in **3(a)** had a substantial impact on this part too. While most candidates scored at least one mark for correctly calculating the change in kinetic energy very few could set up a correct work-energy equation to find the speed of the block at B. Although the question explicitly said that an energy method must be used, several used Newton's second law and the formulae for constant acceleration and so therefore could only score a maximum of 2 (of the 4 available) marks.

Question 4

(a) Most candidates scored the first two marks in this part for correctly differentiating the expression for s to find an expression for the velocity of P. However, several candidates interpreted 'minimum velocity' as when the velocity 'equals zero' and therefore scored no further marks. Of those that did realise that they needed to differentiate again and find the value of t when the acceleration was zero most were successful in doing so. One common error at the end of this part was to give the velocity as –5.07 even though the question asked for the speed (and so therefore must be positive). A second common error was to take the correct value of –40.7 and to change the sign to positive even though the question explicitly asked for the value of s.

(b) Although many candidates appreciated that the direction of motion of P would change when $v = 0$, some candidates considered instead the situation when $s = 0$. Of those that did correctly solve the equation $v(t) = 0$, most then went on to substitute the correct positive value of t into the expression for the acceleration they'd found in **4(a)**.

Question 5

(a) The first part of this question was a routine pulleys problem involving finding the tension in the string and the magnitude of the acceleration of the particles. Most candidates scored at least three of the four marks available with the final mark often lost for giving an acceleration that was negative. Surprisingly, some candidates tried to use the formulae for constant acceleration in this part.

(b) Many candidates struggled with this and the next part. Many believed that the acceleration of the particles was g initially rather than the value found in **5(a)** and some had mathematical statements that implied that the particles were released from rest. While some candidates did use an appropriate method to find the required time, many attempts contained sign errors. Of those that did use an appropriate method many were not efficient ones; candidates are reminded that in this type of problem (in which a particle is moving vertically; first up and then coming back down) the acceleration is constant throughout the motion and therefore the single SUVAT equation of $-3.2 = 1.2t + 0.5(-5)t^2$ (or equivalent) can be used to find the required time (as indicated by only 2 marks being available for this part). Many candidates attempted this part by considering the motion of B in two stages and very few were successful.

(c) As mentioned in **5(b)**, candidates found this final part of this question particularly difficult with very few obtaining the correct distance. Many made little progress apart from correctly finding the distance that A travels from its initial position to the time when it first comes to rest. It was quite

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common to see candidates simply doubling this value and not realising that when *B* hits the ground the string will become slack and therefore the distance travelled by *A* after *B* hits the ground (using an acceleration of $-g$) needed to be considered.

Question 6

(a) Although many candidates correctly applied the principal of conservation of linear momentum to find an expression for the velocity of *Q* after the collision, many of these attempts contained sign errors or incorrect values. Of those that did obtain the correct expression, many did not realise that to show the required result that $m > 4.5$, all that was required was to set their expression for the velocity > 0 and solve for *m*.

(b) This, along with **Question 5(c)**, were the least well answered parts of the paper with most candidates making little progress apart from finding an expression for the speed of *Q* after impact with the wall. Many then believed that to find the largest possible value of *m* they either had to use an energy consideration or apply the conservation of linear momentum again. Those few candidates that did realise that for no further collisions to occur, $\frac{1}{3}u \geqslant \frac{1}{4}v$ where *v* was the expression from **6(a)**, most were successful.

Question 7

(a) Some candidates struggled with this final question, and it was clear that many were unsure where to begin with this type of unstructured response. Examiners commented that it was very unclear at times what candidates were doing and what mechanical principles were being applied. It would be extremely beneficial if candidates stated which principle (for example, resolving parallel to the plane) they were applying at each stage of their working.

One common error was to assume that the normal contact force of the plane on the particle was a component of the weight only when in fact it was a combination of this component and a component of the 12 N force. A second common error was to have the frictional force acting in the wrong direction (as *P* was on the point of slipping down the plane the friction should be acting up the plane).

While it was pleasing to see that a good number of candidates obtained a correct expression for the coefficient of friction in terms of sine and cosine, e.g. $\mu = \frac{6g\sin\theta - 12\cos\theta}{6g\cos\theta + 12\sin\theta}$ many then simply stated the given answer in terms of the tan ratio without any indication of how this was achieved.

(b) This part was also often left blank with very few candidates realising that all that was required was to set $\mu > 0$ (using the given expression in **7(a)**) and solve to obtain the required set of possible values for theta. The common misconception that $\mu < 1$ appeared all too often in this part.

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MATHEMATICS

Paper 9709/45
Mechanics

Key messages

- Non-exact numerical answers are required correct to three significant figures (or correct to one decimal places for angles in degrees) as stated on the question paper. Candidates would be advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.
- In questions such as **Question 4** in this paper, where velocity is given as a function of time, calculus must be used, and it is not possible to apply the equations of constant acceleration.
- When answering questions involving any system of forces, a well-annotated force diagram could help candidates to include all relevant terms when forming either an equilibrium situation or a Newton's law equation. Such a diagram would have been particularly useful in **Questions 3(b)** and **5**.

General comments

The requests were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 1** and **3(a)** were found to be the easiest questions whilst **Questions 2(c), 5** and **6** proved to be the most challenging.

In **Question 3** the angle was given exactly as $\sin^{-1} 0.15$. There is no need to evaluate the angle in this case and doing so can often lead to inexact answers (and therefore a possible loss of accuracy). It is pleasing to note that on this occasion most candidates did not evaluate the angle, but used 0.15 in their working.

One of the rubric points on the front cover of the question paper was to take $g = 10$ and it was noted that almost all candidates followed this instruction.

Comments on specific questions

Question 1

The vast majority of candidates scored full marks in this question. Occasionally, candidates rounded the value of Q, or the values of the trigonometric functions, and therefore found a value for P which was not accurate correct to three significant figurers. Candidates can use a calculator to solve simultaneous equations (despite the fact that one of the equations only involved Q), but if this is done incorrectly and no working is shown in a question such as this, candidates should be aware that marks cannot be awarded. There were a few candidates who, having resolved correctly and with correct angles in one direction, when they resolved in the other direction, used a different angles. There was a small number of candidates who, when resolving, missed one or two forces or got sine and cosine mixed up.

Question 2

(a) Almost all candidates drew a trapezium, but some did not label the horizontal axis with 246 and/or the vertical axis with V.

(b) Many good answers were seen here with candidates using a wide variety of methods. The most common method was to use the total distance to form an equation from which T could be found. The expression for T could then be substituted into an equation for the total time to obtain the given equation. A rather less popular but still fairly common method was to use the total time to form an equation from which T could be found and then substitute this into an equation for distance. This part did however prove rather challenging for a number of candidates.

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(c) Almost all candidates solved the quadratic equation given in **2(b)**, but many simply discarded the solution $V = \frac{450}{7}$ without giving a justification, and some simply gave both answers to the equation which was not accepted.

Question 3

(a) This question was well answered with most candidates correctly finding the relationship between the constant resistance force R and the power P on the horizontal section of the road. Many then went on to form an equation either in R or more usually in P to evaluate both variables, although some then made errors such as including an extra force of one sort or another.

(b) This question was also fairly well answered, although some candidates omitted either the resistance force or the weight component or occasionally both when trying to find the acceleration.

Question 4

(a) Many candidates scored full marks in this part. Even those who did not know how to show that $k > 4.5$, still did not tend to have any issues with correctly integrating the acceleration and finding the constant of integration. There were just a few candidates who did not try to find the constant of integration, or thought that they should use equations of motion for constant acceleration.

The most common error was setting the discriminant to be greater than 0, rather than less than 0. Some candidates seemed to think that because the velocity is always greater than zero, the discriminant should also be greater than zero. They did not take into account that because P is never at instantaneous rest, this meant there could be no solutions to the equation where $v = 0$. Despite setting the discriminant to be greater than zero, errors within negatives and inequalities meant that many candidates still somehow managed to show that $k > 4.5$ via a wrong method.

(b) Those candidates who obtained a three term quadratic in **4(a)**, usually managed to correctly integrate the expression for velocity with $k = 2.5$ substituted. The most common error here was in not taking into account that the velocity could now be less than zero, as the question stated that now $k = 2.5$. Many candidates just used limits of $t = 1$ and $t = 3$ and subtracted the resulting displacements.

The equation formed when setting $v = 0$ was often not considered at all. Some candidates realised they needed to think carefully about which limits to use, but unfortunately set $a = 0$ rather than $v = 0$. Other candidates set $v = 0$, found the times of 0.4 and 2 when $v = 0$, but then used limits of 0, 0.4, 1, 2 and 3 in their calculation for the total distance travelled.

Question 5

(a) This question proved rather more challenging for many candidates. Those who realised that they should ignore the 6 kg mass in this part and just concentrate on the other two masses usually correctly found the two equations $5g - T_{PQ} = 5a$ and $T_{PR} - 2g = 2a$. Most then went on to use the fact that $T_{PQ} = 2T_{PQ}$ to find the acceleration and the two tensions, although some forgot to give the value of T_{PQ} and others did not express the tensions in terms of g. In fact, it seemed clear that some candidates did not understand what this instruction meant. A few candidates who did involve the 6 kg as well, managed to get to the correct answers, often finding the value of the coefficient of friction on the way (thus obtaining the answer to **5(b)**).

(b) This question again proved challenging for many candidates, with those who were unsuccessful in **5(a)** making little progress. Candidates who had wrong values for the acceleration and/or the tension in **5(a)** were still able obtain marks if they correctly followed through their values but few did so, as often one or more terms were omitted.

(c) A considerable number of candidates omitted this question, but of those who did not, it was well answered as candidates simply had to use constant acceleration equations with the acceleration that they had found in **5(a)** to find the total distance. Those with the wrong acceleration could still gain full credit on follow through.

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Question 6

This question was found to be the most challenging question on the paper for the vast majority of candidates.

Many candidates realised that consideration of the loss of total kinetic energy and the conservation of momentum both needed to be considered. Careful presentation would have helped some candidates achieve a higher score here, due to the rather complex algebra that was required. One common error was in using 75 per cent of the initial kinetic energy, rather than 65 per cent.

Another common error was only considering the loss in the total kinetic energy and not considering conservation of momentum at all. This meant that the speed of B could only be found in terms of u and k, which is not what the question required.

Those candidates who successfully set up two equations in k, u, and the speed of B, tended to be more successful if they replaced the speed of B with an expression in terms of k and u, rather than replacing k with an expression in terms of u and the speed of B.

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MATHEMATICS

Paper 9709/51
Probability & Statistics 1

Key messages

Good solutions were characterised by clear communication and often featured a helpful list or diagram. Candidates must be aware of the need to provide both workings and explanations to support their answers. It is important to include all the relevant steps including the mathematical operations necessary to reach their final solutions. Where there are various scenarios, these should be identified clearly. Candidates should state only non exact answers correct to 3 significant figures and exact answers should be stated exactly. When using working values to support further processes these should not be rounded prematurely. To justify a final answer correct to 3 significant figures working values correct to at least 4 significant figures should be used in the calculations. There is no requirement to convert fractions into decimals.

General comments

The use of helpful diagrams, sketches, lists and tables were frequently seen in successful solutions. These aided the efficient organisation of responses.

Most candidates used the answer space well; when extra space is required candidates should use the additional page in the first instance. If a candidate has more than one attempt at a question the one intended for marking should be clearly identified. Candidates should be reminded that they should present only one solution for marking.

Many good solutions were provided for the earlier parts of **Questions 1, 2, 3, 6** and **7**. Frequently the latter parts of **Questions 1, 3** and **7** seemed challenging for most. Sufficient time seems to have been available for most candidates to complete the paper using efficient methods. A few candidates did not appear to have been prepared well for all topic areas of the syllabus.

Comments on specific questions

Question 1

Many candidates were able to use the given formula to find the required probabilities but using them to find the required conditional probability was more challenging.

(a) Many candidates were able to find the probabilities in terms of *k* with most who did so using the sum of probabilities being 1 to find *k*. A good number of correct probability distribution tables were seen with *k* substituted. Some assumed an inversely proportional relationship and gave probabilities as 4/*k* etc.

(b) Where the table in **1(a)** was correct, an accurate response usually followed with a small number of errors in sign noted for the probability that $x = –2$.

(c) Those who identified that the required outcome involved finding the probability of *X* being 1 or 3 were usually able to reach the correct answer although some neglected the necessary division by the probability of *X* being 1, 2 or 3.

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Question 2

A number of candidates rounded their answers to 3 decimal places rather than 3 significant figures.

(a) Most solutions involved the use of the Geometric Distribution with weaker attempts including a Binomial coefficient. Early rounding to 0.047 without any more accurate answer was seen.

(b) A good number of candidates realised that the 7th throw needed to be a six and that the preceding throws needed to be 2 sixes and 4 non sixes but many did not deal with the number of ways that the preceding throws could be combined.

Question 3

Candidates must be aware of the requirement to identify what they are finding in their solutions; the median in **3(a)** and the standard deviation in **3(b)**. In both parts some solutions neglected the fact that the salaries were in thousands of dollars, and the necessary place value was lost.

(a) This part was well done by many. Most knew how to find the median (although some did not identify it as such) and the quartiles. On occasion the necessary subtraction to find the interquartile range was missing.

(b) Candidates are reminded of the requirement to show their working. Many solutions were seen where the necessary calculations were not present. Rounding too early resulted in inaccurate final answers, particularly when using a rounded value for the joint mean. Many found this challenging and simply added the mean and standard deviation for each group of employees.

Question 4

(a) Most candidates were able to follow the framework provided to complete and label the branches for bag *B* correctly. Many found it difficult to follow the instructions for the designated placement of the chosen marble particularly when a blue marble was first selected from bag *A*. A small minority omitted the branch labels throughout.

(b) Vital stages in working were sometimes omitted. The best solutions worked with fractions throughout. A small number of candidates correctly found the probability of 3 red marbles and 3 blue marbles but did not add them to find the required probability.

Question 5

(a) Most candidates recognised this as a binomial distribution application and many of these were able to add the correct terms with sufficient accuracy to get an answer within the required range. Some did not interpret 'at least 5 days' to mean that 5 days must be included while others just gave the term relating to 5 days. Some subtracted their answer from 1 thus finding the probability of less than 5 days. Candidates must be aware of the necessity to use values corrected to at least 4 significant figures in their working as using rounded or truncated values often lead to a final answer outside the range. Weaker solutions omitted the binomial coefficient. A few instances of the Normal distribution being used in this part were seen.

(b) Many good clear solutions were provided by candidates who interpreted the phrase 'suitable approximation' correctly as a direction to use the normal distribution. If identified, the variance or standard deviation must be designated appropriately. Many appreciated that a continuity correction was needed with most using 21.5 correctly. A minority of candidates then subtracted their probability from 1 to find the probability of fewer than 22 days. Many solutions were seen where good practice in drawing a helpful diagram helped to avoid this. Weaker candidates attempted to use terms from the binomial distribution which was both incorrect and time consuming.

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Question 6

Candidates should be reminded that the standardisation formulae must be seen and that there must be evidence to support their working to find the required probability. Some were unable to attempt this question.

(a) Strong solutions were often seen in this standard application of the Normal distribution. Supporting diagrams were a feature of successful methods, helping the selection of the correct area. Inclusion of the standardisation formulae and the calculation of the required probability must be present if good candidates are to gain full credit; in some cases, these were omitted. A significant number of candidates did not complete the question by finding the expected number of chocolate bars, this application often occurs. Candidates are reminded that they need to show 4 significant figure probabilities in their calculations and provide a single integer answer with no approximation sign or reference to rounding.

(b) Questions on finding the parameters for the normal distribution are often seen. Again, a supporting diagram was a feature of successful solutions. Good solutions provided two standardisation formulae and used the correct probabilities, 0.8 and 0.2, to find the appropriate z values. When using the normal distribution tables, a fair number were not sufficiently accurate when finding the z values and this impacted the accuracy of their final answers. Weaker candidates used the percentages as z values or in a few cases miscalculated 100 percent – 8 percent. A few instances of continuity corrections being incorrectly applied were seen. Having got the two formulae, some were unable to successfully solve them.

Question 7

(a) A lot of correct answers were seen here. Many realised that there were 10! Ways of arranging the 10 letters and the necessity to divide by 2! × 3! × 2! to account for the repetition of S, E and L. Weaker candidates omitted the division.

(b) Most successful candidates considered the groups SC_S or S_CS as a single item where the gap could be filled by one of the remaining letters. The seven items were then arranged in 7! ways, dividing by 2! X 3! to account for the repeated Ls and Es. Some did not deal with the repeated letters or the fact that there were two possible outcomes for the group. A fair number of attempts considered the groups SC_S and SC_S separately with each of the remaining letters Y, H, E and L and summed their results. A significant number of weaker solutions started with the unrestricted number of ways of arranging all ten letters. A few instances of providing an unrequested probability were seen.

(c) Good candidates began this question in organised fashion by placing an S at the start and end of the arrangement; this was frequently seen in a helpful list which enabled them to find the correct number of items for calculation. Subtracting the number of ways that the Es could be together from the total number of possible arrangements of the eight items followed. Some again omitted the necessary division by 3! X 2! to deal with the repeated Es and Ls. Weaker candidates started by arranging all ten letters or were considering too few items.

(d) Good solutions efficiently considered the number of ways of selecting all three Es and two other letters from the remaining seven letters then dividing by the total number of ways of selecting any five letters from ten. Some weaker candidates attempted to list all the possible outcomes. This was rarely successful.

Some successful solutions were also seen where the product of the probability of getting three Es and the number of ways the two other letters could be chosen was found. In both cases some good candidates, when considering the relevant combinations, did not state where their values came from.

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MATHEMATICS

Paper 9709/52
Probability & Statistics 1

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially when there are errors earlier in the solution. The use of algebra to communicate processes is anticipated at this level and enables candidates to review their method effectively. When errors are corrected, candidates would be well advised to cross through and replace the term.

There should be a clear understanding of how significant figures work for decimal values less than 1. It is important that candidates realised the need to work to at least 4 significant figures throughout to justify a 3 significant figure value. Many candidates rounded prematurely in normal approximation questions which produced inaccurate values from the tables and lost accuracy in their solutions. It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent; there is no requirement for probabilities to be stated as a decimal.

The interpretation of success criteria is an essential skill for this component. Candidates would be well advised to include this within their preparation.

General comments

Although many well-structured responses were seen, some candidates made it difficult to follow their thinking within their solution by not using the response space in a clear manner. The best solutions often included some simple notation to clarify the process that was being used.

It was encouraging that the labelling of the statistical diagram included units more frequently, but the failure by candidates to use a ruler to draw the straight lines was disappointing at this level, as many histograms were not drawn accurately. The number of correct unsimplified calculations that were evaluated inaccurately was unexpected.

The use of simple sketches and diagrams can help to clarify both context and the information provided. These were often seen in successful solutions. Similarly, when listing scenarios, it is advisable to use a systematic approach to ensure that all possibilities are considered.

Sufficient time seems to have been available for candidates to complete all the work they were able to, although some candidates may not have managed their time effectively. A few candidates did not appear to have prepared well for some topics, in particular when more than one technique was required within a solution. Many good solutions were seen for **Questions 3** and **6**. The context in **Questions 2, 4** and **7** was found to be challenging for many.

Comments on specific questions

Question 1

Most candidates recognised that the geometric approximation was the appropriate technique to use. A very small number of candidates attempted to use the binomial approximation but were not able to interpret the success criteria appropriately.

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(a) Almost all candidates showed the correct calculation, but many failed to give the exact answer that was required. The instructions on the front of the examination paper state that non-exact answers should be rounded to 3 significant figures, rather than all answers.

(b) Although many correct solutions using the more efficient 0.6^6 approach was seen, a significant number of candidates used 1 – P(1,2,3,4,5,6) accurately. A common misconception was to interpret the success criteria as allowing the head to be obtained on the fifth throw.

Question 2

The context of this probability question was found challenging by many candidates. Allowing replacement was seen occasionally, but a more common error was not to account for the different arrangements that the marbles could be selected.

(a) Almost all solutions had an attempt at a probability distribution table, the best having ruled lines to provide the structure for results. The best solutions considered $P(X = 1)$ and $P(X = 5)$ initially, then $P(X = 2)$ and $P(X = 4)$, as having the least possible arrangements and used the property that the probabilities would sum to 1 to obtain $P(X = 5)$. These solutions often listed the possible scenarios rather than simply working with combinations. A surprising number of solutions had probabilities that did not sum to 1, which should be a standard check at this level to confirm accuracy. Common misconceptions were that the random variable X was the number of other marbles selected, and it was possible to select 6 marbles to obtain a green marble.

(b) A good number of very clear solutions were seen which stated the calculations required for both $E(X)$ and $Var(X)$, with accurate evaluations. Weaker solutions often failed to justify their values, especially where the answer of **2(a)** was incorrect. Candidates should be reminded that calculations for supporting work is expected, and simply stating a value for $E(X)$ will gain no credit, particularly if from previous incorrect work.

Question 3

Almost all candidates identified the appropriate techniques to use in this standard normal distribution question. The increase in sketches of the normal distribution curve to help interpret the success criteria was encouraging.

(a) It was encouraging to see more candidates use the given criteria directly in the normal standardisation formula, rather than calculating the upper and lower heights that are expected. Good solutions recognised that the probability area would be symmetrical and effectively worked with a single standardisation calculation and then used this property to find the area. There were an unexpected number of arithmetical errors in evaluating correct calculations. A significant number of candidates found the required probability but failed to find the number of members. Candidates are well advised to read the question again once they think they have completed their answer to confirm that all the demands of the question have been met. At this level, candidates are expected to interpret their answer to state the number of members, rather than give an approximation or simply round.

(b) Most solutions formed an equation using the expected standardisation formula, although not all were equated to a z-value. Candidates need to be confident in reading the normal tables provided to find the necessary values. As σ is the denominator in the equation, it is expected that the final answer would not be presented as an improper fraction. A small number of candidates formed equations which produced a negative solution.

Question 4

This probability question based around a data table was found challenging by many candidates. There was a large amount of information to consider, and many better solutions highlighted sections of the question to help clarify the problem. The fact that Ivy had x cards in their pack was not used by a surprising number of candidates.

(a) The best solutions calculated P(CC) initially, as values were present in the table and the P(BB). A clear equation was then formed and the 'quadratic' solved using factorisation to obtain the solution. Many candidates recognised that as the denominators were equal on both sides of the equation,

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only the numerators needed to be equated and solve in a linear equation. A common misconception was that the probabilities needed to be added to obtain P(CC) or P(BB). A common misunderstanding was that the buses having twice the probability of the cars required an equation $2 \times$ P(BB) = P(CC). A small number of candidates stated that $x = 0$ was a possible solution, which should have been clearly rejected. Almost all candidates who formed an equation gained some credit for solving, although it was unexpected that decimal solutions were seen so frequently given the context.

(b) This probability question was found challenging by many candidates and was not attempted by over 25%. The best solutions identified clearly the possible scenarios, showed the unsimplified probability calculations using *their x* from **4(a)**, and then summed these probabilities accurately. Some weaker solutions did not use *their x* from **4(a)** but assumed that there were also 18 cards in Ivy's pack. Candidates who did not show their calculations for the probabilities could gain little credit if their value of *x* was incorrect as the method was not present to check.

Question 5

Although this was a quite standard statistical question, many candidates found it challenging because of the accuracy expected in statistical diagrams and determining the data accurately to estimate the mean and standard deviation.

(a) Many candidates found drawing the histogram challenging. The most successful graphs used a scale of 1 cm = 10 minutes on the horizontal axis and 2 cm = 'frequency density 2' on the vertical axis. These enabled the bars to be drawn with accuracy. Good candidates often translated the scale for time so that the bar ends were on the grid lines, which is acceptable. The best candidates then ensured that the corner of the grid was labelled 0 for frequency density and 0.5 for time. Weaker candidates either plotted the frequencies or determined the class width inaccurately by simply finding the difference of the stated ranges. Although there was a pleasing increase in well labelled axes, a high proportion of candidates omitted the units on the time axis or omitted all labels. It is anticipated that a ruler is used to draw the straight lines required in the bars to ensure that appropriate accuracy is maintained throughout the diagram.

(b) Many excellent solutions were seen, with the most efficient being presented in a table to minimise errors and communicating clearly the processes that are being used. The majority of candidates used the formula approach, stating the calculations required before evaluating. This was quite a standard style question and was often successfully answered by candidates who found other topics challenging. The most common errors were using the class width or upper boundary as the mid-point value. Almost all candidates stated the mean formula accurately, but again arithmetical errors were noted at this stage.

Question 6

This binominal and normal approximation question was quite standard in approach, so it was unexpected that so many candidates did not attempt **6(b)** or **6(c)**.

(a) The interpretation of the context was challenging for many candidates. Good solutions recognised that the probability of not having a birthday on Saturday or Sunday was $\frac{5}{7}$, and so calculated $\left(\frac{5}{7}\right)^{10}$. Weaker solutions assumed that 1 – P(birthday on Saturday or Sunday)^{10} was equivalent, however it does not include the occasions where one or more person had their birthday on those days and the remainder did not.

(b) Over 10% of candidates did not attempt this question, which was quite standard in style. The most common approach was to calculate P(0, 1, 2), stating the unsimplified calculation and a final answer with no intermediate steps. Weaker solutions included intermediate calculations which seemed to encourage premature approximation leading to inexact answer. Again, there were a surprising number of completely correct unsimplified calculations evaluated inaccurately. A small number of candidates worked with indices that totalled 7, related to the number of days in the week, or 3, the number of people in the success criteria. Weaker candidates were more likely to

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interpret the success criteria to include 3 people. Candidates would be well advised to focus on interpreting success criteria in their preparation for the examination.

(c) 16% of candidates did not attempt this routine normal approximation for the binomial question. Where attempted, most successfully found the mean and variance, although a small number incorrectly identified the variance as the standard deviation. Using the correct vocabulary is an expectation of the component. Good solutions clearly stated the standardisation formula, identified that a continuity correction was required as the data was discrete and found the appropriate probability area. It was encouraging to see simple sketches of the normal distribution used to interpret the success criteria. Weaker solutions omitted the continuity correction or found the complement of the probability area. Far few candidates were seen to use the incorrect continuity correction than in previous sessions, which was encouraging.

Question 7

This permutations and combinations question was found challenging by many. Candidates who used simple diagrams to illustrate their approach or listed logically possible scenarios often achieved good solutions. Candidates do need to ensure that they read the conditions for each part carefully and consider the different approaches that could be used rather than trying to use the same technique in each part.

(a) The subtraction method was the most common approach to this part but was often unsuccessful as candidates were unsure as to when there were repeated letters affecting the results as a common error was dividing each term by 3!. The more successful approach was to consider ^ ^ ^ OOO ^ ^ to calculate the number of ways that the letters without Ls could be arranged and then identifying the number of ways that the 2 Ls can be inserted separately into the spaces. A few candidates attempted to consider the different scenarios that were possible, but these were rarely successful because of the complexity of the context.

(b) The most successful approach was considering the number of arrangements of the 8 remaining letters with the Ls fixed L ^ ^ ^ ^ ^ L ^ ^ ^ and then determining the number of places that the Ls can be positioned. The majority of candidates divided by 3! to remove the effect of the repeated Os. Less successful approaches focused on how the Os were positioned in relation to the Ls, with candidates not identifying all possible scenarios or not appreciating how their scenario allowed letters to be placed.

(c) Many candidates found this part challenging, with 13% not attempting the question. There were many different approaches that could be used, and all were seen completed successfully. The most common method was to calculate 1 – P(2 letters the same), and clear methods were shown in the majority of solutions. A common error was to assume the first letter chosen was replaced. Less successful were candidates who attempted P(2 letters are different) as P(OL) was often omitted in the calculation, especially if combinations were used to obtain the probabilities.
A small number of candidates worked assuming that all letters were ‘identifiable’ and were successful providing they applied this condition consistently.

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MATHEMATICS

Paper 9709/53
Probability & Statistics 1

Key messages

Candidates are to be reminded of the need to read the question carefully to ensure all parts have been addressed in their answer and to show all necessary working clearly as no marks are given for unsupported answers from a calculator.

Where a diagram is requested, candidates would be well advised to be precise in its execution. It should be clearly and accurately drawn and appropriately labelled.

Candidates should state only non-exact answers correct to 3 significant figures; exact answers should be stated exactly. To justify a final answer correct to 3 significant figures, working values correct to at least 4 significant figures should be used in candidates' calculations throughout. There is no requirement for fractions to be converted to decimals.

The interpretation of success criteria is an essential skill for this component. Candidates would be well advised to include this within their preparation.

General comments

Most candidates used the answer spaces effectively.

Where there is more than one attempt at a question, care should be taken to identify the intended solution.

The use of supporting diagrams, sketches and tables were often a feature of correct solutions, helping to organise the information efficiently.

It was encouraging that many candidates used a ruler to construct box-and-whisker plots and scales that enabled the five key-values to be plotted accurately. Candidates should be aware that the axes should be labelled with both the variable and the units.

Sufficient time seems to have been available for most candidates to attempt all the paper. A few candidates did not appear to have prepared well for some topics.

Comments on specific questions

Question 1

A very accessible opening question. Most candidates performed well on both parts of the question.

(a) Candidates who recognised this as a one-mark question realised that the solution would not involve a complex method and most established an appropriate method to obtain the correct probability.

(b) Most candidates recognised this as a problem requiring the binomial distribution, and they were able to produce an expansion leading to the correct answer to the required 3 significant figure accuracy. The most common approach was to add the probability of 9, 10 and 11 candidates and then subtract this value from 1. It was pleasing to see that most candidates used brackets appropriately to show this method clearly. Those who opted for the longer calculation of adding all the probabilities of fewer than 9 candidates' travel were infrequent.

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Question 2

This question was answered well by candidates. The most common error was to not identify the three required scenarios in their working when finding the final answer.

Some weaker candidates added the correct combinations rather than multiplying them. Only a very few candidates used permutations rather than combinations. A few candidates used the longer subtraction method and although usually successful in the calculation, again some failed to list the scenarios required.

Question 3

(a) Most successful candidates used the method involving finding the arrangements with D's at the ends and then subtracting the arrangements with D's at the end and the F's together obtaining $\frac{7!}{2!} - 6!$. A common mistake in this method was subtracting an incorrect value from $\frac{7!}{2!}$. Use of the method involving placing the D's at the end and inserting the F's not next to each was less common but still with many candidates having full or partial success in obtaining the final answer.

(b) This proved to be the most challenging question on the paper for most candidates. However, a significant number did identify the correct number of arrangements. Unfortunately, many of these candidates then left the question at that point missing that they were required to find a probability and not the number of arrangements. Those that did go on and find the probability tended to obtain the correct answer. Some candidates made an error with the number of arrangements but used the correct expression or value as the denominator (total number of arrangements) of their probability.

Question 4

(a) This question was answered well by most candidates with all candidates attempting to give their answers in a table. Many candidates used a sample space diagram to find the possible outcomes and then transferred their findings to a table with the correct X-values and corresponding probabilities given in fraction form. A few candidates also used exact decimal values which usually worked well. The most common error was to add the scores from the 2 spinners and therefore achieve X-values from 0 to 7.

(b) This question was answered well by many candidates but proved more challenging for weaker candidates. Successful candidates wrote down complete calculations for both $E(X)$ and $E(X^2)$ and then used their results to find $Var(X)$. Most candidates could use their probability distribution table to calculate $E(X)$ although a few failed to write down a complete calculation. $Var(X)$ proved more difficult for some candidates but those who used their table to write down a complete calculation were usually successful. Errors included subtracting $E(X)$ rather than $(E(X))^2$ or squaring the probability rather than the X-value. A few thought that $E(X^2)$ is the variance or gave the standard deviation rather than the variance.

(c) This question was answered well by more able candidates, but some weaker candidates struggled to understand what was required. Most successful candidates used $1 - \left(\frac{3}{4}\right)^7$ although some candidates did not accurately interpret the success criteria 'before the 8th spin' and used power of 8 rather than 7. A common error was to find $1 - \frac{1}{4}\left(\frac{3}{4}\right)^7$ or just $\frac{1}{4}\left(\frac{3}{4}\right)^7$. Many candidates used the long method of summing the probabilities for scoring a 4 on the 1st to 7th trials and although many were successful, several candidates failed to sum their values accurately.

Question 5

(a) Overall, this question was well done provided the candidate understood the principles of a tree diagram. The most common mistake was not considering this as a three-stage problem, stopping the tree diagram after the first two stages (choosing a counter and one marble). Another less common error was to not apply the condition 'without replacement' stated in the question. Some

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candidates condensed the tree diagram by combining the two marble choices into a single probability which again for some led to confusion in answering **5(b)** and **5(c)**.

(b) In this question, most candidates completed the question correctly but did not identify which outcomes were being used either by clearly marking the diagram or writing them out. This was similar to the error of not identifying scenarios seen in **Question 2**. Candidates should be encouraged to communicate their thinking clearly by identifying which scenarios are being used to construct their solution if they want to gain all available marks.

(c) Candidates who understood the concept of conditional probability did this question very well, correctly using their answers from **5(b)** although some started again rather than realising they could use calculations from **5(b)**.

Question 6

(a) This question was attempted well by many candidates although some had missing components or a poor layout of their work. Almost all candidates provided a correct stem although a few candidates gave an 'upside down' stem or repeated the values (a method only recommended where there are large amounts of data). Most candidates followed the instruction 'with Linnets on the left-hand side' although some failed to include the team headings on their diagram. Almost all candidates entered the data for Linnets appropriately from right to left. Common mistakes on this question included the non-alignment of data vertically (particularly where an error was made and corrected by crossing out with the replacement to the left of the error, use of commas between values, or in a few cases, missing or incorrect data values.

Most candidates attempted to include a key with their stem and leaf diagram but in many cases failed to include 'minutes' in their definition stating only a team name with a numerical value. A few candidates did not use a single key.

(b) This question was answered well by the majority of candidates. Most candidates could identify the upper and lower quartiles and subtract correctly although 65 – 54 = 9 was a surprisingly frequent error.

(c) Most candidates found this question challenging although the majority of candidates did produce two labelled box-and-whisker plots. The most common error was the failure to label the scale and those who did label, frequently mentioned only 'time' or 'minutes' rather than both. A few candidates used scales other than the obvious 2 cm:10 or 20 minutes with a common scale being 2 cm: 6 minutes which allowed the full range of values (45 to 75) to be plotted but which caused difficulties in plotting the 5 key values accurately. The best answers produced two labelled box plots drawn using a ruler and a sharp pencil although unlabelled freehand and pen drawings were not uncommon. Some candidates produced only one box plot, frequently unlabelled.

(d) Candidates found this question challenging. Many comments merely compared the medians, ranges or inter-quartile ranges (and in a few cases, the means) of the two sets of data without interpretation within the context of the question. Good answers were usually concise, such as 'Puffins are faster (on average)' or 'Linnets times were more consistent (than Puffins times)'. Many candidates failed to interpret that the higher median meant that Linnets were the slower team on average. The question only asked for one comparison between the teams, but many candidates tried to produce two comparisons with many generic answers seen referring only to the data rather than specifically to the spread of times or speed for the teams.

Question 7

(a) This question was done very well. Almost all candidates established the correct standardisation formula and obtained the probability value of 0.7029. However, a minority of candidates did not subtract their value from 1 to obtain the probability area required by the success criteria. Those candidates may have been more able to see the probability required if they had sketched the distribution diagram as an aid.

(b) Candidates lost marks in this question for not using the correct z-value of 1.282, failing to recognise this as the critical value required. However, many did use the value of 1.281 and were then able to use their z-value with the correct standardisation formula. Some candidates did not give the correct

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probability area and candidates would be advised to make use of a diagram to help with this decision.

(c) This question was done very well. Some candidates did not show a complete method, for example missing out the standardisation formula or not fully showing the steps to working out the probability. Several good candidates who obtained the correct probability did not write the number of days as an exact integer, but, instead, often gave an approximation.

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MATHEMATICS

Paper 9709/55
Probability & Statistics 1

Key messages

Candidates should be reminded that Statistics is about interpreting data and describing the information to the reader. When drawing diagrams, as in **Question 4**, candidates should use a pencil and ruler, label the diagram and give appropriate units. In **Questions 3(c)** and **5**, it is important to include all the relevant steps including the mathematical operations necessary to reach their final solutions.

General comments

Candidates are reminded to be careful when rounding a number to 3 significant figures, especially when the first figure after the decimal point is a zero. When using working values to support further processes these should not be rounded prematurely. To justify a final answer correct to 3 significant figures working values correct to at least 4 significant figures should be used in calculations. There is no requirement to convert fractions into decimals.

Although we welcome the use of calculators in this paper, we do need to see the working written down as well as the answer, especially when a standardisation expression is involved. It is also a requirement of the syllabus that candidates are familiar with statistical tables and that they remember to use the critical values for the normal distribution.

Comments on specific questions

Question 1

(a) Almost all candidates wrote down the correct product, appreciating that the first six throws each had a probability of $\frac{5}{6}$ before the seventh throw of a 6. A number of candidates counted the zero in the tenths column as significant and gave the incorrectly rounded answer 0.056.

(b) The most common and successful approach was to continue the method of **1(a)** and to add the probabilities of X = 3, 4 and 5. The alternative correct approach where they subtract $P(X \leqslant 2)$ from $P(X \leqslant 5)$ usually appeared as $(1-(\frac{5}{6})^5) - (1-(\frac{5}{6})^2)$. However, a significant number applied this method incorrectly and subtracted $P(X \leqslant 3)$ instead of $P(X \leqslant 2)$.

Question 2

(a) This question was answered well with most candidates knowing that the four probabilities summed to 1 and quickly finding the numerical value of p (0.15) before they went on to substitute into the formulae for E(X) and Var(X). A few forgot to square the value of E(X) when they subtracted from E(X^2) and others incorrectly divided the formulae by 4. Some chose to work in algebra and found algebraic expressions for E(X) and Var(X) before they substituted the numerical value of p.

(b) Most candidates recognised this as a question on the binomial distribution with a probability of 0.4 and the question was answered well. Very few chose the longer and less efficient method of adding the probabilities of $X < 8$ and those who did often forgot to include zero. Some omitted the binomial coefficients while others assumed that each of the four numbers was equally likely and used the

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probability 0.25. Notation has improved with only a few candidates either omitting brackets round the terms to be subtracted or not putting minus signs in front of each of the three terms.

A few candidates confused this with a geometric distribution, and a commonly seen incorrect response was: $1 - 0.6^7 0.4 - 0.6^8 0.4 - 0.6^9 0.4$.

Question 3

(a) The structure of the tree diagram was generally well displayed. The majority realised that three stages were needed leading to 8 final branches. Mistakes in structure came with some candidates using only two events: tossing a coin and then picking from one bag or just picking from two bags. Some did not separate the events of picking from one bag and then the other and presented a two-stage tree diagram – H or T followed by RR, RB or BR and BB. One of the most common mistakes was in labelling the first set of branches *X* and *Y* instead of H and T, not realising that these labels did not describe the outcomes of the event. Other mistakes occurred in the probabilities when the with/without replacement element was confused or alternatively giving values with a denominator of 13 when selection from the bags was double counted.

(b) Almost all understood that they needed to calculate P(HBB) + P(TBB) and some candidates with an incorrect tree diagram were able to start from scratch and produce the necessary calculation. A few forgot about the toss of the coin and omitted the factor of $\frac{1}{2}$ in their calculation.

(c) This proved to be one of the most challenging parts of the paper. Many misunderstood the question and found the probability of selecting two discs from bag *X* with at least one of them red (0.4) as their final answer. Of those who did understand what was required, only a minority used their answer to the previous part (probability of 2 blues) and subtracted it from 1 to find the probability of at least one red which was the required denominator. Most produced a sum of up to 6 products without describing what they were trying to find. Strong candidates explained that they were dividing the probability of at least one red from bag *X* by the probability of at least one red and found the most succinct way of producing this calculation: (P(H) – P(HBB)) divided by (1 – their answer to **3(b)**).

Question 4

(a) This question was answered well with most following the instruction to put Pelicans on the left-hand side. Some candidates struggled with misalignment of the values and would be advised to use a ruler to support with alignment. Most diagrams were correctly labelled with Pelicans and Swans and almost all remembered to include a key although a significant number missed out the units (cm).

(b) This was very well answered with almost all understanding what was required. It is expected for candidates to show the value of the median identified and, for the interquartile range, the subtraction of the of upper and lower quartile.

(c) This question was less well answered. Strong candidates used a sensible scale, usually one small square representing 1 cm, knew to label their scaled line with both 'height' and 'cm' and plotted two accurate Box Plots clearly labelled Pelicans and Swans. As in the previous question, use of a ruler and sharp pencil would have improved the presentation and accuracy of many diagrams. Many did not label their scaled line, and a commonly seen incorrect response was a single unlabelled Box Plot.

(d) Although this style of question is common, candidates are not always giving a comment in the context of the question and merely comparing statistical measures such as medians, interquartile ranges or ranges. They also need to be aware that when considering consistency, comparing interquartile ranges takes priority over comparing ranges. Those candidates who made their comment having compared the medians, were generally more successful as long as they made their comment in the context of height.

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Question 5

(a) This question was very well answered by the majority of candidates with almost all appreciating that they needed to describe the six correct scenarios (6S, 5S 1H, 5S 1T, 4S 2H, 4S 1H 1T, 3S 2H 1T) as well as show the products of combinations that they used to find the number of ways each time. Common errors came from the omission or addition of scenarios. Extra scenarios were occasionally produced when the number of 'throwers' allowed was exceeded. Omissions were usually as a result of missing one of the '4S' situations. That said, 3S 2H 1T was routinely accounted for. On correct identification of the scenarios, weaker candidates sometimes produced a sum of combinations rather than a product and a few stopped at describing the scenarios and gave 6 as their answer.

(b) This question was fairly well answered with most opting for the first method of subtracting the number of ways where the 3 hurdlers are together from the number of ways the 8 athletes could be arranged. A significant number of these responses did not account for the fact that the 3 hurdlers could be arranged in 3! ways and only subtracted 6! from 8!. Those who opted for the second method, needed to sum the number of ways of arranging the athletes with the hurdlers all separated and the number of ways with two of them together and one separate. Many who chose this method often only considered the 'all separate' situation. A common error was to treat the athletes as if they were identical: $\frac{8!}{3!4!} - \frac{6!}{4!} = 250$.

A few candidates misunderstood that the 8 athletes consisting of 1 sprinter, 3 hurdlers and 4 throwers had already been chosen and added an extra complication to their response by multiplying the correct answer by the number of ways 8 such athletes could be chosen from the 25.

(c) This was a complicated question, and many candidates found it confusing. The most straightforward way was to consider the four possible positions of the hurdlers:

1. H - - H - - H -
2. H - - - H - - H
3. H - - H - - - H
4. - H - - H - - H

The Hurdlers can be arranged in 3! ways for each arrangement and the other athletes can be arranged in 5! ways. Strong candidates quickly arrived at the solution 5! × 3! × 4. Many others attempted to use the method from the previous question of subtracting from 8!, the total number of ways of arranging the athletes. This was a very challenging way to approach this question and very few made any significant progress towards the correct final answer. A common error seen was candidates only considering two hurdlers when they still needed to consider all three.

Question 6

(a) This question was answered well with most candidates scoring full marks. Only a few chose the incorrect probability area.

(b) It is a requirement of the syllabus that candidates are able to use the tables provided and, where appropriate, that they use the 'Critical values for the normal distribution'. This question involved a probability of 0.9 and we needed to see the z-value 1.282. Most candidates knew to standardise and equate their standardisation to a z-value with only a few using the tables the wrong way round and equating to a probability.

(c) Most candidates understood the question and realised that the value of the standard deviation found in **6(b)** was not applicable in this part of the question. This part concerns 150 runners as opposed to 'a large number' and the number of runners who took more than 48.6 seconds is binomially distributed with n = 150 and p = 0.1. They understood that a normal approximation with those values could be applied and usually completed the question well, with very few forgetting to apply the continuity correction.

Most of those who misunderstood the question tried to work with the standard deviation found in **6(b)** and produced some very muddled working.

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MATHEMATICS

Paper 9709/61
Probability and Statistics 2

There were too few candidates for a meaningful report to be produced.

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MATHEMATICS

Paper 9709/62
Probability & Statistics 2

Key messages

- Candidates should be encouraged to read questions carefully and extract the relevant information.
- Candidates should be encouraged to reread the question when they have completed their solution to ensure that all the requirements have been fulfilled.
- Candidates should be reminded that in all questions, sufficient method must be shown to justify answers.
- When the question requires an answer to be shown, it is particularly important that all relevant working is presented with no steps in the solution omitted.
- Candidates need to keep to at least 4 significant figures accuracy during working in order to ensure a final answer is correct to 3 significant figures.
- If questions are continued on additional pages, the question number must be clearly identified.
- Clear presentation of work is of vital importance; in particular digits must be clear and unambiguous.

General comments

This was a reasonably well attempted paper. There were some very good and well-presented scripts, but there were also some candidates who were not fully prepared for the demands of the paper. Questions that were well attempted were **Questions 1(b)**, **1(c)** and **5(a)** whilst **Questions 1(a)**, **3(b)**, **5(b)(ii)** and **6(c)** were found to be more demanding. On the whole adequate working was shown, but there were occasions where candidates did not fully justify their answers. There did not appear to be any issues with timing.

It should be noted that the conclusion to a hypothesis test must not be written in definite terms (i.e. a phrase such as 'There is sufficient evidence to suggest….' should be used rather than 'The test proves that ….') Comments on individual questions follow, but it should be noted that there were some good fully correct solutions offered as well as the common errors highlighted below.

Comments on specific questions

Question 1

(a) It was important here that solutions were given in the context of the question (i.e. that computers were donated independently or singly or randomly). Answers given in general terms were often seen; merely saying that the events occurred independently or singly or randomly with no indication of what the 'events' were, was not sufficient. Some candidates said that computers must be donated at a constant average rate, but this had already been stated in the question.

(b) This was a particularly well attempted question. Most candidates used the correct value for λ and gave the correct Poisson expression. Candidates should note that the Poisson expression must be clearly shown to justify their final answer.

(c) Again, this part was well answered. Common errors included a lack of, or an incorrect, continuity correction and finding the wrong probability area, i.e. >0.5 when <0.5 was required.

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Question 2

(a) In general, this part was well attempted, though there was some confusion seen between the variance and standard deviation of Y. Some candidates incorrectly found the standard deviation of $X + Y$ by adding the standard deviations of X and Y together rather than the correct method of adding the variances of X and Y together then taking the square root to find the standard deviation of $X + Y$. There were also candidates who left their answer as $\text{Var}(X + Y)$; candidates are always advised to re-read the question after completing their solution.

(b) There was a similar confusion here between variance and standard deviation, and the same method error when calculating the standard deviation was noted. When finding $\text{var}(5X)$ many candidates calculated 5×3^2 rather than $5^2 \times 3^2$. There was evidence in both parts of this question of candidates not showing what their calculated values represented, making their solutions difficult to follow.

Question 3

(a) This was reasonably well attempted. Candidates either found the width of the confidence interval or found the sample mean then used a suitable formula to find σ. Errors included use of an incorrect z value in their formula, or a factor of 2 error in the formula for the width. Longer methods involving simultaneous equations (rather than calculating the sample mean) were also seen.

(b) This was not well attempted with many candidates just stating that the population distribution was unknown, i.e. considering why it was used rather than why it was valid to be used. A statement saying that the sample size was large was required.

(c) This was a particularly poorly answered question. Whilst there were some correct answers seen, the majority of candidates were unclear on how to approach the question.

Question 4

(a) Many candidates did not state a necessary assumption as requested in the question. Candidates are advised to read the question carefully. Those who did state an assumption did not always relate it to the question. Hypotheses were mostly stated correctly. It was important that the Binomial expression for $P(\leqslant 3)$ was clearly stated; errors included calculating $P(X < 3)$ or $P(X = 3)$ and using an incorrect value for p. A valid comparison should be clearly stated, followed by a conclusion in context and non-definite. Some candidates were too definitive in their conclusion; use of the word 'prove' is not advisable.

(b) Some candidates correctly stated that $np = 7.2$ which is greater than 5 (or equivalent statement using $n = 40$ or $p = 0.18$). Some candidates did not relate their answer to the question merely saying $np > 5$ (or $n < 50$ or $p > 0.1$) without justifying this with the values in this question.

Question 5

(a) This part was particularly well answered. As this was a question where the answer was given, it was important that all working out was clearly shown. It was pleasing to note that most candidates did this, though it would have be preferable here to include the fact that $\int f(x) = 1$ clearly in the solution and at an early stage; some candidates stated '= 1' very late in their solution.

(b) (i) Many candidates correctly stated that the probability was 0.5, however some candidates thought they needed to find the value for m rather than just state the required probability.

(ii) Many candidates successfully found the value of $E(X)$, but then went on to try and find the value of m (or incorrectly thought that m was 0.5 or 1). It was not necessary to find m as the required probability could be found by finding $P(X < E(X))$ then calculating 0.5 minus this probability. Few candidates attempted this method.

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Question 6

(a) As in **Question 4(a)**, the required assumption was not always stated. Some candidates carried out the test well, but errors included standard deviation/variance errors and omission of $\sqrt{35}$ when standardising and accuracy errors. Incorrect comparison with *z* values of 2.326 or 2.54 or 2.05 rather than 2.054 were seen and conclusions often lacked the required level of uncertainty.

(b) Some candidates gave the correct answer of 0.02, but many gave the answer 0.0062 found in **6(a)** or defined a Type I error.

(c) This part was not well attempted with many candidates unclear of the method to use. Many candidates did not find the critical value, and errors when standardising such as standard deviation/variance errors and omission of $\sqrt{35}$ were often seen as well as incorrect probability areas (> 0,5 rather than < 0.5).

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MATHEMATICS

Paper 9709/63
Probability & Statistics 2

Key messages

Throughout the paper it was necessary to show complete methods and full working.

When asked to use an approximating distribution candidates should choose carefully between Poisson, binomial and normal distributions.

General comments

For questions involving significance tests the conclusion does need to follow certain guidelines. In particular it should not be expressed in definite form. Many candidates do use the recommended wording such as 'there is evidence to suggest that … '.

In this paper because of the two tail nature of the hypotheses in **Question 2** this type of wording for the conclusion was more likely to be of the form 'there was insufficient evidence that …..'

Comments on specific questions

Question 1

(a) Using the relevant Poisson distribution $P_o(3)$ the required probabilities were P(X = 3 and X = 4). Most candidates found these correctly and wrote down the terms. A few candidates included an extra unwanted term such as P(X = 5).

(b) For the new variable $X + Y$ the parameter was 5. To find P($X + Y > 2$) the appropriate method was to find the probability 1 – P(0, 1, 2). Again, it was necessary to write down the terms, which most candidates did correctly. A few candidates tried a longer method using the original distributions and combinations, but relevant terms were often omitted.

(c) For the 100 values of X and the 150 values of Y the combined Poisson parameter was 600. Hence a suitable approximating distribution was the normal N(600, 600) and to use the continuity correction factor leading to 599.5 for the standardisation. Many candidates did this correctly. Some candidates omitted the factor or used an incorrect factor.

Question 2

This significance test question required candidates to follow all of the required steps, namely finding unbiased estimates, standardising, comparing to the 5 per cent level and finding a conclusion. Many candidates did most of this work clearly and correctly. The unbiased estimates were 499.5 and 4.83 or $\frac{285}{59}$.

It was essential to include $\sqrt{60}$ in the standardisation. The standardised value (1.762) could then be compared to the critical value for the two-tailed 5 per cent test (1.96) or each of these could be changed to the probability/area values (0.0390 > 0.025). It was necessary to write down these comparisons and many candidates did so. Finally, a conclusion could be written down which was in context, not definite and had no contradictions. Wording such as 'there was insufficient evidence that …' was to be recommended.

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Question 3

(a) For this large value of *n* and this small value of *p* the mean *np* was equal to 1.45. Thus, a suitable approximating distribution was P_o(1.45). Then P(X < 4) required P(0,1,2,3). Many candidates found this probability correctly. A few candidates included the extra term P(X = 4) or omitted the first term.

(b) The new distribution was P_o(14500p) and this was to be set equal to $2 \times e^{-1.45}$. The resulting equation could then be solved. Some candidates had the 2 on the wrong side of the equation. Other candidates made errors when taking ln's of each side.

Question 4

(a) The general formula for the width of a confidence interval could be quoted or deduced. For the 90 percent confidence interval the critical value was 1.645. When this width was multiplied by 1.414 the critical value for the unknown confidence interval could be found to be 2.326 leading to a percentage of 99 percent. From this it could be deduced that the size of the new confidence interval was to be 98 per cent. Some candidates incorrectly thought that the 99 per cent was the new confidence interval. Other candidates multiplied the wrong side of the equation by the 1.414.

(b) The conditional probability was required here. This reduced to finding $\frac{90}{98}$ with a possible follow through allowed.

Question 5

(a) In order to find P (Type I error) it was necessary to find the probabilities of the sums of the early sets of terms in the binomial distribution B (40, 0.2) and to compare these to 2.5 percent. These calculations showed that P (0,1,2) < 0.025 and that P (0,1,2,3) > 0.025. Thus, the significant values of X were 0,1,2. All of these calculations needed to be written down to gain full marks.

(b) Candidates could use the calculated results from **5(a)** to state the rejection region for the test.

(c) For this question concerning the possibility or otherwise of a Type II error the explanation could refer to the previous work already done in **5(a)** and **5(b)**.

Question 6

(a) The normal distribution parameters for the sum of a large bag and a small bag of potatoes were required. These were N(3.3, 0.07). These were then to be used in the standardisation and in the selection of the correct area and probability. If required a sketch could be useful.

(b) To find the required probability that L < 3S it was necessary to consider the new variable L – 3S. This variable followed the normal distribution N(0.1, 0.23). This was then to be used in the standardisation and in the selection of the correct area and probability. If required a sketch could be useful.

Question 7

(a) To find the probability that a candidate would take longer than 4.5 minutes it was necessary to integrate the given probability density function between the limits 4.5 and 5. Many candidates did this clearly and correctly.

Instead of this some candidates incorrectly found the mean and variance and then attempted to use a normal distribution.

(b) Because of the symmetry of the given function it was possible to write down the value of the median without calculation. A few candidates wrongly suggested the answer as 0.5.

(c) The question required this part to be found without further integration.

Because of the symmetry of the given function, it was possible to write down the value of P(3 < × < 3.5) as also being 0.156(25) and hence the required probability as 1 – 2 × 0.156(25).

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MATHEMATICS

Paper 9709/65
Probability & Statistics 2

Key messages

- In all questions, sufficient method must be shown to justify answers.
- It is important that candidates read the question carefully and refer to it when they have completed the question to ensure they have answered it in full.
- Candidates are strongly advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.
- For answers that are required 'in context', quoting general textbook statements will not be sufficient.
- All working should be done in the correct question space of the answer booklet. If answers need to be continued on the Additional page, it must be clearly labelled with the correct question number.
- Candidates should make corrections by crossing through and replacing the work, not by over-writing their answer.
- Only one solution should be offered.

General comments

The paper reflected a generally competent effort, with candidates showing a sound understanding of statistical methods and their practical use. Candidates performed better in **Questions 1a**, **4(b)(ii)**, **5**, and **6(a)**. They found **Questions 1(b)** and **4(b)(i)** more challenging.

In general, the working was laid down neatly, and the explanations were clear throughout.
Candidates are reminded that the conclusions of a test should be in context and should not be definite.

Comments on specific questions

Question 1

(a) This part question was answered well by most candidates. A common error was to use an incorrect value for z (2.54 instead of 2.054).

(b) Some candidates found this part question challenging. Stronger candidates realised that a small sample size made it impossible to apply the Central Limit Theorem and explained this in a clear and concise way.

Question 2

(a) This part question was answered correctly by most candidates, who explained that the researcher was looking for evidence of 'different from' or, conversely, he was not looking for 'less than' or 'more than'.

(b) This part question was answered correctly by most candidates. A common error was the use of the variable μ instead of p to describe the proportion of the population in the whole country.

(c) This part question proved more challenging than the two previous parts for many candidates. Some of them compared 1.82 with 1.645 instead of 1.96 (or, conversely 0.0344 with 0.05). Another common error was to write a conclusion that was either definite, or not in context.

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Question 3

(a) This part question was well attempted by most candidates, who correctly wrote the hypotheses (either using the average hits per hour, or per 10 minutes), showed all the calculations in full, performed a suitable comparison, and finally drew the appropriate conclusions in context. A common error was to consider $P(X > 6)$ instead of $P(X \geqslant 6)$ in carrying out the test.

(b) This question was generally answered well by most candidates, who realised that they had to consider the conclusion of their test and the definition of Type I and Type II errors. Some answers were very detailed and showed mastery of the statistical terms. A common error was to consider only Type I or Type II, but not both, in the answer.

Question 4

(a) (i) Most candidates answered this question correctly, showing their working well. The most common error was to consider $X \sim N(4.5, 4.5)$ or $X \sim N(4.5, 4.492)$ instead of $X \sim Po(4.5)$.

(ii) To answer this part question candidates had to write down the conditions that justified the use of $Po(4.5)$, i.e. $2430 > 50$ and either $4.5 < 5$ or $0.00185 < 0.01$. Weaker candidates wrote only one condition or wrote generic statements like $n > 50$, $np < 5$ without stating what $\boldsymbol{n}$ and $\boldsymbol{np}$ were equal to.

(b) (i) Many candidates found this question challenging. The best answers stated the condition in context, e.g. they mentioned X or referred to the number of copies sold, or the magazine sales.

(ii) This part question was generally well attempted, with most candidates scoring full marks. They often showed all their working clearly and demonstrated a good understanding of the topic. A few candidates did not calculate an area that was consistent with their working.

Question 5

(a) This part question was well attempted by most candidates, who correctly identified the distribution and its parameters, performed the standardisation and often obtained the correct answer. As in **Question 4(b)(ii)**, a few candidates did not calculate an area that was consistent with their working.

(b) This part question too was well attempted by most candidates, who described the distribution they were considering, e.g. $L + S \sim \mathrm{N}\left(4.3 \times (2.10 + 1.51), 4.3^2 \times (0.12 + 0.09)\right)$ and carried out the calculations flawlessly. The most common error was to consider the variance of the total price of the tomatoes to be equal to $4.3 \times (0.12 + 0.09)$.

Question 6

(a) Most candidates answered this question correctly, often in a very compact and elegant way. A few candidates worked out the equation of the straight line going through $(0, a)$ and $(b, 0)$ and used integral calculus to obtain an answer, which was not always correct.

(b) To answer this questions the candidates used a variety of approaches, often with good success. Some of them expressed $f(x)$ in terms of a only $\left(f(x) = a - \frac{a^2}{2}x\right)$, set the integral of $xf(x)$ (between 0 and $\frac{2}{a}$) equal to $\frac{4}{9}$, and correctly solved the resulting equation in a. Another popular approach was to express $f(x)$ in terms of b only $\left(f(x) = \frac{2}{b} - \frac{2}{b^2}x\right)$, integrate $xf(x)$ between 0 and b, set the integral equal to $\frac{4}{9}$, substitute $b = \frac{2}{a}$ and solve the resulting equation in a. Another

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approach was to integrate $xf(x) = ax - \frac{a}{b}x^2$ between 0 and either $\frac{2}{a}$ or b, set the resulting expression in a and b equal to $\frac{4}{9}$, substitute $b = \frac{2}{a}$ and solve the resulting equation in a.

(c) This last part question was well attempted by many candidates, who showed good algebraic manipulative skills. A common error was to correctly exclude the solution $k = 2$ but without justification.

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