

Parallel teaching of A Level Mathematics and Further Mathematics

Cambridge International AS & A Level Mathematics 9709 and Further Mathematics 9231

For examination from 2020



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About this document

The syllabus and schemes of work for Cambridge International AS & A Level Mathematics 9709 and Further Mathematics 9231 help you to teach each course in isolation (one after the other).

This document provides two possible routes to teach the **A Level** of these courses alongside each other over a two-year period. (The AS Level of each course is not covered explicitly here.)

We suggest you use the schemes of work available for each course alongside this document to facilitate planning.

The suggested co-teaching routes are based on the following assumptions:

- The A Level courses will be taught by two teachers, and are fully timetabled.
- Candidates will take the Paper 4 Mechanics and Paper 5 Probability & Statistics options of the **AS & A Level** Mathematics 9709 course.
- The **AS & A Level** Mathematics 9709 course is prerequisite material for the **A Level** Further Mathematics 9231 course.
- There are around 380 teaching hours for Mathematics 9709 and 345 teaching hours for Further Mathematics 9231.
- There are 30 teaching weeks in each school year; 14 hours of teaching per week are assigned to Mathematics/Further Mathematics (14 hours total for all maths).
- The school year is split into 30 weeks.
- The final six weeks of Year 2 are used only for revision and sitting examinations.

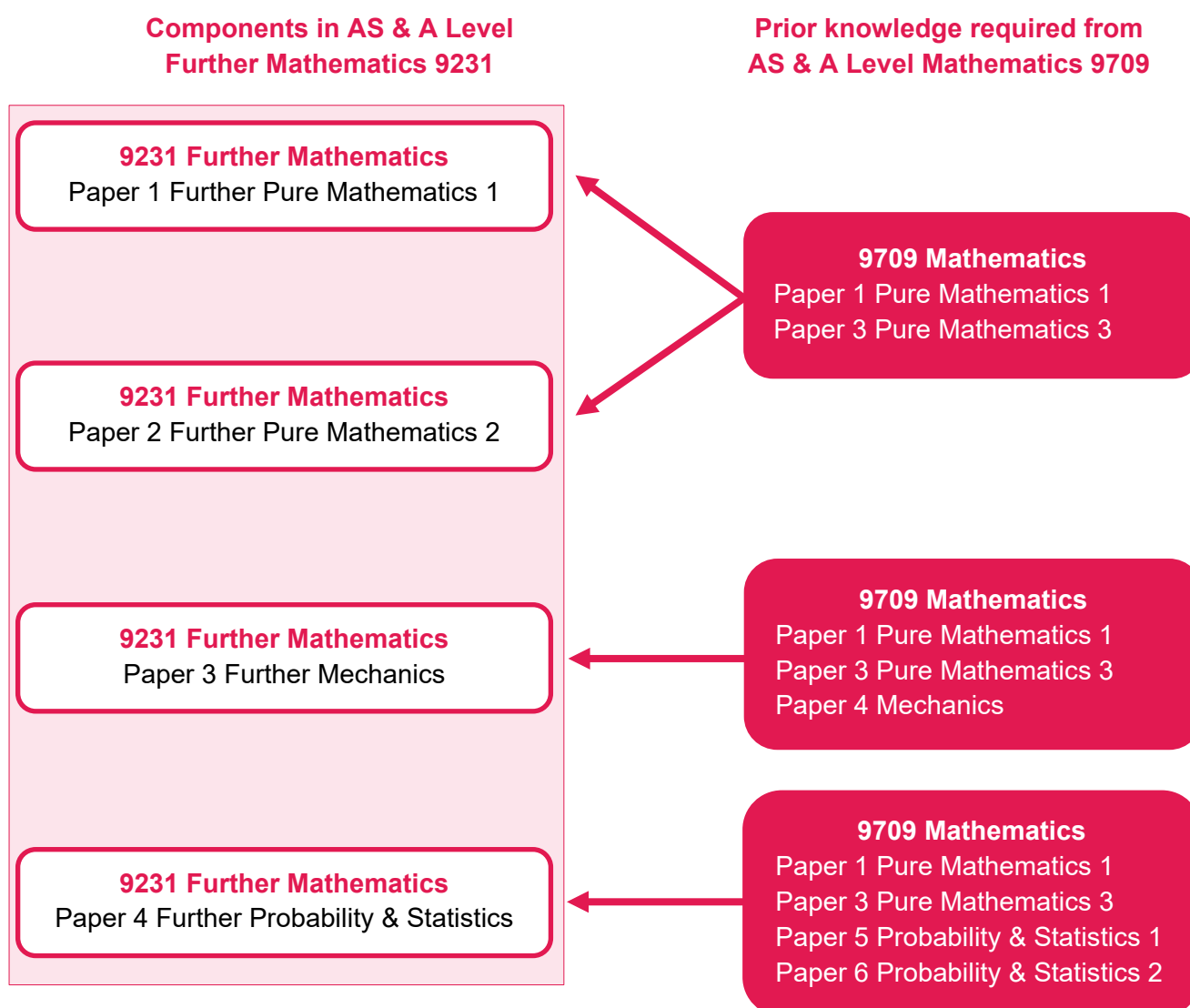
The routes provided in this document are just two possible routes of many. It is important that you plan appropriately for your centre and learners.

The suggested teaching routes provided in this document are only relevant to candidates taking examinations in **both** A Level courses; if the candidate is not taking the AS & A Level Further Mathematics 9231 course then the suggested teaching routes given here for AS & A Level Mathematics 9709 might not be appropriate.

Relationship between the A Level courses

The content of the Cambridge International **A Level** Mathematics 9709 course which is required prior knowledge for the **A Level** Further Mathematics 9231 course is listed in the 9231 syllabus.

Note that for the A Level course, candidates do not need to learn the content for 9709 Paper 2 Pure Mathematics 2.



The full content of the 9709 Mathematics course is **not** required for the **AS Level** Further Mathematics course. For the AS Level only, 9231 candidates take *either* Paper 3 Further Mechanics *or* Paper 4 Further Probability & Statistics and therefore only need to know the content from the corresponding component (mechanics or statistics) in the 9709 Mathematics course.

The full content of the 9709 Mathematics Paper 6 Probability & Statistics 2 component is not essential for the 9231 Further Mathematics Paper 4 Further Probability & Statistics component. Please see the School Support Hub www.cambridgeinternational.org/support for more information.

Which route?

Use the information below to help you identify which route might be more suitable for you.

You should also read the topic order within each route in more detail, before making your decision.

Route 1	Route 2
Both teachers cover pure, statistics and mechanics across both A Levels.	Both teachers cover pure. Teacher 1 specialises in statistics and Teacher 2 specialises in mechanics.
<i>This is only suitable if both teachers are equally confident at teaching mechanics and probability/statistics.</i>	<i>This is suitable if one teacher is more confident teaching mechanics, and the other more confident in teaching probability/statistics.</i>
Teachers and learners would be able to comfortably move between topics on pure, mechanics and probability/statistics.	Teachers and learners would be more comfortable covering statistics first alongside pure, and then moving onto teaching mechanics alongside more complex pure topics later.
The two teachers have very similar or identical teaching styles.	The two teachers can have different teaching styles.
<i>If teachers do not teach in a similar way, then learners might struggle to move confidently between pure, statistics and mechanics.</i>	<i>As statistics and mechanics are covered at completely different times in the year, it should not be too confusing for learners to cope with different teaching styles.</i>
Transport of textbooks is not an issue.	Transport of textbooks is an issue.
<i>Moving between topics from mechanics and probability/statistics might require learners to bring in multiple textbooks in one day or week. Consider if this would be an issue.</i>	<i>As learners do not need to move between topics from mechanics and probability/statistics with the same day or week, they would need to carry fewer textbooks into school each day.</i>
AS Level Further Mathematics topics (Papers 1 and 3) can be covered in the first three terms, meaning that Paper 1 and Paper 3 Further Mechanics can be taken in the November series.	Paper 1 and Paper 4 Further Probability & Statistics can be taken in the November series.
<i>This might be useful if you have AS Level learners who want to take Paper 3.</i>	<i>This might be useful if you have AS Level learners who want to take Paper 4.</i>

Both teaching routes require learners to:

- write clear notes
- work in an organised manner
- see and understand the connections within pure, mechanics and statistics.

Co-teaching Route 1

Teacher 1 and Teacher 2 both cover pure, mechanics and statistics across the two year period. Both teachers and learners will need to be very organised and able to move comfortably between pure, mechanics and probability/statistics content.

The number of hours to spend on each topic is given in brackets after each syllabus topic heading. Please be aware that this is just a very **general starting point** and the exact timing will depend on how much content you decide to cover within a lesson compared to homework; your learners' previous knowledge and ability; available resources; class size and so on.

In some weeks, a given teacher might be covering one topic from each course in the same week. Note that these could be taught one after the other so that learners are not coping with multiple different topics at the same time (and juggling notes and textbooks).

Learners will need clear guidance as to what is on the syllabus for each of 9709 and 9231, especially initially as past papers (old syllabus) will be misleading.

Through this route, AS Level Further Mathematics candidates would take 9231 Papers 1 and 3 (Further Pure Mathematics 1 and Further Mechanics). Therefore 3.9 (Complex Numbers) and content exclusively for Papers 5 & 6 of the AS & A Level Mathematics 9709 course are not needed for the **AS Level**.

The route

The route is laid out on the pages that follow.

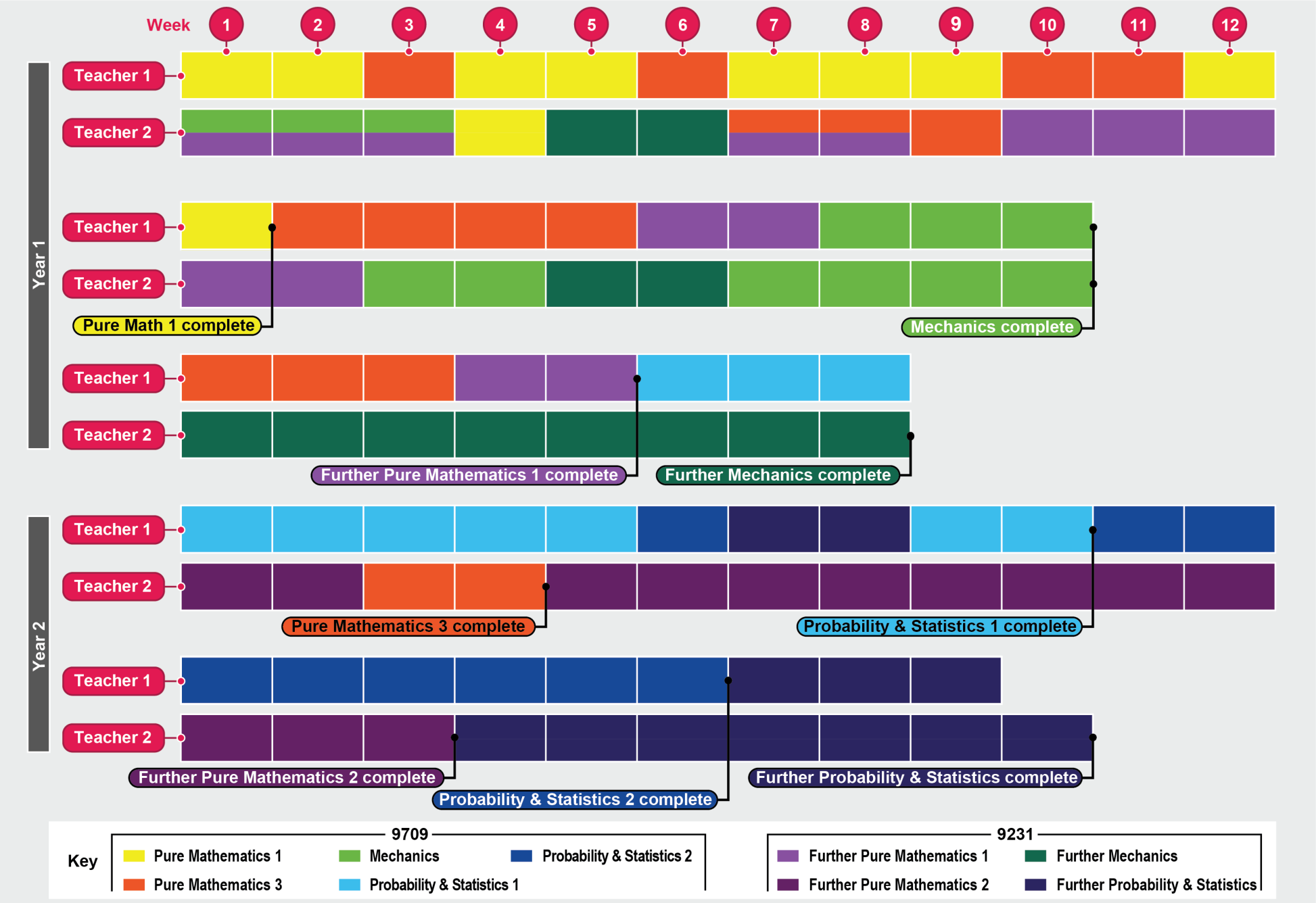


This diagram gives a summary of the spread of topics across the two-year period.

Route 1 Year 1: Weeks 1–12

Teacher 1		Teacher 2	
9709	9231	9709	9231
1 1.1 Quadratics (7 hours)		1 4.1 Forces and equilibrium (11 hours)	1.1 Roots of polynomial equations (10 hours)
2 1.5 Trigonometry (7 hours)		2	
3 3.3 Trigonometry (7 hours) (Note that some learners might find this challenging this early on.)		3	
4 1.3 Coordinate Geometry (7 hours)		4 1.4 Circular measure (7 hours)	
5 1.6 Series (7 hours)		5	3.2 Equilibrium of a rigid body (14 hours)
6 3.1 Algebra (7 hours)		6	
7		7 3.6 Numerical solution of equations (4 hours)	1.3 Summation of series (10 hours)
8 1.7 Differentiation (14 hours)		8	
9		9 3.2 Logarithmic and exponential functions (7 hours)	
10 3.4 Differentiation (14 hours)		10	1.4 Matrices (14 hours)
11 1.8 Integration (7 hours, continues in Term 2)		11	
12		12	1.2 Rational functions and graphs (7 hours)

The tables provide the detail of which topics are taught in which weeks, and by which teacher.



Route 1 Year 1: Weeks 1 – 12

Teacher 1			Teacher 2		
9709		9231	9709		9231
1	1.1 Quadratics (7 hours)		1	4.1 Forces and equilibrium (11 hours)	1.1 Roots of polynomial equations (10 hours)
2	1.5 Trigonometry (7 hours)		2		
3	3.3 Trigonometry (7 hours) <i>(Note that some learners might find this challenging this early on.)</i>		3		
4	1.3 Coordinate Geometry (7 hours)		4	1.4 Circular measure (7 hours)	
5	1.6 Series (7 hours)		5		3.2 Equilibrium of a rigid body (14 hours)
6	3.1 Algebra (7 hours)		6		
7	1.7 Differentiation (14 hours)		7	3.6 Numerical solution of equations (4 hours)	1.3 Summation of series (10 hours)
8			8	<i>(Note, you might prefer to teach 3.6 then 1.3 or vice versa, rather than running these two topics side by side.)</i>	
9	1.2 Functions (7 hours)		9	3.2 Logarithmic and exponential functions (7 hours)	
10	3.4 Differentiation (14 hours)		10		1.4 Matrices (14 hours)
11	<i>(Note that some learners might find this challenging this early on.)</i>		11		
12	1.8 Integration (7 hours, continues in Term 2)		12		1.2 Rational functions and graphs (7 hours)

Route 1 Year 1: Weeks 13 – 22

Teacher 1			Teacher 2		
9709		9231	9709		9231
1	1.8 Integration (7 hours, continued from Term 1)		1		1.7 Proof by induction (14 hours)
2	3.5 Integration (14 hours)		2		
3	<i>(Note that some learners might find this challenging this early on.)</i>		3	4.2 Kinematics of motion in a straight line (14 hours)	
4			4	<i>(This requires 1.7 and 1.8, so if required, you could move 3.4 and 3.5 to later in the year.)</i>	
5	3.7 Vectors (14 hours)		5		3.1 Motion of a projectile (14 hours)
6		1.6 Vectors (14 hours)	6		
7			7		
8	4.5 Energy, work and power (21 hours)		8	4.4 Newton's laws of motion (21 hours)	
9	<i>(Questions on this topic may also require use of 4.4 so beware of any of timing with Teacher 2. In Route 2 these are both taught by Teacher 2, which may be preferable.)</i>		9		
10			10	4.3 Momentum (7 hours)	

Route 1 Year 1: Weeks 23 – 30

	Teacher 1			Teacher 2	
	9709	9231		9709	9231
1	3.8 Differential Equations (21 hours)		1		3.6 Momentum (14 hours)
2			2		
3			3		
4		1.5 Polar coordinates (14 hours)	4		3.4 Hooke's law (14 hours)
5			5		
6	5.1 Representation of data (21 hours)		6	3.3 Circular motion (14 hours)	
7			7		
8			8	3.5 Linear motion under a variable force (14 hours)	

Route 1 Year 2: Weeks 1 – 12

	Teacher 1			Teacher 2	
	9709	9231		9709	9231
1	5.2 Permutations and combinations (7 hours)		1		2.2 Matrices (14 hours)
2	5.3 Probability (14 hours)		2		
3			3	3.9 Complex numbers (14 hours)	
4	5.4 Discrete random variables (14 hours)		4		
5	6.3 Continuous random variables (7 hours)		5		2.5 Complex numbers (21 hours)
6			6		
7		7	2.1 Hyperbolic functions (7 hours)		
8		8			
9	5.5 The normal distribution (14 hours)	9	2.3 Differentiation (14 hours)		
10		10			
11	6.1 The Poisson distribution (7 hours)	11	2.4 Integration (14 hours)		
12	6.2 Linear combinations of random variables (7 hours)	12			

Route 1 Year 2: Weeks 13 – 22

Teacher 1			Teacher 2	
9709		9231	9709	9231
1	6.4 Sampling and estimation (21 hours)		1	2.6 Differential Equations (21 hours)
2			2	
3			3	
4	6.5 Hypothesis tests (21 hours)		4	4.5 Probability generating functions (14 hours)*
5			5	
6			6	
7		4.2 Inference using normal and t -distributions (21 hours)	7	4.4 Non-parametric tests (21 hours)*
8			8	
9			9	
10			10	4.3 χ^2 -tests (14 hours)*

* Note that 4.3, 4.4 and 4.5 could be taught any order.

Co-teaching Route 2

Teacher 1 and Teacher 2 both cover pure content. Teacher 1 teaches statistics and Teacher 2 teaches mechanics. The course starts with Teacher 2 covering only pure content, whilst Teacher 1 covers probability & statistics content interspersed with some pure content. Later it changes so that Teacher 2 is mainly covering mechanics and Teacher 1 is now focussing on more complex pure content.

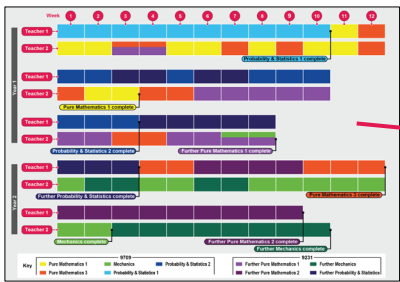
The number of hours to spend on each topic is given in brackets after each syllabus topic heading. Please be aware that this is just a very **general starting point** and the exact timing will depend on how much content you decide to cover within a lesson compared to homework; your learners' previous knowledge and ability; available resources; class size; and so on. In some weeks, Teacher 1 or 2 is teaching two different topics in the same week(s). Note that these could be taught one after the other so that learners are not coping with three different topics at the same time (and juggling notes and textbooks).

Learners will need clear guidance as to what is on the syllabus for each of 9709 and 9231, especially initially as past papers (old syllabus) will be misleading.

Through this co-teaching route, AS Level Further Mathematics candidates would take 9231 Papers 1 and 4 (Further Pure Mathematics 1 and Further Probability & Statistics). Therefore 3.8, 3.9 and the content exclusively for Paper 4 from the ASA & Level Mathematics 9709 course are not needed for the AS Level.

The route

The route is laid out on the pages that follow.

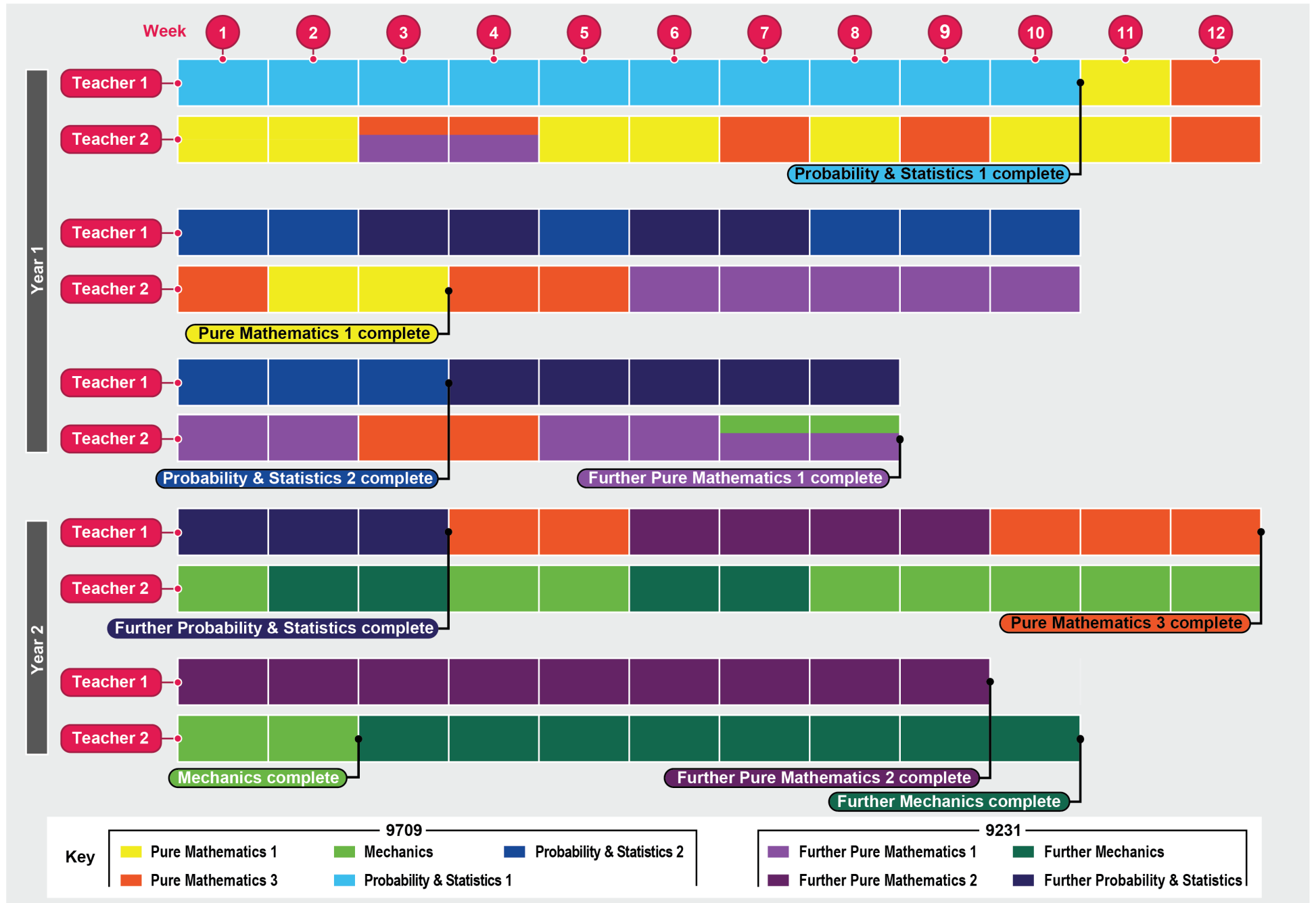


This diagram gives a summary of the spread of topics across the two-year period.

Route 2 Year 1: Weeks 1 – 12

Teacher 1		Teacher 2	
9709	9231	9709	9231
1		1.1 Quadratics (7 hours)	
2	5.1 Representation of data (21 hours)	1.3 Coordinate geometry (7 hours)	
3		3.6 Numerical solution of equations (4 hours)	1.1 Roots of polynomial equations (10 hours)
4	5.2 Permutations and combinations (7 hours)	(Note, you might prefer to teach 3.6 then 1.1 or vice versa, rather than running these two topics side by side.)	
5	5.3 Probability (14 hours)	1.4 Circular measure (7 hours)	
6		1.5 Trigonometry (7 hours)	
7	5.4 Discrete random variables (14 hours)	3.3 Trigonometry (7 hours)	
8		(Note that some learners might find this challenging this early on.)	
9	5.5 The normal distribution (14 hours)	1.6 Series (7 hours)	
10		3.2 Logarithmic and exponential functions (7 hours)	
11	1.2 Functions (7 hours)	1.7 Differentiation (14 hours)	
12	3.1 Algebra (7 hours)	3.4 Differentiation (7 hours continues in Term 2)	

The tables provide the detail of which topics are taught in which weeks and by which teacher.



Route 2 Year 1: Weeks 1 – 12

Teacher 1			Teacher 2		
9709		9231	9709		9231
1	5.1 Representation of data (21 hours)		1	1.1 Quadratics (7 hours)	
2			2	1.3 Coordinate geometry (7 hours)	
3			3	3.6 Numerical solution of equations (4 hours)	1.1 Roots of polynomial equations (10 hours)
4			4	<i>(Note, you might prefer to teach 3.6 then 1.1 or vice versa, rather than running these two topics side by side.)</i>	
5			5	1.4 Circular measure (7 hours)	
6			6	1.5 Trigonometry (7 hours)	
7			7	3.3 Trigonometry (7 hours) <i>(Note that some learners might find this challenging this early on.)</i>	
8			8	1.6 Series (7 hours)	
9			9	3.2 Logarithmic and exponential functions (7 hours)	
10			10	1.7 Differentiation (14 hours)	
11			11		
12			12	3.4 Differentiation (7 hours continues in Term 2)	
	5.2 Permutations and combinations (7 hours)				
	5.3 Probability (14 hours)				
	5.4 Discrete random variables (14 hours)				
	5.5 The normal distribution (14 hours)				
	1.2 Functions (7 hours)				
	3.1 Algebra (7 hours)				

Route 2 Year 1: Weeks 13 – 22

Teacher 1			Teacher 2		
9709		9231	9709		9231
1	6.1 The Poisson distribution (7 hours)		1	3.4 Differentiation (7 hours continued from Term 1)	
2	6.2 Linear combinations of random variables (7 hours)		2	1.8 Integration (14 hours)	
3		4.5 Probability generating functions (14 hours)	3		
4			4	3.5 Integration (14 hours)	
5	6.3 Continuous random variables (7 hours)		5		
6		4.1 Continuous random variables (14 hours)	6		1.5 Polar coordinates (14 hours)
7			7		
8					1.4 Matrices (14 hours)
9	6.4 Sampling and estimation (21 hours)				
10					1.2 Rational functions and graphs (7 hours)

Route 2 Year 1: Weeks 23 – 30

		Teacher 1			Teacher 2	
		9709	9231		9709	9231
1	6.5 Hypothesis tests (21 hours)			1		1.7 Proof by induction (14 hours)
2				2		
3				3		
4		4.2 Inference using normal and <i>t</i> -distributions (21 hours)		4	3.7 Vectors (14 hours)	
5				5		
6				6		
7		4.3 χ^2 -tests (14 hours)		7	4.1 Forces and equilibrium (4 hours, continues in Term 4)	1.3 Summation of series (10 hours)
8				8	(Note, you might prefer to teach 4.1 then 1.3 or vice versa, rather than running these two topics side by side.)	

Route 2 Year 2: Weeks 1 – 12

		Teacher 1		Teacher 2		
		9709	9231	9709	9231	
1		4.4 Non-parametric tests (21 hours)		1	4.1 Forces and equilibrium (7 hours, continued from Term 3)	
2				2		3.2 Equilibrium of a rigid body (14 hours)
3				3		
4	3.9 Complex numbers (14 hours)			4	4.2 Kinematics of motion in straight line (14 hours)	
5				5		
6		2.5 Complex numbers (21 hours)		6		3.1 Motion of a projectile (14 hours)
7				7		
8		2.1 Hyperbolic functions (7 hours)		8	4.4 Newton's laws of motion (21 hours)	
9	9					
10	3.8 Differential equations (21 hours)		10	4.5 Energy, work and power (14 hours, continues in Term 5)		
11			11			
12			12			

Route 2 Year 2: Weeks 13 – 22

Teacher 1			Teacher 2		
9709		9231	9709		9231
1		2.3 Differentiation (14 hours)	1	4.5 Energy, work and power (7 hours, continued from Term 4)	
2			2	4.3 Momentum (7 hours)	
3		2.4 Integration (14 hours)	3		3.6 Momentum (14 hours)
4			4		
5		2.6 Differential equations (21 hours)	5		3.4 Hooke's law (14 hours)
6			6		
7			7		3.3 Circular motion (14 hours)
8		2.2 Matrices (14 hours)	8		
9			9		3.5 Linear motion under a variable force (14 hours)
10			10		

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