Cambridge Assessment
International Education

Example Responses – Paper 3

Cambridge International AS & A Level Further Mathematics 9231

For examination from 2025

Cambridge Pathway

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Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge International AS & A Level Further Mathematics 9321.

This booklet contains responses to all questions from June 2025 Paper 34, which have been written by a Cambridge examiner. Responses are accompanied by a brief commentary highlighting common errors and misconceptions where they are relevant.

The question papers and mark schemes are available to download from the School Support Hub

9321 June 2025 Question Paper 34

9321 June 2025 Mark Scheme 34

Past exam resources and other teaching and learning resources are available from the School Support Hub

Question 1

1 A light spring of natural length a and modulus of elasticity $20mg$ is placed so that it stands vertically on a horizontal plane. The lower end of the spring is fixed to the plane. A particle of mass m is attached to the upper end of the spring.

The particle is pushed vertically downwards until the length of the spring is $\frac{3}{5}a$. The system is then released from rest.

Find the maximum extension of the spring in the subsequent motion. [5]

let x be the maximum extension in the string

initial EPE $= \frac{1}{2} \times \frac{20mg}{a} \times \left(a - \frac{3}{5}a\right)^2$ final EPE $= \frac{1}{2} \times \frac{20mg}{a} \times x^2$

taking initial position of particle as base for gravitational potential energy:

initial GPE $= mg \times \frac{3}{5}a$ final GPE $= mg\,(a + x)$

the particle has zero velocity both initially and when extension is maximum, so no kinetic energy to consider

using Conservation of Energy:

$$\frac{1}{2} \times \frac{20mg}{a} \times \left(a - \frac{3}{5}a\right)^2 + mg \times \frac{3}{5}a = \frac{1}{2} \times \frac{20mg}{a} \times x^2 + mg\left(a + x\right)$$

$$\frac{8}{5}a + \frac{3}{5}a = \frac{10}{a}x^2 + a + x$$

$$50x^2 + 5ax - 6a^2 = 0$$

$$\left(10x - 3a\right)\left(5x + 2a\right) = 0$$

this means: $x = \frac{3}{10}a$ or $x = -\frac{2}{5}a$

$x = -\frac{2}{5}a$ is rejected (corresponds to initial position)

the maximum extension is $\frac{3}{10}a$

Examiner comment

- Some candidates used an incorrect number of terms in the energy equation. Usually this was due to the omission of a final elastic potential energy term. Candidates needed to consider the dynamics of the situation carefully and ensure that they either included, or could justify not including, each of elastic potential energy, gravitational potential energy and kinetic energy terms at the beginning and at the end of the motion. In this example, the particle is moving from rest to rest, so there were no kinetic energy terms, but the remaining four energy terms should all have been present.
- Sometimes, an incorrect formula was used for the elastic potential energy. Candidates needed to learn the correct formulae. It can be helpful to write a general formula down before the substitution of particular values.

- An energy equation must be dimensionally correct for it to be awarded marks. Having written down an energy equation, candidates should check carefully that each term has the correct dimensions.
- It is often helpful to draw a simple diagram when reading through a question. This helps to understand what is happening and can help in determining which energy terms are required.

Question 2

2 A particle P of mass m kg moves along a horizontal straight line against a resistive force of magnitude $2mv^3$ N, where $v\,\mathrm{m\,s^{-1}}$ is the velocity of P at time t s. When $t = 0$, $v = 1$.

(a) Find an expression for v in terms of t. [4]

$$m\frac{dv}{dt} = -2mv^3$$

$$\int \frac{1}{v^3}\,dv = \int -2\,dt$$

$$-\frac{1}{2}v^{-2} = -2t + A$$

when $t = 0$, $x = 1$, so $A = -\frac{1}{2}$

$$-\frac{1}{2}v^{-2} = -2t - \frac{1}{2}$$

$$v^{-2} = 4t + 1$$

$$v^2 = \frac{1}{4t+1}$$

$$v = \frac{1}{\sqrt{4t+1}}$$

(b) Find the displacement of P from its initial position when $t = 6$. [3]

$$\frac{dx}{dt} = \frac{1}{\sqrt{4t+1}}$$

$$\int dx = \int (4t+1)^{-\frac{1}{2}}\,dt$$

$$x = \frac{1}{2}\sqrt{4t+1} + B$$

when $t = 0$, $x = 1$, so $B = -\frac{1}{2}$

$$x = \frac{1}{2}\sqrt{4t+1} - \frac{1}{2}$$

when $t = 6$:

$$x = \frac{1}{2}\sqrt{25} - \frac{1}{2}$$

$$= \frac{5}{2} - \frac{1}{2}$$

$$= 2\ (\text{metres})$$

Examiner comment

- The mass m needed to be seen in the expression for Newton's second law, even though it then cancels.
- In both parts, it was equally acceptable to use definite integration to apply the initial condition. Such an approach was used by a minority of candidates.

Question 3

3 A particle P is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at any subsequent time t are denoted by x and y respectively.

(a) Derive the equation of the trajectory of P in the form

$$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha.$$ [4]

use $s = ut + \frac{1}{2}at^2$ horizontally and vertically

horizontally: $x = (u\cos\alpha)t$

vertically: $y = (u\sin\alpha)t - \frac{1}{2}gt^2$

from first equation: $t = \dfrac{x}{u\cos\alpha}$

substitute for t in the second equation:

$$y = u\sin\alpha\left(\frac{x}{u\cos\alpha}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$$

$$= x\frac{\sin\alpha}{\cos\alpha} - \frac{gx^2}{2u^2}\left(\frac{1}{\cos^2\alpha}\right)$$

$$= x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha.$$

(b) Find the possible values of $\tan\alpha$. [3]

$$8 = 64\tan\alpha - \frac{128}{5}(1+\tan^2\alpha)$$

$$5 = 40\tan\alpha - 16 - 16\tan^2\alpha$$

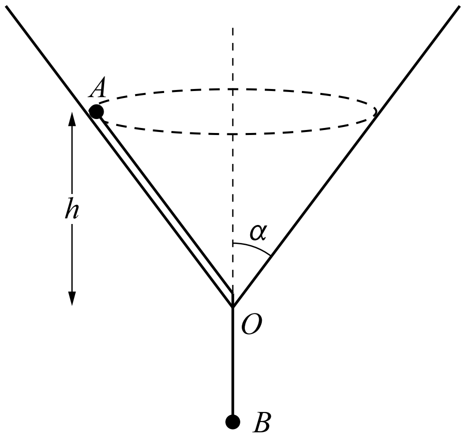
$$16\tan^2\alpha - 40\tan\alpha + 21 = 0$$

$$(4\tan\alpha - 3)(4\tan\alpha - 7) = 0$$

$$\tan\alpha = \frac{3}{4}, \frac{7}{4}$$

Question 4

4



A hollow cone with a smooth inner surface is fixed with its vertex O downwards. The semi-vertical angle of the cone is α, where $\tan\alpha = \frac{3}{4}$. A light inextensible string has a particle A of mass m attached to one end and a particle B of mass m attached to the other end. The string passes through a small hole in the cone at O. Particle B hangs in equilibrium below O. Particle A is on the inner surface of the cone at a height h above the level of O and moves in horizontal circles with constant angular speed ω (see diagram).

Find ω in terms of g and h. [6]

let T be the tension in the string and R be the normal reaction between particle A and the cone

resolving vertically for particle B which is in equilibrium:

$T = mg$

since $\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$.

resolving vertically for particle A:

$R\sin\alpha = T\cos\alpha + mg$

$R\sin\alpha = mg\cos\alpha + mg$

$$\frac{3}{5}R = mg\left(\frac{4}{5} + 1\right)$$

$$R = 3mg$$

resolving horizontally for particle A:

$R\cos\alpha + T\sin\alpha = mr\omega^2$

since $r = h\tan\alpha$:

$$3mg \times \frac{4}{5} + mg \times \frac{3}{5} = mh \times \frac{3}{4}\omega^2$$

$$\frac{15}{5}g = \frac{3}{4}h\,\omega^2$$

$$\omega^2 = \frac{4g}{h}$$

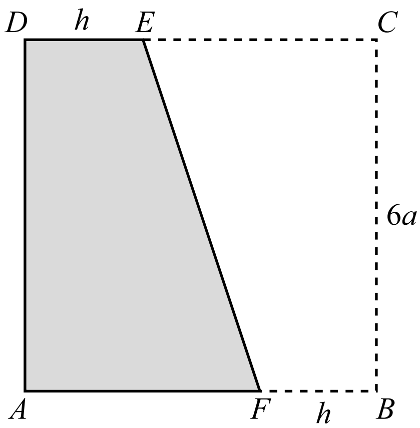
$$\omega = 2\sqrt{\frac{g}{h}}$$

Examiner comment

- A common error was to omit any normal reaction between particle B and the cone. Resolving horizontally for particle A then incorrectly simplifies to $T\sin\alpha = mr\omega^2$, from which an incorrect expression for ω in terms of g and h can be found without any need to resolve vertically for particle A. The mechanics of the situation is important: there must be a normal reaction for particle B to stay in contact with the surface of the cone.
- Candidates needed to carefully consider all the forces that were relevant for a physical situation to exist. It is helpful to mark all these forces on the given diagram as the beginning of a solution.
- Another common error in an otherwise correct solution was the omission of the equation for resolving vertically for particle B. In these scenarios, candidates often began by resolving vertically for particle A with T already substituted as mg. While this was correct, it was essential that candidates did not omit steps in their working, even when these steps were simple ones. It is a key feature of this scenario that particle B is in equilibrium, and this is why the tension in the string is equal to mg.

Question 5

5



ABCD is a uniform square lamina of side $6a$. Points *E* and *F* are on *DC* and *AB* respectively and are such that $DE = FB = h$. The quadrilateral *BCEF* is removed from the square lamina (see diagram).

(a) Show that the distance of the centre of mass of the resulting lamina *AFED* from *AD* is $\frac{h^2 - 6ah + 36a^2}{18a}$ and find a corresponding expression for the distance of the centre of mass from *AB*. [5]

	area	distance of COM from AD	distance of COM from AB
rectangle EDAG	$6ah$	$\frac{1}{2}h$	$3a$
triangle GFE	$18a^2 - 6ah$	$\frac{1}{3}h + 2a$	$2a$
lamina AFED	$18a^2$	$\bar{x}$	$\bar{y}$

take moments about AD:

$$18a^2\bar{x} = 6ah\left(\frac{1}{2}h\right) + \left(18a^2 - 6ah\right)\left(\frac{1}{3}h + 2a\right)$$

$$18a^2\bar{x} = 3ah^2 + 6a^2h - 2ah^2 + 36a^3 - 12a^2h$$

$$18a^2\bar{x} = 36a^3 - 6a^2h + ah^2$$

$$\bar{x} = \frac{a\left(36a^2 - 6ah + h^2\right)}{18a^2}$$

$$\bar{x} = \frac{h^2 - 6ah + 36a^2}{18a}$$

take moments about AB:

$$18a^2\bar{y} = 6ah(3a) + (18a^2 - 6ah)(2a)$$

$$18a^2\bar{y} = 18a^2h + 36a^3 - 12a^2h$$

$$18a^2\bar{y} = 36a^3 + 6a^2h$$

$$\bar{y} = 2a + \frac{1}{3}h$$

When the lamina *AFED* is suspended from the point *D*, the edge *DA* makes an angle θ with the downward vertical, where $\tan\theta = \frac{7}{15}$.

(b) Find, in terms of a, the two possible values of h. [3]

$$\tan\theta = \frac{\bar{x}}{6a - \bar{y}}$$

$$\frac{7}{15} = \frac{\dfrac{h^2 - 6ah + 36a^2}{18a}}{6a - 2a - \frac{1}{3}h}$$

$$7\left(6a - 2a - \frac{1}{3}h\right) = 15\left(\frac{h^2 - 6ah + 36a^2}{18a}\right)$$

$$42a\left(4a - \frac{1}{3}h\right) = 5\left(h^2 - 6ah + 36a^2\right)$$

$$5h^2 - 30ah + 180a^2 = 168a^2 - 14ah$$

$$5h^2 - 16ah + 12a^2 = 0$$

$$(h - 2a)(5h - 6a) = 0$$

$$h = \frac{6}{5}a,\ 2a$$

Examiner comment

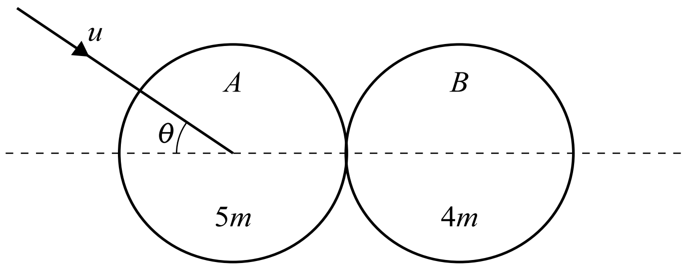
- There were many ways of approaching part **(a)**. This solution shows the simplest, with the division of the given shape into two simple shapes. It was also possible to divide the shape into two triangles. Many candidates chose to use a 'subtraction' method, by considering the whole square *ABCD* and subtracting a rectangle and a triangle, or two triangles, from it. All suitable methods were acceptable as solutions, though the more complicated the method, the more likely that algebraic errors would occur. Candidates are advised to choose the simplest division of the shape.
- Some of the divisions of the shape, particularly those involving the subtraction method, led to lengthy algebra and almost inevitably errors occurred. However, as always with a given answer, the correct result appeared as the final line of almost all solutions. Examiners always check solutions carefully, and candidates who have incorrect work on the penultimate line of their solution should go back and attempt to trace their error.
- Before writing down a moments equation in problems of this type, it is advisable to find expressions for all the areas and distances of the centres of mass (COM) of the component parts from the lines about which moments are being taken, as in the table above. Candidates who took such an approach were prone to making fewer errors.
- 'Since the lamina is uniform, mass is proportional to area, so we can consider areas instead of masses throughout.' This statement was not often seen in the solutions offered by candidates, although for completeness

it is necessary. It is good practice for candidates to be aware of the assumption they are making and to state it, although examiners are generous with its omission.

- The common error in part **(b)** was an incorrect expression for $\tan\theta$, usually $\tan\theta = \frac{\bar{y}}{\bar{x}}$. Candidates needed to study the geometry of the diagram to determine the correct expression, in this case, $\tan\theta = \frac{\bar{x}}{6a - \bar{y}}$, and not assume a more elementary result.

Question 6

6



Two uniform smooth spheres A and B of equal radii have masses $5m$ and $4m$ respectively. Sphere A is moving with speed u on a horizontal surface when it collides with sphere B which is at rest. Immediately before the collision, A's direction of motion makes an angle θ with the line of centres (see diagram). The coefficient of restitution between the spheres is e.

(a) Show that the speed of B after the collision is $\frac{5}{9}u(1+e)\cos\theta$. [3]

denote the speeds of A and B along line of centres after collision as v_1 and v_2 respectively

conservation of momentum along line of centres:

$$5mv_1 + 4mv_2 = 5mu\cos\theta$$

$$5v_1 + 4v_2 = 5u\cos\theta \qquad (1)$$

Newton's experimental law:

$$e = \frac{v_2 - v_1}{u\cos\theta}$$

$$v_2 - v_1 = eu\cos\theta$$

$$5v_2 - 5v_1 = 5eu\cos\theta \qquad (2)$$

adding equations (1) and (2):

$$9v_2 = 5u\cos\theta + 5eu\cos\theta$$

$$v_2 = \frac{5}{9}u\left(1+e\right)\cos\theta$$

After the collision the kinetic energy of A is equal to the kinetic energy of B.

(b) Given that $\tan\theta = \frac{2}{3}$, find the value of e. [6]

using $v_2 - v_1 = eu\cos\theta$ from **(a)**

$$v_1 = v_2 - eu\cos\theta$$

$$= \frac{5}{9}u\cos\theta + \frac{5}{9}ue\cos\theta - eu\cos\theta$$

$$= \frac{5}{9}u\cos\theta - \frac{4}{9}ue\cos\theta$$

$$= \frac{1}{9}u(5-4e)\cos\theta$$

speed of A perpendicular to line of centres is

$\tan\theta = \frac{2}{3}$, $\sin\theta = \frac{2}{\sqrt{13}}$ and $\cos\theta = \frac{3}{\sqrt{13}}$

equate kinetic energies after collision, then substitute for v_1, v_2, $\cos\theta$ and $\sin\theta$:

$$\frac{1}{2}(5m)\left(v_1^2 + (u\sin\theta)^2\right) = \frac{1}{2}(4m)v_2^2$$

$$\frac{5}{2}\left(\left(\frac{1}{9}u(5-4e)\cos\theta\right)^2 + (u\sin\theta)^2\right) = \frac{4}{2}\left(\frac{5}{9}u(1+e)\cos\theta\right)$$

$$5\left(\left(\frac{1}{9}u(5-4e)\frac{3}{\sqrt{13}}\right)^2 + \left(\frac{2}{\sqrt{13}}u\right)^2\right) = 4\left(\frac{5}{9}u(1+e)\frac{3}{\sqrt{13}}\right)^2$$

$$5\left(\frac{9}{13\times 81}u^2(5-4e)^2 + \frac{4}{13}u^2\right) = 4\times\frac{25\times 9}{13\times 81}u^2(1+e)^2$$

$$(5-4e)^2 + 36 = 20(1+e)^2$$

$$25 - 40e + 16e^2 + 36 = 20 + 40e + 20e^2$$

$$4e^2 + 80e - 41 = 0$$

$$(2e-1)(2e+41) = 0$$

$$e = \frac{1}{2}$$

note that $e = -\frac{41}{2}$ is rejected because e must be positive

Examiner comment

A common error in part **(b)** was to omit the perpendicular component of the velocity of sphere A when finding the kinetic energy of A after the collision.

Question 7

7 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle moves in complete vertical circles with centre O, with the string taut. When the string makes an angle θ with the downward vertical through O the speed of P is $\sqrt{4ag}$. The ratio of the greatest and least tensions in the string during the motion is 11 : 1.

Find the value of $\cos\theta$. [8]

let the speed at highest point H be v and the speed at lowest point L be u

energy change from H to L:

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = 2amg$$

$$v^2 = u^2 - 4ag$$

energy change from point at angle θ with the downward vertical through O to L:

$$\frac{1}{2}mu^2 - \frac{1}{2}m(4ag) = mg(a - a\cos\theta)$$

$$u^2 = ag\left(6 - 2\cos\theta\right)$$

Newton's second law at highest and lowest points:

$$T_L - mg = \frac{m}{a}u^2$$

$$T_H + mg = \frac{m}{a}v^2$$

it is given that $T_L = 11T_H$.

$$\frac{m}{a}u^2 + mg = 11\left(\frac{m}{a}v^2 - mg\right)$$

$$u^2 + ag = 11v^2 - 11ag$$

$$11v^2 = u^2 + 12ag$$

$$11\left(u^2 - 4ag\right) = u^2 + 12ag$$

$$10u^2 = 56ag$$

$$10ag\left(6 - 2\cos\theta\right) = 56ag$$

$$6 - 2\cos\theta = 5.6$$

$$\cos\theta = \frac{1}{5}$$

Examiner comment

- There were five unknowns in this problem: the tensions at the highest and lowest points, the velocities at the highest and lowest points, and the angle θ. Therefore, five equations were required. These came from two energy equations, applying Newton's second law at the highest and lowest points, and the given ratio between the tensions. These five equations could then be solved in various ways to find θ.

- Some common errors were in applying Newton's second law to find expressions for the tensions at the highest and lowest points. The least tension is at the highest point, and the greatest tension is at the lowest point. A common error was to assign the least tension to the lowest point and the greatest tension to the highest point. Other candidates believed that the least and greatest tensions must involve the angle θ.
- Other common errors were sign errors in the energy equations and algebraic errors in combining the various equations.

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